CBSE Class 12 - Mathematics
Sample Paper 08

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. If A= $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

   a. $A^2 = A$

   b. $A^2 = O$

   c. $A^3 = O$

   d. $A^2 = I$

2. If A is a non singular matrix of order 3, then |adj(adj A)|

a. $|A|^3$

b. $|A|^4$

c. None of these

d. $|A|^6$

3. If f (x) = $\tan^{-1}x$ and g (x) = $\tan^{-1}\left(\frac{x+1}{1-x}\right)$, then

a. f ‘ (x) = g ‘ (x)

b. f (x) = g(x)

c. None of these

d. $D_f = D_g$

4. Let f (x + y) = f(x) + f(y) $\forall$ x, y $\in \mathbf{R}$. Suppose that f (6) = 5 and f ‘ (0) = 1, then f ‘ (6) is equal to

a. 1

b. 30

c. None of these

d. 25

5. General solution of$cos^2x\frac{dy}{dx} + y = \ \tan x \ \left(0 \leqslant x < \frac{\pi}{2}\right)$ is

a. $y = (\tan x - 1) + Ce^{-\tan x}$

b. $y = (\tan x + 1) + Ce^{-\tan x}$

c. $y = (\tan x + 1) - Ce^{-\tan x}$

d. $y = (\tan x - 1) - Ce^{-\tan x}$

6. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ holds good for all

a. None of these

b. $0 \leqslant x \leqslant 1$

c. $|x| \leqslant \frac{1}{2}$

d. $|x| \leqslant 1$

7. Compute P(A|B), if P(B) = 0.5 and P (A ∩ B) = 0.32

a. P(A|B) = $\frac{16}{33}$

b. P(A|B) = $\frac{15}{27}$

c. P(A|B) = $\frac{16}{25}$

d. P(A|B) = $\frac{16}{29}$

8. An antiderivative of $\frac{x}{\cos^2 x}$ is equal to

a. $x \cos x + \log|\tan x| + C$

b. cot x + C

c. $x \tan x + \log|\cos x| + C$

d. x tanx + C

9. Two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are coplanar if

a. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(-\overrightarrow{b_1} \times \overrightarrow{-b_2}\right) = 0$

b. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$

c. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(-\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$

d. $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(\overrightarrow{b_1} \times -\overrightarrow{b_2}\right) = 0$

10. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k} \ and \ \vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

a. $8\sqrt{3}$

b. $19\sqrt{3}$

c. $19\sqrt{5}$

d. $17\sqrt{2}$

11. Fill in the blanks:

A relation R defined on a set A is said to be ________, if (x, x) $\in$ R, where x $\in$ A

12. Fill in the blanks:

In an experiment, an outcome having highest probability is called ________ outcome.

13. Fill in the blanks:

If A and B are symmetric matrices, then AB - BA is a ________ matrix.

14. Fill in the blanks:

The value of $\int \frac{dx}{1+\cos x}$.

OR

Fill in the blanks:

$\int f(x)dx$ =F(x)+c, these type of integrals are called ________ integrals.

15. Fill in the blanks:

The process of obtaining the optimal solution of the linear programming problem is called ________.

OR

Fill in the blanks:

Any point in the feasible region that gives the optimal value of the objective function is called an ________ solution.

16. Write the adjoint of the following matrix.
$\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

17. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes.

18. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx.$

OR

Evaluate $\int 2^x \, dx.$

19. Find the minimum value of the function
f(x) = 2|x - 2| +5|x - 3|, $\forall \ x \in R$

20. Find the sum of the vectors.

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}, \vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

Section B

21. Proof that whether f_2 = {(2, a), (3, b), (4, c)}; A = {2, 3, 4}, B = {a, b, c] function from A to B is one-one and onto?

22. Find the value of k so that the function f is continuous at the indicated point:
$f(x) = \begin{cases} 3x - 8, \ if \ x \leqslant 5 \\ 2k, \ if \ x > 5 \end{cases}$ at x = 5.

OR

Check the continuity of the following function at the indicated point.

$f(x) = \begin{cases} 3x + 5, \ if \ x \geqslant 2 \\ x^2, \ if \ x > 2 \end{cases}$ at x = 2.

23. Find the vector joining the points P (2, 3, 0) and Q (-1, -2, -4) directed from P to Q. Also find direction ratio and direction cosine.

24. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

OR

Prove that the function given by f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$

25. Find the angle between the line $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

26. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?

Section C

27. Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1} 2x; |2x| < \frac{1}{\sqrt{3}}$.

28. If y = $\sin^{-1}\left\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right\}$ and 0< x < 1, then find $\frac{dy}{dx}$.

OR

Check the continuity of the following function at the indicated point.

$$.f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2}, \; if \; x \neq 0 \\ 5, \; if \; x = 0 \end{cases}$$ at x = 0

29. An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

30. Two tailors A and B, earn Rs 300 and Rs 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

31. Solve the differential equation $(\tan^{-1}x - y)dx = (1 + x^2)dy$.

OR

Solve the differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0, x \neq 0$.

32. Evaluate $\int \frac{2}{(1-x)(1+x^2)}dx$.

Section D

33. Prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$ is divisible by (a + b + c) and find the quotient.

OR

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

34. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi^3 h \tan\alpha$.

35. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$

OR

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

36. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

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Solution
Section A

1. (c) $A^3 = O$

   Explanation:

   If any row or column of a square matrix is 0, then its product with itself is always a zero matrix.

2. (b) $|A|^4$

   Explanation:

   |adj.(adj.A) = $|A|^{n+1}$, where n is order of matrix. Here n = 3.

3. (a) f ' (x) = g ' (x)

   Explanation:

   $g(x)=\tan^{-1}\left(\frac{1+x}{1-x}\right) \Rightarrow g'(x)=\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\frac{(1-x).1-(1+x).(-1)}{(1-x)^2}=\frac{1}{(1+x^2)}$

4. (a) 1

   Explanation:

   $f'(6)=\lim\limits_{h\to 0}\frac{f(6+h)-f(6)}{h}$ = $\lim\limits_{h\to 0}\frac{f(6+h)-f(6+0)}{h}$

   $=\lim\limits_{h\to 0}\frac{f(6)+f(h)-\{f(6)+f(0)\}}{h}$

   $=\lim\limits_{h\to 0}\frac{f(h)-f(0)}{h}=f'(0)=1$

5. (a) $y=(\tan x-1)+Ce^{-\tan x}$

   Explanation:

   $\frac{dy}{dx}+\sec^2 x.y=\tan x.\sec^2 x \Rightarrow P=\sec^2 x, Q=\tan x.\sec^2 x$

$\Rightarrow I.F. = e^{\int \sec^2 x dx} = e^{\tan x}$

$\Rightarrow y.e^{\tan x} = \int \tan x \sec^2 x e^{\tan x} dx \Rightarrow y.e^{\tan x} = (\tan x - 1)e^{\tan x} + C$

$\Rightarrow y = (\tan x - 1) + Ce^{-\tan x}$

6. (c) $|x| \leqslant \frac{1}{2}$

   Explanation:

   Let $sin^{-1} = \theta \Rightarrow \sin \theta = x$

   Consider,

   RHS: $\sin^{-1}(3\sin\theta - 4\sin^3\theta) = \sin^{-1}(\sin 3\theta) = 3\theta$ if and only if $3\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

   $= 3sin^{-1}\theta$

   Here, it is given $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

   i.e, $\frac{-\pi}{2} \leqslant 3\theta \leqslant \frac{\pi}{2} \Rightarrow \frac{-\pi}{6} \leqslant \theta \leqslant \frac{\pi}{6}$

   $\Rightarrow \sin\left(\frac{-\pi}{6}\right) \leqslant \sin\theta \leqslant \sin\left(\frac{\pi}{6}\right)$

   $\Rightarrow \frac{-1}{2} \leqslant x \leqslant \frac{1}{2} \Rightarrow |x| \leqslant \frac{1}{2}$

7. (c) P(A|B) = $\frac{16}{25}$

   Explanation:

   We have , P(B) = 0.5 and P (A ∩ B) = 0.32

   $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{16}{25}$

8. (c) $x \tan x + \log|\cos x| + C$

   Explanation:

   $\int \frac{x}{\cos^2 x} dx$

   $= \int x \sec^2 x dx$ (Using By Part taking x as I function)

   $= x \tan x - \int \tan x dx$

$= x \tan x + \log|\cos x| + C$

9. (b) $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$

   Explanation:

   In vector form: Two lines $\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$ *and* $\vec{r} = \overrightarrow{a_2} + \mu \overrightarrow{b_2}$ are coplanar if $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) . \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$

10. (a) $8\sqrt{3}$

    Explanation:

    $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix} = 8\hat{i} - 8\hat{j} - 8\hat{k}$$

    $$\therefore\ |\vec{a} \times \vec{b}| = \sqrt{8^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$$

11. reflexive

12. most likely

13. skew symmetric

14. $\tan \frac{x}{2} + C$

    OR

    Indefinite

15. optimisation technique

    OR

    optimal

16. We have to find the adjoint of the following matrix.

    $$\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

Let A = $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, then adj (A)= $\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}$, where

C_{ij} denotes the cofactor of a_{ij} .

Therefore adj (A)= $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

17. According to the question, the normal to the plane is equally inclined with coordinates axes, and the distance of the plane from origin is $5\sqrt{3}$ units

$\therefore$ the direction cosines are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$

The required equation of plane is

$\frac{1}{\sqrt{3}} \cdot x + \frac{1}{\sqrt{3}} \cdot y + \frac{1}{\sqrt{3}} \cdot z = 5\sqrt{3}$

$\Rightarrow x + y + z = 5 \times 3$

$\Rightarrow \quad x + y + z = 15$

[$\because$ If l, m and n are direction cosines of normal to the plane and P is a distance of a plane from origin, then the equation of plane is given by $lx + my + nz = p$]

18. Let I = $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx - \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \sec^2 x dx - \int \cos ec^2 x dx$

= tan x + cot x + C

OR

According to the question , $I = \int 2^x \, dx$

$= \frac{2^x}{\log 2} + C \left[\because \int a^x dx = \frac{a^x}{\log a} + C\right]$

19. we have,

f(x) = 2|x - 2| + 5|x - 3|

= 2(2 - x) + 5(3 - x) = 19 - 7x, if x < 2

= 5, if x = 2

= 2(x - 2) + 5(3 - x) = 11 - 3x, if 2 < x < 3

= 2(3 - 2) = 2, if x = 3

= 2(x - 2) + 5(x - 3) = 7x - 19, if x > 3

Thus, we find that f(x) has a minimum value 2 at x = 3.

20. $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

$= 0\hat{i} - 4\hat{j} - \hat{k}$

Section B

21. F_2 = {(2, a), (3, b), (4, c)}

A = {2, 3, 4} B = {a, b, c}
It is clear that different elements of A have different images in B
$\therefore$ f_2 is one -one
Again, each element of B is the image of some element of A.
$\therefore$ f_2 in onto.

22. We have, $f(x) = \begin{cases} 3x - 8, \ if \ x \leqslant 5 \\ 2k, \ if \ x > 5 \end{cases}$

Since, f(x) is continuous at x = 5.

$\therefore LHL = RHL = f(5)$

Now, $LHL = \lim\limits_{x \to 5^-} (3x - 8) = \lim\limits_{h \to 0}[3(5 - h) - 8]$

$= \lim\limits_{h \to 0}[15 - 3h - 8] = 7$

$RHL = \lim\limits_{x \to 5^+} 2k = \lim\limits_{h \to 0} 2k = 2k$

Since LHL=LHL, therefore 2k=7,which gives

k=$\frac{7}{2}$

OR

We have, $f(x)=\begin{cases} 3x+5,\ if\ x\geqslant 2 \\ x^2,\ if\ x>2 \end{cases}$

At x = 2, $LHL=\lim\limits_{x\to 2^-}(x)^2$

$=\lim\limits_{h\to 0}(2-h)^2=\lim\limits_{h\to 0}\left(4+h^2-4h\right)=4$

And $RHL=\lim\limits_{x\to 2^+}(3x+5)$

$=\lim\limits_{h\to 0}[3\,(2+h)+5]=11$

Since, $LHL\neq RHL$ at x = 2

So, f(x) is discontinuous at x = 2.

23. $\overrightarrow{PQ}=(-1-2)\,\hat{i}+(-2-3)\,\hat{j}+(-4-0)\,\hat{k}$

$=-3\hat{i}-5\hat{j}-4\hat{k}$

DR are $-3,-5,-4$

$\left|\overrightarrow{PQ}\right|=\sqrt{9+25+16}=\sqrt{50}$

D.C are $\frac{-3}{\sqrt{50}},\frac{-5}{\sqrt{50}},\frac{-4}{\sqrt{50}}$

24. We have, rate of decrease of the volume of spherical ball of salt at any instant is proportional to surface area. Let the radius of the spherical ball of the salt be r.

$\therefore$ Volume of the ball (V) $=\frac{4}{3}\pi r^2$

and surface area (s) $=4\pi r^2$

$\because \frac{dV}{dT}\propto s$

$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)\propto 4\pi r^2$

$\Rightarrow \frac{d}{dt}\pi 3r^2\frac{dr}{dt}\propto 4\pi r^2$

$\Rightarrow \frac{dr}{dt} \propto \frac{4\pi r^2}{4\pi r^2}$

$\Rightarrow \frac{dr}{dt} = k.1$ [where, k is the proportionality constant]

$\Rightarrow \frac{dr}{dt} = k$

Hence, the radius of ball is decreasing at a constant rate.

OR

$f'(x) = \frac{1}{\sin x}.\cos x$
$f'(x) = \cot x$
$f'(x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$
and $f'(x) < 0 \forall x \in \left(\frac{\pi}{2}, \pi\right)$
Hence, f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$

25. $\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k}$

$\vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$

$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

$= \left|\frac{2(-1)+5(8)+(-3)(4)}{\sqrt{4+25+9}\sqrt{1+64+16}}\right|$

$= \frac{26}{9\sqrt{38}}$

$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

26. Consider the following events:

A = Getting a white ball in first draw, B = Getting a black ball in second draw.

Required probability

= Probability of getting a white ball in first draw and a black ball in second draw

= P (A and B)

= P(A ∩ B)

= P(A) P($\frac{B}{A}$) [By Multiplication Theorem] ....(i)

Now, P(A) = $\frac{^{10}C_1}{^{25}C_1}$ = $\frac{10}{25}$ = $\frac{2}{5}$

And, P($\frac{B}{A}$) = Probability of getting a black ball in second draw when a white ball has already been drawn in first draw

$\Rightarrow$ P($\frac{B}{A}$) = $\frac{^{15}C_1}{^{24}C_1}$ = $\frac{15}{24}$ = $\frac{5}{8}$ [$\because$ 24 ball are left after drawing a white ball in first-draw out of which 15 are black]

Substituting these values in (i), we obtain

Required probability = P(A $\cap$B) = P(A) P($\frac{B}{A}$) = $\frac{2}{5}$ $\times$ $\frac{5}{8}$ = $\frac{1}{4}$

Section C

27. We need to prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x; |2x|<\frac{1}{\sqrt{3}}$

We consider, $\text{LHS}=\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)$

$$=\tan^{-1}\left[\frac{\left(\frac{6x-8x^3}{1-12x^2}\right)-\left(\frac{4x}{1-4x^2}\right)}{1+\left(\frac{6x-8x^3}{1-12x^2}\right)\left(\frac{4x}{1-4x^2}\right)}\right]$$

$$\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right) : xy>-1\right]$$

$$=\tan^{-1}\left[\frac{\frac{\left(6x-8x^3\right)\left(1-4x^2\right)-4x\left(1-12x^2\right)}{\left(1-12x^2\right)\left(1-4x^2\right)}}{\frac{\left(1-12x^2\right)\left(1-4x^2\right)+\left(6x-8x^3\right)(4x)}{\left(1-12x^2\right)\left(1-4x^2\right)}}\right]$$

$$=\tan^{-1}\left(\frac{6x-24x^3-8x^3+32x^5-4x+48x^3}{1-4x^2-12x^2+48x^4+24x^2-32x^4}\right)$$

$$=\tan^{-1}\left(\frac{2x+16x^3+32x^5}{16x^4+8x^2+1}\right)$$

$$=\tan^{-1}\left[\frac{2x\left(16x^4+8x^2+1\right)}{\left(16x^4+8x^2+1\right)}\right]=\tan^{-1}2x=\text{RHS}$$

Hence proved.

28. According to the question, if, $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$.

We have to find the derivative of y w.r.t x. We shall make use of the substitution to find the derivative.

Now,y can be rewritten as $y=\sin^{-1}\left[x\sqrt{1-(\sqrt{x})^2}-\sqrt{x}\sqrt{1-x^2}\right]$

Now, put $\sqrt{x}=\sin\theta$ and x =$\sin\phi$, so that

y = $\sin^{-1}\left[\sin\phi\sqrt{1-\sin^2\theta}-\sin\theta\sqrt{1-\sin^2\phi}\right]$

$\Rightarrow \quad y=\sin^{-1}[\sin\phi\cos\theta-\sin\theta\cos\phi]\left[\because \sqrt{1-\sin^2 x}=\cos x\right]$

$\Rightarrow \quad y=\sin^{-1}\sin(\phi-\theta)\,[\because \sin\phi\cos\theta-\cos\phi\sin\theta=\sin(\phi-\theta)]$

$\Rightarrow \quad y=\phi-\theta\left[\because \sin^{-1}\sin\theta=\theta\right]$

$\Rightarrow \quad y=\sin^{-1}x-\sin^{-1}\sqrt{x}\;\left[\because \phi=\sin^{-1}x \text{ and } \theta=\sin^{-1}\sqrt{x}\right]$

On differentiating both sides w.r.t x, we get

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-(\sqrt{x})^2}}\times\frac{d}{dx}(\sqrt{x}\,)\left[\because \frac{d}{d\theta}\left(\sin^{-1}\theta\right)=\frac{1}{\sqrt{1-\theta^2}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$$

Hence, $\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2\sqrt{x-x^2}}$

OR

We have $f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2},\ if\ x\neq 0\\ 5,\ if\ x=0\end{cases}$

At x = 0 $LHL=\lim\limits_{x\to 0^-}\frac{1-\cos 2x}{x^2}$

$=\lim\limits_{h\to 0}\frac{1-\cos 2(0-h)}{(0-h)^2}$

$=\lim\limits_{h\to 0}\frac{1-\cos 2h}{h^2}\;[\because \cos(-\theta)=\cos\theta]$

$=\lim\limits_{h\to 0}\frac{1-1+2\sin^2 h}{h^2}\;[\because \cos 2\theta=1-2\sin^2\theta]$

$=\lim\limits_{h\to 0}\frac{2(\sin h)^2}{(h)^2}=2\;\left[\because \lim\limits_{h\to 0}\frac{\sinh}{h}=1\right]$

$\text{RHL}=\lim\limits_{x\to 0^+}\frac{1-\cos 2x}{x^2}$

$=\lim\limits_{h\to 0}\frac{1-\cos 2(0+h)}{(0+h)^2}$

$=\lim\limits_{h\to 0}\frac{1-cos2h}{h^2}$

$=\lim\limits_{x\to 0}\frac{2\sin^2 h}{h^2}=2\;\left[\because \lim\limits_{x\to 0}\frac{\sinh}{h}=1\right]$

And f(0) = 5

Since, $LHL = RHL \neq f(0)$

Hence, f(x) is not continuous at x = 0

29. Let X be a random variable that denotes the number of heads in a toss of coin n times.
Clearly, X follows binomial distribution with parameter n = 4, $p = \frac{1}{2}$ and $q = \frac{1}{2}$
$\therefore P(X = r) = ^4C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{4-r} = ^4C_r\left(\frac{1}{2}\right)^4$
For X = 0, 1, 2, 3, 4 we have
$P(X=0) = {}^4C_0\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4$
$P(X=1) = {}^4C_1\left(\frac{1}{2}\right)^4 = 4\left(\frac{1}{2}\right)^4$
$P(X=2) = {}^4C_2\left(\frac{1}{2}\right)^4 = 6\left(\frac{1}{2}\right)^4$
$P(X=3) = {}^4C_3\left(\frac{1}{2}\right)^4 = 4\left(\frac{1}{2}\right)^4$
and $P(X=4) = {}^4C_4\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4$
Therefore, the probability distribution of X is given by

X	0	1	2	3	4
P(X)	$\left(\frac{1}{2}\right)^4$	$4\left(\frac{1}{2}\right)^4$	$6\left(\frac{1}{2}\right)^4$	$4\left(\frac{1}{2}\right)^4$	$\left(\frac{1}{2}\right)^4$

Now, mean $E(X) = \sum X \cdot P(X)$
$= 0\left(\frac{1}{2}\right)^4 + 1 \cdot 4\left(\frac{1}{2}\right)^4 + 2 \cdot 6\left(\frac{1}{2}\right)^4 + 3 \cdot 4\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^4$
$= \left(\frac{1}{2}\right)^4(4+12+12+4) = \frac{32}{2^4} = 2$
Var (X) = E(X^2) - [E(X)]^2
Now, $E\left(X^2\right) = \sum X^2 P(X)$
$= 0 + (1)^2 \cdot 4 \cdot \left(\frac{1}{2}\right)^4 + (2)^2 \cdot 6 \cdot \left(\frac{1}{2}\right)^4 + (3)^2 \cdot 4 \cdot \left(\frac{1}{2}\right)^4 + (4)^2\left(\frac{1}{2}\right)^4$

$= \left(\frac{1}{2}\right)^4[4+24+36+16] = \frac{80}{16} = 5$
Therefore, from Eq. (i), we have,

Var (X) = 5 - $(2)^2$ = 1[$\because$ E(X = 2]

30. According to the given situation , the given data can be tabularised as following

	Tailor A	Tailor B	Minimum Total No.
No. of Shirts	6	10	60
No. of Trousers	4	4	32
Wage	Rs 300/day	Rs 400/day	

Let tailor A and tailor B work for x days and y days, respectively.Given that the minimum number of shirts that can be stitched per day is 60. The inequality representing the information is given as

$\therefore$ 6x + 10y $\geq$ 60 $\Rightarrow$(shirt constraint) (dividing by 2 we get)

3x + 5y $\geq$ 30

Given that the minimum number of trousers that can be stitched per day is 32.

$\therefore$ 4x + 4y $\geq$ 32 $\Rightarrow$ (trouser constraint) (dividing throughout by 4 we get

x + y = 8

$\therefore$ x $\geq$ 0, y $\geq$ 0 (non negative constarint which restricts the feasible region in the first quadrant only , since it is real world situation and the variables cannot take negative values.

Let z be the objective function representing the total labour cost. Hence the equation for the cost function is given as z = 300x + 400y

So, the given L P.P. is designed as

z = 300x + 400y

x $\geq$ 0, y $\geq$ 0, 3x + 5y $\geq$ 30 and x + y $\geq$ 8

31. Given differential equation, $(\tan^{-1} x - y)dx = (1 + x^2)\, dy$

The above differential equation can be rewritten as,

$\Rightarrow \frac{dy}{dx} = \frac{\tan^{-1} x - y}{(1+x^2)} \Rightarrow \frac{dy}{dx} = \frac{\tan^{-1} x}{1+x^2} - \frac{1}{1+x^2} y$

$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{\tan^{-1} x}{1+x^2}$ ...(i)

which is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, here $P = \frac{1}{1+x^2}$ and $Q = \frac{\tan^{-1} x}{1+x^2}$

Now, IF $= e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x}$

$\therefore$ The general solution is given by

$y \cdot \mathrm{IF} = \int Q \cdot \mathrm{IF} dx + C$

$\Rightarrow \quad y \cdot e^{\tan^{-1} x} = \int \frac{\tan^{-1} x}{1+x^2} \cdot e^{\tan^{-1} x} dx + C$

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$\therefore \quad y e^{\tan^{-1} x} = \int t \cdot e^t dt + C = t \cdot e^t - \int 1 \cdot e^t dt + C$ [using integration by parts]

$\Rightarrow \quad y e^{\tan^{-1} x} = t \cdot e^t - e^t + C$

$\Rightarrow \quad y e^{\tan^{-1}} x = \tan^{-1} x \cdot e^{\tan^{-1} x} - e^{\tan^{-1} x} + C \left[\because t = e^{tan^{-1} x}\right]$

$\Rightarrow y e^{\tan^{-1} x} = \left(\tan^{-1} x - 1\right) e^{\tan^{-1} x} + C$

OR

According to the question ,

Given differential equation is ,

$x \frac{dy}{dx} + y - x + xy \cot x = 0$

Above equation can be written as

$x \frac{dy}{dx} + y(1 + x \cot x) = x$

On dividing both sides with x, we get

$\frac{dy}{dx} + y \left(\frac{1+x \cot x}{x}\right) = 1$

$\Rightarrow \quad \frac{dy}{dx} + y \left(\frac{1}{x} + \cot x\right) = 1$

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$,

where $P = \frac{1}{x} + \cot x$ and $Q = 1$.

we know that ,

$\mathrm{IF} = e^{\int P dx} = e^{\int \left(\frac{1}{x} + \cot x\right) dx} = e^{\log |x| + \log \sin x}$

$\left[\because \int \frac{1}{x} dx = \log |x| \text{ and } \int \cot x dx = \log |\sin x|\right]$

$= e^{\log |x \sin x|} [\because \log m + \log n = \log mn]$

$\Rightarrow$ IF = x sin x

$y \times IF = \int (Q \times IF) dx + C$

$\therefore \quad y \times x \sin x = \int 1 \times x \sin x dx + C$

$\Rightarrow \quad yx \sin x = \int \underset{I}{x} \underset{II}{\sin} x dx + C$

$\Rightarrow \quad y \cdot x \sin x = x \int \sin x dx - \int \left(\frac{d}{dx}(x) \int \sin x dx \right) dx + C$ [using integration by parts]

$\Rightarrow yx \sin x = -x \cos x - \int 1(-\cos x) dx + C$

$\Rightarrow yx \sin x = -x \cos x + \int \cos x dx + C$

$\Rightarrow yx \sin x = -x \cos x + \sin x + C$

On dividing both sides by x sin x, we get

$y = \frac{-x \cos x + \sin x + C}{x \sin x}$

$\therefore \quad y = -\cot x + \frac{1}{x} + \frac{C}{x \sin x}$

which is the required solution.

32. Given, $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Using partial fraction,

$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ ...(i)

$\Rightarrow \frac{2}{(1-x)(1+x^2)} = \frac{A(1+x^2)+(Bx+C)(1-x)}{(1-x)(1+x^2)}$

$\Rightarrow 2 = A(1+x^2) + (Bx+C)(1-x)$

$\Rightarrow 2 = A + Ax^2 + Bx + C - Bx^2 - Cx$

$\Rightarrow 2 = (A-B)x^2 + (B-C)x + (A+C)$

Comparing coefficients of x^2, x and constant terms from both sides,

$\Rightarrow A - B = 0$ ...(ii)

$B - C = 0$ ...(iii)

and A + C=2 ...(iv)

Solving Equations (ii), (iii) and (iv),

A = 1, B = 1 and C = 1

Now, Eq. (i) becomes

$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$

Integrating both sides w.r.t.x,

$\Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x+1}{1+x^2} dx$

$= -\log|1-x| + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$= -\log|1-x| + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$= -\log|1-x| + \frac{1}{2} \log\left|1+x^2\right| + \tan^{-1} x + C$

$\left[\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C\right]$

Section D

33. Let $\Delta = \begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$

$Applying$ we get,
$C_1 \to C_1 - C_2 \; and \; C_2 \to C_2 - C_3,$

$$\Delta = \begin{bmatrix} bc-a^2-ca+b^2 & ca-b^2-ab+c^2 & ab-c^2 \\ ca-b^2-ab+c^2 & ab-c^2-bc+a^2 & bc-a^2 \\ ab-c^2-bc+a^2 & bc-a^2-ca+b^2 & ca-b^2 \end{bmatrix}$$

$= \begin{bmatrix} (b-a)(a+b+c) & (c-b)(a+b+c) & ab-c^2 \\ (c-b)(a+b+c) & (a-c)(a+b+c) & bc-a^2 \\ (a-c)(a+b+c) & (b-a)(a+b+c) & ca-b^2 \end{bmatrix}$ Taking (a + b + c) common from C_1 and C_2 each,we get,

$$\Delta = (a+b+c)^2 \begin{vmatrix} b-a & c-b & ab-c^2 \\ c-b & a-c & bc-a^2 \\ a-c & b-a & ca-b^2 \end{vmatrix}$$

$Applying \; R_1 \to R_1 + R_2 + R_3, we \; get,$

$$\Delta = (a+b+c)^2 \begin{vmatrix} 0 & 0 & ab+bc+ca-\left(a^2+b^2+c^2\right) \\ c-b & a-c & bc-a^2 \\ a-c & b-a & ca-b^2 \end{vmatrix}$$

Now, expanding along R_1,

$= (a + b + c)^2 [(ab + bc + ca - (a^2 + b^2 + c^2))((c - b)(b - a) - (a - c)^2)]$

$= (a + b + c)^2 [(ab + bc + ca - a^2 - b^2 - c^2)(cb - ac - b^2 + ab - a^2 - c^2 + 2ac)]$

$= (a + b + c)^2 (a^2 + b^2 + c^2 - ab - bc - ca)$

$(a^2 + b^2 + c^2 - ac - ab - bc)$

$= \frac{1}{2}$ (a + b + c) [(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]

[(a - b^2) + (b - c)^2 + (c - a)^2]

$= \frac{1}{2}$ (a + b + c) (a^3 + b^3 + c^3 - 3abc) [(a - b)^2 + (b - c)^2 + (c - a)^2]

Hence, given determinant is divisible by (a + b + c) and quotient is

(a^3 + b^3 + c^3 - 3abc)[(a - b^2) + (b - c)^2 + (c - a)^2]

OR

L.H.S. = A^3 - 6A^2 + 7A + 2I

$$= \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$-6\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+2\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$= \begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$$

$$-6\begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$= \begin{bmatrix}5+0+16&0+0+0&10+0+24\\2+0+10&0+8+0&4+4+15\\8+0+26&0+0+0&16+0+39\end{bmatrix}$$

$$-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}7+2&0+0&14+0\\0+0&14+2&7+0\\14+0&0+0&21+2\end{bmatrix}$$

$$= \begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix}$$

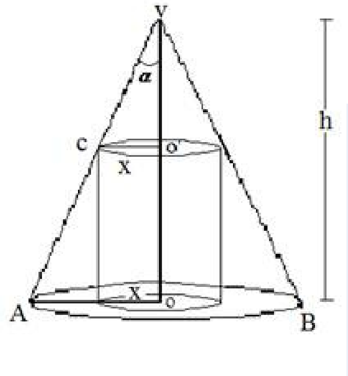
$$= \begin{bmatrix}21-30&0-0&34-48\\12-12&8-24&23-30\\34-48&0-0&55-78\end{bmatrix}+\begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} -9+9 & 0+0 & -14+14 \\ 0+0 & -16+16 & -7+7 \\ -14+14 & 0+0 & -23+23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$ (Zero matrix)

= R.H.S. Proved.

34. 

$\frac{vo'}{x} = \cot\alpha$

$vo' = x\cot\alpha$

$oo' = h - x\cot\alpha$

$V = \pi x^2.(h - x\cot\alpha)$

$V = \pi x^2 h - \pi x^3 \cot\alpha$

$\frac{dV}{dx} = 2\pi x h - 3\pi x^2 \cot\alpha$

for maximum/minimum

$\frac{dV}{dx} = 0$

$2\pi x h - 3\pi x^2 \cot\alpha = 0$

$x = \frac{2h}{3}\tan\alpha$

$\frac{d^2h}{dx^2} = 2\pi h - 6\pi x\cot\alpha$

$\left.\frac{d^2h}{dx^2}\right]_{x=\frac{2h}{3}\tan\alpha} = \pi(2h - 4h)$ = -2πh<0

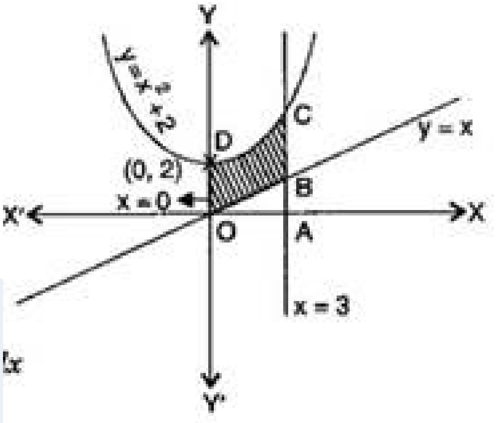
Thus, volume is maximum.

Volume is maximum at $x = \frac{2h}{3}\tan\alpha$

Maximum volume is

$V=\pi . x^{2}(h-x \cot \alpha)$
$=\pi\left(\frac{2 h}{3} \tan \alpha\right)^{2}\left[h-\frac{2 h}{3} \tan \alpha \cot \alpha\right]$
$=\pi \frac{4 h^{2}}{9} \tan ^{2} \alpha . \frac{h}{3}$
$V=\frac{4}{27} \pi h^{3} \tan ^{2} \alpha$
Hence the required result

35. Equation of the given curve is



$y=x^{2}+2$ ...(i)
$\Rightarrow x^{2}=y-2$
Here Vertex of the parabola is (0, 2).
Equation of the given line is y = x ...(ii)
Table of values for the line y = x

x	0	1	2
y	0	1	2

We know that it is a straight line passing through the origin and having slope 1 i.e., making an angle of 45^{0} with x-axis.
Here also, Limits of integration area given to be x = 0 to x = 3
$\therefore$ Area bounded by parabola (i) namely y = x^2 + 2 the x - axis and the ordinates x = 0 to x = 3 is the area OACD and $\int_{0}^{3} y d x=\int_{0}^{3}\left(x^{2}+2\right) d x$
$=\left(\frac{x^{3}}{3}+2 x\right)_{0}^{3}=(9+6)-0=15$...(iii)
Again Area bounded by parabola (ii) namely y = x the x - axis and the ordinates x =

0 to x = 3 is the area OAB and $\int_0^3 y dx = \int_0^3 x dx$

$= \left(\frac{x^2}{2}\right)_0^3 = \frac{9}{2} - 0 = \frac{9}{2}$ ...(iii)

$\therefore$ Required area = Area OBCD = Area OACD – Area OAB

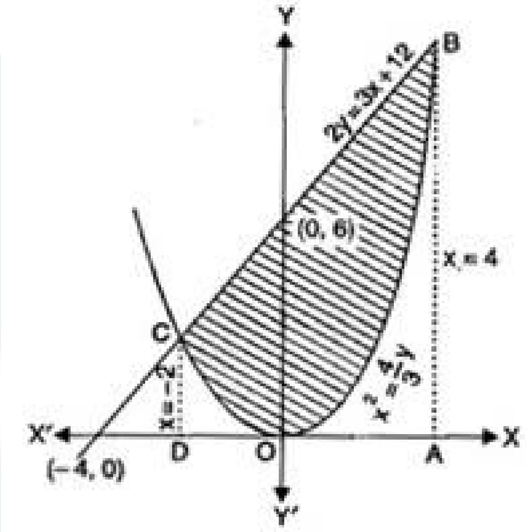
= Area given by eq. (iii) – Area given by eq. (iv)

$= 15 - \frac{9}{2} = \frac{21}{2}$ sq. units

OR

Equation of the parabola is

4y = $3x^2$ ...(i)



$\Rightarrow x^2 = \frac{4}{3}y$

Equation of the line is 2y = 3x + 12 ...(ii)

$\Rightarrow y = \frac{3x+12}{2} = \frac{3x}{2} + 6$

In the graph, points of intersection are B (4, 12) and C (-2, 3).

Now, Area ABCD $= \left| \int_{-2}^{4} \left(\frac{3}{2}x + 6\right) dx \right|$

$= \left[\frac{3}{4}x^2 + 6x\right]_{-2}^{4}$

= (12 + 24) - (3 - 12)

= 45 sq units

Again, Area CDO + Area OAB $= \left| \int_{-2}^{4} \left(\frac{3}{4}x^2\right) dx \right|$

$= \left[\frac{3}{4} \cdot \frac{x^3}{3}\right]_{-2}^{4}$
$= \frac{1}{4}[64-(-8)] = 18$ sq. units
∴ Required area = Area ABCD - (Area CDO + Area OAB)
= 45 - 18 = 27 sq. units

36. Let P(2 3, 4) be the given point and given equation of line be
$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$
Let $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2} = \lambda(\text{ say })$
$x = 3\lambda - 3, y = 6\lambda + 2, z = 2\lambda$
Coordinates of any point T on given line are $(3\lambda - 3, 6\lambda + 2, 2\lambda)$.
Now, DR's of line PT
$= (3\lambda - 3 - 2, 6\lambda + 2 - 3, 2\lambda - 4)$
$= (3\lambda - 5, 6\lambda - 1, 2\lambda - 4)$
since, the line PT is parallel to the plane
3x + 2y + 2 - 5 = 0,then,
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$\left[\begin{array}{l}\text{line is parallel to the plane, therefore normal} \\ \text{to the plane is perpendicular to the line.} \\ \text{where } a_1 = 3\lambda - 5, b_1 = 6\lambda - 1, c_1 = 2\lambda - 4 \\ \text{and } a_2 = 3, b_2 = 2, c_2 = 2\end{array}\right]$
$\Rightarrow (3\lambda - 5)3 + (6\lambda - 1)2 + (2\lambda - 4)2 = 0$
$\Rightarrow \quad 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$
$\Rightarrow \quad 25\lambda - 25 = 0$
$\Rightarrow \quad 25\lambda = 25 \Rightarrow \lambda = 1$
Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda)$
$= (0, 8, 2)$ [∵ λ= 1]
Now, the required distance between points
P(2 3, 4) and T(0, 8, 2) is given by
$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$
$[\because (x_1, y_1, z_1) = (2, 3, 4) \text{ and } (x_2, y_2, z_2) = (0, 8, 2)]$
$= \sqrt{4 + 25 + 4} = \sqrt{33}\,.units$