CBSE Class 12 - Mathematics
Sample Paper 07

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. If A = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   a. none of these

   b. $A^2 = O$

   c. $A^2 = A$

   d. $A^3 = A$

2. If A and B are square matrices of order 3, such that Det.A = –1, Det.B =3 then, the determinant of 3AB is equal to

a. –27

b. -81

c. –9

d. 81

3. $\underset{x \to 0}{Lt} \frac{x(e^{\sin x}-1)}{1-\cos x}$ is equal to

a. 2

b. 1

c. $\frac{1}{2}$

d. 0

4. The conditional probability of the event E', given that F has occurred is given by

a. P (E'|F) = 1

b. P (E'|F) = 1 – P (E|F)

c. P (E'|F) = P (E|F)

d. P (E'|F) = – 1 + P (E|F)

5. Let X be a random variable assuming values x1, x2,....,xn with probabilities p1, p2, ...,pn, respectively such that pi ≥ 0, $\sum_{i=1}^{n} p_i = 1$. If E is the expectation, mean of X is denoted by μ, variance denoted by σ2, is defined as

a. σ2=E(X + μ)2

b. σ2=E(X – μ)2

c. σ2=E(X – μ)3

d. σ2=E(X – μ)

6. The optimal value of the objective function Z = ax + by may or may not exist, if the feasible region for a LPP is

   a. Unbounded

   b. A circle

   c. Bounded

   d. A polygon

7. cos 2θ is not equal to

   a. $1-2\sin^2\theta$

   b. $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

   c. $2\cos^2\theta-1$

   d. $\frac{1+\tan^2\theta}{1-\tan^2\theta}$

8. If f (x) be a function such that $\frac{d}{dx}(f(x))=\log x$,then f (x) is equal to

   a. x log (x e) + C

   b. $x\log\left(\frac{x}{e}\right)$ + C

   c. $x\log\left(\frac{e}{x}\right)$ + C

   d. $\frac{\log x}{x}$ + C

9. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them. 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0

   a. The planes are at 45°

   b. The planes are parallel

c. The planes are at 55°

d. The planes are perpendicular

10. If l, m and n are direction cosines of the position vector OP the coordinates of P are

a. lr, mr and n

b. lr, m and nr

c. lr, mr and nr

d. l, mr and nr

11. Fill in the blanks:

A relation R defined on a set A is said to be ________, if (x, y) $\in$ R and (y, z) $\in$ R $\Rightarrow$ (x, z) $\in$ R, where x, y, z $\in$ A

12. Fill in the blanks:

The value of c in Mean value theorem for the function f(x) = x(x - 2), x $\in$ [1, 2] is ________.

13. Fill in the blanks:

In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeros in one or more, then A^{-1} ________.

14. Fill in the blanks:

The vector equation of a line that passes through two points whose positions vectors are $\vec{a}$ and $\vec{b}$ is ________.

OR

Fill in the blanks:

Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is ________.

15. Fill in the blanks:

If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} . \vec{b}\right|^2$ = 144 and $\left|\vec{a}\right|$ = 4, then $\left|\vec{b}\right|$ is equal to _______.

OR

Fill in the blanks:

If $\vec{r}.\vec{a} = 0, \vec{r}.\vec{b} = 0$, and $\vec{r}.\vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}(\vec{b} \times \vec{c})$ is _______.

16. If, $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$ write the value of |AB|.

17. Evaluate $\int_2^3 \frac{1}{x} dx$.

OR

Evaluate, $\int \sec^2(7 - 4x)dx$.

18. Find the value of $\int \frac{dx}{x^2+16}$.

19. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

20. Find the differential equation of the family of curves $y = Ae^{2x} + B.e^{-2x}$.

Section B

21. Write the following function in the simplest form: $\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}$, $-a < x < a$.

OR

Let S = {0, 1, 2, 3, 4} and * be an operation on S defined by a * b = r, where r is the least non-negative remainder when a + b is divided by 5. Prove that * is a binary operation on S.

22. A stone is dropped into a quite lake and waves move in circles at the rate of 5 cm/sec. At the instant when radius of the circular wave is 8 cm, how fast is the enclosed area

increasing?

23. If $x = e^{\frac{x}{y}}$, prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$

24. Consider two point P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the positions vector of a point R which divides the line joining P and Q in the ratio 2:1

i. internally
ii. externally.

OR

Show that $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right), \vec{b} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right), \vec{c} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$ are mutually $\perp$ unit vectors.

25. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.

26. Bag I contains 3 black and 2 white balls, Bag II contains 2 black and 4 white balls. A bag and a ball is selected at random. Determine the probability of selecting a black ball.

Section C

27. Let Z be the set of integers. Show that the relation R = {(a, b): a, b $\in$ Z and a + b is even} is an equivalence relation on Z.

28. Find $\frac{dy}{dx}$, if $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

OR

If y = a sin x + b cos x, then prove that $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$.

29. Solve, x cos $\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

30. Evaluate $\int \frac{x+1}{x(1+xe^x)^2}dx$

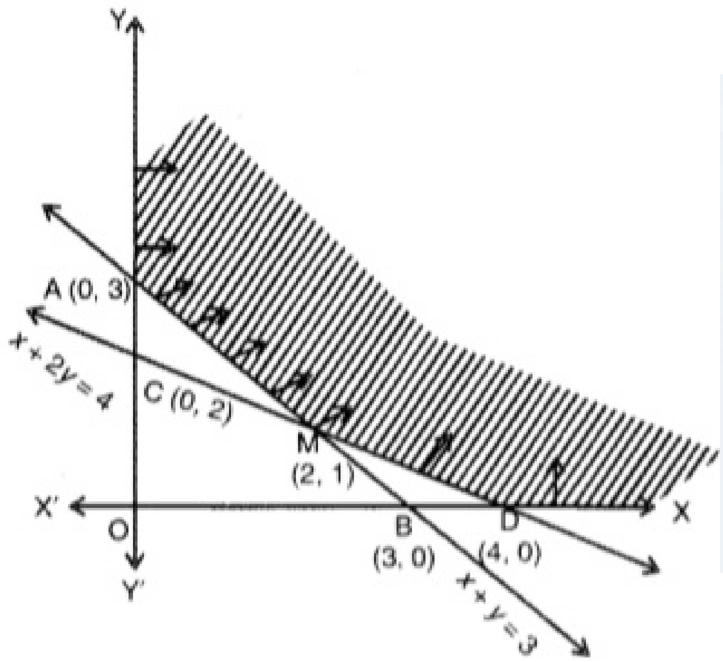
31. In a meeting 70% of the members favour and 30% oppose a certain proposal. A

member is selected at random and we take x = 0 if he opposed and x = 1 if he is in favour. Find E (x) and var (x).

OR

For a loaded die, the probabilities of outcomes are given as under: P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3. The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more', respectively. Determine whether or not A and B are independent.

32. The feasible region for an LPP is shown in fig. Evaluate Z = 4x + y at each of the corner points of this region. Find the minimum value of Z, if it exists.



Section D

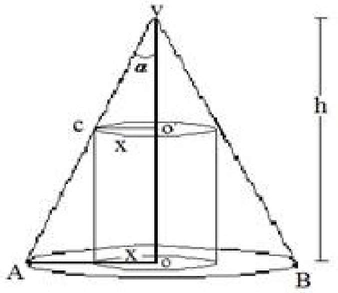
33. Find x and y, if 2x + 3y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3x + 2y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

OR

Solve that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ = (a - b) (b - c) (c - a) (ab + bc + ca)

34. Find Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the lines x + y = 2.

35. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi^3 h \tan\alpha$



OR

Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

36. Find the equation of plane passing through the line of intersection of planes 2x + y - z = 3 and 5x - 3y + 4z + 9 = 0 and Parallel to line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

Solution
Section A

1. (b) A^2 = O

   Explanation:

   If any row or column of a square matrix is 0, then its product with itself is always a zero matrix.

2. (b) -81

   Explanation:

   $|3AB| = 3^3|A||B| = 27(-1)(3) = -81$

3. (a) 2

   Explanation:

   $$\lim_{x\to0}\frac{x(e^{\sin x}-1)}{1-\cos x}$$
   $$=\lim_{x\to0}\frac{\frac{(e^{\sin x}-1)}{x}}{\frac{1-\cos x}{x^2}}=\lim_{x\to0}\frac{(e^{\sin x}-1)}{\sin x}.\frac{\sin x}{x}.2=2\left(\because\lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}\right)$$

4. (b) P (E'|F) = 1 – P (E|F)

   Explanation:

   We know that, P(S/F) =1

   $\implies$ P(E U E'|F)=1 Since, E U E' = S

   $\implies$ P(E/F) +P(E'/F) =1 (Since E and E' are disjoint events)

   $\implies$ P(E'/F) = 1 - P(E/F)

5. (b) σ2=E(X – μ)2

   Explanation:

Let X be a random variable assuming values x1, x2,....,xn with probabilities p1, p2, ...,pn, respectively such that pi ≥ 0,$\sum_{i=1}^{n} p_i = 1$. If E is the expectation, mean of X is denoted by μ, variance denoted by σ2, is defined as : σ2=E(X – μ)2

6. (a) Unbounded

   Explanation:

   The optimal value of the objective function Z = ax + by may or may not exist, if the feasible region for an LPP is unbounded. This is because the maximum or minimum value of the objective function may not exist. Even if it exists it must occur in a corner point of the feasible region.

7. (d) $\frac{1+\tan^2\theta}{1-\tan^2\theta}$

   Explanation:

   $\cos 2\theta$ is equals to $cos(\theta + \theta) = cos\theta . cos\theta - sin\theta . sin\theta = \cos^2\theta - \sin^2\theta$

   $= \frac{cos^2\theta - sin^2\theta}{cos^2\theta + sin^2\theta} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

8. (b) $x \log\left(\frac{x}{e}\right)$ + C

   Explanation:

   Integrating both sides, we get $\int \frac{d}{dx}(f(x))dx = \int \log x \ dx \ we \ get \ f(x) = \int \log x dx$

   $\int \log x dx = x \log x - x = x(\log x - 1)$

   $= x(\log x - \log e) = x\{\log\left(\frac{x}{e}\right)\}$

9. (b) The planes are parallel

   Explanation:

   We have, 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z = 0. Here ,
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{2}{3}$
   Therefore , the given planes are parallel.

10. (c) lr, mr and nr

Explanation:

If l , m and n are the direction cosines of vector $\overrightarrow{r}$ denoted by $\overrightarrow{OP}$, then , the coordinates of point P are given by : lr ,mr and nr respectively.

11. transitive
12. $\frac{3}{2}$
13. does not exist
14. $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$

    OR

    $\vec{r}=\vec{a}+\lambda\vec{b}$
15. 3

OR

0

16. If, $A=\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -1 & 1\end{bmatrix}$ then,we have to find the value of |AB|.

    Clearly, $|A|=\begin{vmatrix}1 & 2\\ 3 & -1\end{vmatrix}=-1-6=-7$
    and $|B|=\begin{vmatrix}1 & 3\\ -1 & 1\end{vmatrix}=1+3=4$
    $\therefore|AB|=|A|\cdot|B|=(-7\times 4)=-28$
17. According to the question , $I=\int_2^3 \frac{1}{x} dx$

    $=[\log|x|]_2^3$

    $=\log 3-\log 2$
    $=\log\frac{3}{2}\left[\because \log m-\log n=\log\frac{m}{n}\right]$

OR

Let $I=\int \sec^2(7-4x)dx$

put 7 - 4x = t

$\Rightarrow \quad -4dx = dt \Rightarrow dx = \frac{-1}{4}dt$

$\therefore \quad I = \frac{-1}{4}\int \sec^2 t dt = \frac{-1}{4}\tan t + C$

$= -\frac{\tan(7-4x)}{4} + C$

18. Let I = $\int \frac{dx}{x^2+16} = \int \frac{dx}{x^2+(4)^2}$

$= \frac{1}{4}\tan^{-1}\frac{x}{4} + C\left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

19. Let f(x)= $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$

$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \ \forall\, x \in R$

So, f(x) is increasing.

Hence, f(x)=0 has only one solution.

Or according to the rule of signs, f(x) has the sign change in the coefficients only once (between the coefficient of x and the constant term) . So there is exactly only one positive real root/solution for f(x).

20. Given y = $Ae^{2x} + B.e^{-2x}$.

Differentiating on both sides with respect to x

$\frac{dy}{dx} = 2Ae^{2x} - 2B.e^{-2x}$ and $\frac{d^2y}{dx^2} = 4.Ae^{2x} + 4Be^{-2x}$

Thus, $\frac{d^2y}{dx^2} = 4y$ i.e., $\frac{d^2y}{dx^2} - 4y = 0$

Section B

21. Putting x = a sin θ, we obtain

$\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}$

$= \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right\}$

$= \tan^{-1}\left\{\frac{a\sin\theta}{a\cos\theta}\right\}$

$= \tan^{-1}(\tan\theta) = \theta = \sin^{-1}\frac{x}{a}\left[\because x = a\sin\theta \Rightarrow \sin\theta = \frac{x}{a} \Rightarrow \theta = \sin^{-1}\frac{x}{a}\right]$

OR

In order to prove that * is a binary operation on S, it is sufficient to show that a * b ∈ S for all a, b ∈ S.
We have,
0 * 0 = (Remainder when 0 + 0 = 0 is divided by 5) = 0
3 * 4 = (Remainder when 3 + 4 = 7 is divided by 5) = 2
2 * 3 = (Remainder when 2 + 3 = 5 is divided by 5) = 0
Similarly, we have
0 * 1 = 1 , 0 * 2 = 2, 0 * 3 = 3, 0 * 4 = 4
1 * 0 = 1, 1 * 1 = 2, 1 * 2 = 3, 1 * 3 = 4, 1 * 4 = 0
2 * 0 = 2, 2 * 1 = 3, 2 * 2 = 4 , 2 * 3 = 0, 2 * 4 = 1
3 * 0 = 3, 3 * 1 = 4, 3 * 2 = 0 , 3 * 3 = 1, 3 * 4 = 2
4 * 0 = 4, 4 * 1 = 0, 4 * 2 = 1, 4 * 3 = 2 and 4 * 4 = 3
Clearly, a * b ∈ S for all a, b ∈ S. So, * is a binary operation on S.

22. Let x cm be the radius and y be the enclosed area of the circular wave at any time t.

Rate of increase of radius of circular wave = 5 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 5 cm/sec

$\Rightarrow \frac{dx}{dt} = 5cm/$ sec ...(i)

$y = \pi x^2$

$\therefore$ Rate of change of area $= \frac{dy}{dt} = \pi\frac{d}{dt}x^2$

$= \pi.2x\frac{dx}{dt} = 2\pi x\left(5\right)$ (from (i)

$= 10\pi xcm^2/$ sec

Putting $x = 8cm$ (given),

$\frac{dy}{dt} = 10\pi\left(8\right) = 80\pi cm^2/$ sec

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of

$80\pi cm^2/$ sec.

23. We have, $x = e^{\frac{x}{y}}$

$\therefore \frac{d}{dx} x = \frac{d}{dx} e^{x/y}$

$\Rightarrow 1 = e^{x/y} . \frac{d}{dx}\left(\frac{x}{y}\right)$

$\Rightarrow 1 = e^{x/y} . \left[\frac{y.1 - x.dy/dx}{y^2}\right]$

$\Rightarrow y^2 = y . e^{x/y} - x . \frac{dy}{dx} . e^{x/y}$

$\Rightarrow x . \frac{dy}{dx} . e^{x/y} = ye^{x/y} - y^2$

$\therefore \frac{dy}{dx} = \frac{y\left(e^{x/y} - y\right)}{x . e^{x/y}}$

$= \frac{\left(e^{x/y} - y\right)}{e^{x/y} . \frac{x}{y}}$

$= \frac{x - y}{x . \log x} \left[\therefore x = e^{\frac{x}{y}} \Rightarrow logx = \frac{x}{y}\right].$

Hence Proved.

24. i. $\overrightarrow{OR} = \frac{2\left(\vec{a} + \vec{b}\right) + 1\left(3\vec{a} - 2\vec{b}\right)}{2+1}$
$= \frac{5\vec{a}}{3}$

ii. $\overrightarrow{OR} = \frac{2\left(\vec{a} + \vec{b}\right) - \left(3\vec{a} - 2\vec{b}\right)}{2-1}$
$= \frac{2\vec{a} + 2\vec{b} - 3\vec{a} + 2\vec{b}}{1}$
$= 4\vec{b} - \vec{a}$

OR

$|\vec{a}| = \frac{1}{7}\sqrt{2^2 + 3^2 + 6^2} = 1$

$\left|\vec{b}\right| = \frac{1}{7}\sqrt{6^2 + 2^2 + 3^2} = 1$

$|\vec{c}| = \frac{1}{7}\sqrt{3^2 + 6^2 + 2^2} = 1$

Hence they are unit vectors

$$\vec{a}.\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}).\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{1}{49}(12 + 6 - 18) = 0$$

Similarly $\vec{b}.\vec{c} = 0$

and $\vec{c}.\vec{a} = 0$

$\vec{a} \perp \vec{b}, \vec{b} \perp \vec{c}$ and $\vec{c} \perp \vec{a}$

So they are $\perp$ to each other.

25. $x_1 = -3, y_1 = 1, z_1 = 5$

$a_1 = -3, b_1 = 1, c_1 = 5$

$x_2 = -1, y_2 = 2, z_2 = 5$

$a_2 = -1, b_2 = 2, c_2 = 5$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 2(5 - 10) + 1(-5 + 15) = 0$$

Therefore, lines are coplanar.

26. Bag I = {3B, 2W}, Bag II = {2B, 4W}

Let E_1= Event that bag I is selected

E_2= Event that bag II is selected

And E = Event that a black ball is selected

$$\Rightarrow P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{E}{E_1}\right) = \frac{3}{5}, P\left(\frac{E}{E_2}\right) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$

$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{2}{6} = \frac{3}{10} + \frac{2}{12}$$

$$= \frac{18+10}{60} = \frac{28}{60} = \frac{7}{15}$$

Section C

27. We have Z be set of integers and

R = {(a, b): a, b $\in$Z and a + b is even } be a relation on Z.

Reflexivity: Let a $\in$ Z

$\Rightarrow$ a + a is even $\left[\begin{array}{l}\text{if } a \text{ is even } \Rightarrow a+a \text{ is even} \\ \text{if } a \text{ is } odd \Rightarrow a+a \text{ is even}\end{array}\right]$

$\Rightarrow$ (a, a) $\in$ R

$\Rightarrow$ R is reflexive

Symmetric: Let a, b $\in$ Z and (a, b) $\in$ R

$\Rightarrow$ a + b is even

$\Rightarrow$ b + a is even

$\Rightarrow$ (b, a) $\in$ R,

$\Rightarrow$ R is symmetric

Transitivity: Let (a, b) $\in$ R and (b, c) $\in$ R for some a, b, c $\in$ Z

$\Rightarrow$ a + b is even and b + c is even

$\Rightarrow$ a + c is even $\left[\begin{array}{l}\text{if } b \text{ is odd, then } a \text{ and } c \text{ must be odd } \Rightarrow a+c \text{ is even,} \\ \text{If } b \text{ is even, then } a \text{ and } c \text{ must be even } \Rightarrow a+c \text{ is even}\end{array}\right]$

$\Rightarrow$ (a, c) $\in$ R

$\Rightarrow$ R is transitive

Hence, R is an equivalence relation on Z.

28. Let $y=u+v$

where $u=(x\cos x)^x, v=(x.\sin x)^{\frac{1}{x}}$

$u=(x\cos x)^x$

Taking log both sides

$\log u=\log(x\cos x)^x$

$\log u=x.\log(x.\cos x)$

Differentiate

$\frac{1}{u}\cdot\frac{du}{dx}=x.\frac{1}{x\cos x}(-x\sin x+\cos x.1)+\log(x\cos x).1$

$\frac{du}{dx}=u\left[-x\tan x+1+\log(x.\cos x)\right]$

$v=(x.\sin x)^{\frac{1}{x}}$

Taking log both sides

$\log v = \log (x.\sin x)^{\frac{1}{x}}$

$\log v = \frac{1}{x}.\log(x.\sin x)$

Differentiating

$\frac{1}{v}\cdot\frac{dv}{dx} = \frac{1}{x}\cdot\frac{1}{x.\sin x}(x\cos x + \sin x.1) + \log(x.\sin x)\left(-\frac{1}{x^2}\right)$

$\frac{dv}{dx} = v\left[\frac{\cot x}{x} + \frac{1}{x^2} - \frac{\log(x\sin x)}{x^2}\right]$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= (x\cos x)^x\left[-x.\tan x + 1 + \log(x.\cos x)\right] + (x.\sin x)^{\frac{1}{x}}$
$\left[\frac{\cot x}{x} + \frac{1}{x^2} - \frac{\log(x.\sin x)}{x^2}\right]$

OR

Given, $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$.......(i)

We shall first find $\frac{dy}{dx}$ and then use it in L.H.S of (i).
Now, y = a sin x + b cos x..........(ii)
Therefore,on differentiating both sides of Eq.(ii) w.r.t x, we get,
$\frac{dy}{dx}$ = a cos x - b sin x
Now, let us take LHS of Eq.(i),
Here, LHS = $y^2 + \left(\frac{dy}{dx}\right)^2$
Therefore,on putting the value of y and dy/dx, we get

LHS = $(a \sin x + b \cos x)^2 + (a \cos x - b \sin x)^2$

$= a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x + a^2 \cos^2 x - b^2 \sin^2 x + 2ab \sin x \cos x$

$= a^2 \sin^2 x + b^2 \cos^2 x + a^2 \cos^2 x - b^2 \sin^2 x$

$= a^2 (\sin^2 x + \cos^2 x) + b^2 (\sin^2 x + \cos^2 x)$

$= a^2 + b^2$

= RHS

29. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

$\frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$ ...(i)

let y = vx

$\frac{dy}{dx} = v.1 + x.\frac{dv}{dx}$ ...(ii)

Put $\frac{dy}{dx}$ in eq ...(i)

$v + x\frac{dv}{dx} = \frac{vx\cos v+x}{x\cos v}$

$v + x\frac{dv}{dx} = \frac{v\cos v+1}{\cos v}$

$x\frac{dv}{dx} = \frac{v\cos v-1}{\cos v} - v$

$x\frac{dv}{dx} = \frac{v\cos v+1-v\cos v}{\cos v}$

$x\frac{dv}{dx} = \frac{1}{\cos v}$

$\int \cos v dv = \int \frac{dx}{x}$

$\sin v = \log x + \log c$sin v = log x + log c

$\sin v = \log|cx|$ sin v = log | cx | $[\because y = vx]$[y = vx]

$\sin\left(\frac{y}{x}\right) = \log|cx|$

30. Let $I = \int \frac{x+1}{x(1+xe^x)^2}dx = \int \frac{(x+1)e^x}{xe^x(1+xe^x)^2}dx$

Let $xe^x = t$. Then $d(xe^x) = dt$ or, $(x + 1)e^x dx = dt$

$\therefore \quad I = \int \frac{1}{t(1+t)^2}dt$

Let $\frac{1}{t(1+t)^2} = \frac{A}{t} + \frac{B}{1+t} + \frac{C}{(1+t)^2}$ ...(i)

$\Rightarrow 1 = A(1 + t)^2 + Bt(1 + t) + Ct$ ...(ii)

A = 1, C = -1

Now, putting t = 1 in (ii) we get

1 = 4A + 2B + C $\Rightarrow$ 1 = 4 + 2B -1 $\Rightarrow$ B = -1

Substituting the values of A, B and C in (i), we get

$\frac{1}{t(1+t)^2} = \frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}$

$\therefore \quad I=\int \frac{1}{t(1+t)^2} dt=\int\left(\frac{1}{t}-\frac{1}{1+t}-\frac{1}{(1+t)^2}\right) dt$

$\Rightarrow$ I = log |t| - log |1 + t| + $\frac{1}{1+t}$ + C = log (xe^x) - log (1 + xe^x) + $\frac{1}{1+xe^x}$ + C

31.

X	**0**	**1**
P(X)	$\frac{30}{100}$	$\frac{70}{100}$

$E(X)=\sum XP(X)=0\times\frac{30}{100}+1\times\frac{70}{100}=\frac{7}{10}$
$E(X^2)=\sum X^2P(X)=0^2\times\frac{30}{100}+1^2\times\frac{70}{100}=\frac{70}{100}=\frac{7}{10}$
$Var(X)=E(X^2)-(E(X))^2=\frac{7}{10}-\frac{49}{100}=\frac{70-49}{100}=\frac{21}{100}$

OR

For a loaded die, it is given that

P(1) = P(2) = 0.2,

P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3

Also, die is thrown two times

Here, A = Same number each time and B = Total score is 10 or more

$\therefore$ A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

So, P(A) = [P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + P(5, 5) + P(6, 6)]

$=[P(1)\cdot P(1)+P(2)\cdot P(2)+P(3)\cdot P(3)+P(4)\cdot P(4)+P(5)\cdot P(5)+P(6)$

$=[0.2\times0.2+0.2\times0.2+0.1\times0.1+0.3\times0.3+0.1\times0.1+0.1\times0.1]$

= 0.04 + 0.04 + 0.01 + 0.09 + 0.01 + 0.01 = 0.20

And B = {(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)}

P(B) = P(4, 6) + P(6, 4) + P(5, 5), P(5, 6), (6, 5), (6, 6)}

$=P(4)\cdot P(6)+P(6)\cdot P(4)+P(5)\cdot P(5)+P(5)\cdot P(6)+P(6)\cdot P(5)+P(6)\cdot$

$=0.3\times0.1+0.1\times0.3+0.1\times0.1+0.1\times0.1+0.1\times0.1+0.1\times0.1$

= 0.03 + 0.03 + 0.01 + 0.01 + 0.01 + 0.01 = 0.10

Also, $A\cap B=\{(5,5),(6,6)\}$

$\therefore P(A\cap B)=P(5,5)+P(6,6)=P(5)\cdot P(5)+P(6)\cdot P(6)$

$=0.1\times0.1+0.1\times0.1=0.01+0.01=0.02$

We know that, for two events A and B, if $P(A\cap B)=P(A)\cdot P(B)$, then both are independent events.

Here, $P(A \cap B) = 0.02$ and $P(A) \cdot P(B) = 0.20 \times 0.10 = 0.02$

Thus, $P(A \cap B) = P(A) \cdot P(B) = 0.02$

Hence, A and B are independents events.

32. Consider $x + y = 3$

When x = 0, then y = 3 and

when y = 0, then x = 3

So $A(0,3)$ *and* $B(3,0)$ are the points on the line $x + y = 3$

Consider $x + 2y = 4$

When x = 0, then y = 2 and when y = 0, then x = 4.

So C (0, 2) and D (4, 0) are the points on the line x + 2y = 4

The two lines $x + y = 3$ and$x + 2y = 4$, intersect each other at $M(2,1)$.

So the feasible region is unbounded. Therefore, minimum value may or may not occur. If it occurs, it will be on the corner point.

The comer points are $(4,0),(2,1)$ and $(0,3)$ $Z = 4x + y$

At $(4,0),\ Z = 4(4) + 0 = 16$

At$(2,1),\ Z = 4(2) + 1 = 9$

At $(0,3),\ Z = 4(0) + 3 = 3$ (minimum)

If we draw the graph of $4x + y < 3$, we see that open half plane determined by $4x + y < 3$ and feasible region do not have a point in common other than (0, 3).

Hence, 3 is the minimum value of Z at (0, 3).

Section D

33. Given equations are

2x + 3y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ...... (i)

3x + 2y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ .....(ii)

eqn. (i) $\times$ 3 - eqn. (ii) $\times$ 2

$$6x+9y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$$

$$6x+4y = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$- \; - \quad = \quad -$$

$$5y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$y = \frac{1}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

eqn. (i) $\times$ 2 - eqn. (ii) $\times$ 3

$$x = \frac{1}{5}\begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$$

OR

Let $\triangle$ = $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

$\triangle$ = $\frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & abc & abc \end{vmatrix}$ [Multiplying C_1, C_2, and C_3 by a,b and c respectively]

$\Rightarrow \triangle$ = $\frac{abc}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$ [Taking abc common from R_3]

$\Rightarrow \triangle$ = -$\begin{vmatrix} a^2 & b^2 & c^2 \\ 1 & 1 & 1 \\ a^3 & b^3 & c^3 \end{vmatrix}$ [Applying $R_2 \leftrightarrow R_3$]

$\Rightarrow \triangle$ = $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ [Applying $R_1 \leftrightarrow R_2$]

$\Rightarrow \triangle$ = $\begin{vmatrix} 1 & 0 & 0 \\ a^2 & b^2 - a^2 & c^2 - a^2 \\ a^3 & b^3 - a^3 & c^3 - a^3 \end{vmatrix}$ [Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$]

$\Rightarrow \triangle$ = $\begin{vmatrix} 1 & 0 & 0 \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \\ a^3 & (b-a)\left(b^2+ba+a^2\right) & (c-a)\left(c^2+ca+a^2\right) \end{vmatrix}$

$\Rightarrow \triangle$ = (b - a) (c - a) $\begin{vmatrix} 1 & 0 & 0 \\ a^2 & b+a & c+a \\ a^3 & b^2+a^2+ab & c^2+ac+a^2 \end{vmatrix}$ [Taking (b - a) and (c - a) common from C_2 and C_3 respectively]

$\Rightarrow \triangle$ = (b - a) (c - a) $\begin{vmatrix} b+a & c+a \\ b^2+a^2+ab & c^2+a^2+ac \end{vmatrix}$ [Expanding along R_1]

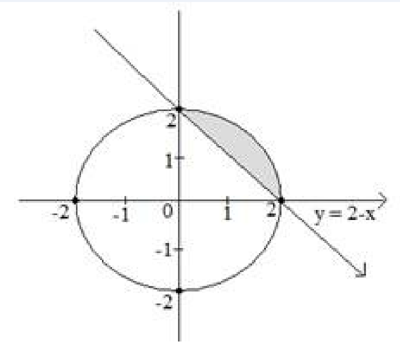
$\Rightarrow \triangle$ = (b - a) (c - a) $\begin{vmatrix} b-c & c+a \\ b^2-c^2+ab-ac & c^2+a^2+ac \end{vmatrix}$ [Applying $C_1 \rightarrow C_1 - C_2$]

$\Rightarrow \triangle$ = (b - a) (c - a) $\begin{vmatrix} b-c & c+a \\ (b^2-c^2)+a(b-c) & c^2+a^2+ac \end{vmatrix}$

$\Rightarrow \triangle$ = (b - a) (c - a) (b - c) $\begin{vmatrix} 1 & c+a \\ b+c+a & c^2+a^2+ac \end{vmatrix}$ [Taking (b - c) common from C_1]

$\Rightarrow \triangle$ = (b - a) (c - a) (b − c) ($c^2 + a^2$ + ac − bc − c^2 − ac − ab − ac − a^2)

$\Rightarrow \triangle$ = (b - a) (c - a) (b − c) (−bc − ab − ac) = (a − b) (b − c) (c − a) (ab + bc + ca).

34. 

$x^2 + y^2 = 4$ ...(1)

x + y = 2 ...(2)

From (2), y = 2 - x

Put this value of y in (1), we get,

$x^2 + (2 - x)^2 = 4$

$\Rightarrow 2x^2 - 4x = 0$

$\Rightarrow$ 2x(x - 2) = 0

$\Rightarrow$ x = 0, 2

When x = 0, y = 2 - 0 = 2

When x = 2, y = 2 - 2 = 0

$\therefore$ points of intersection are (0, 2) and (2, 0)

Required area = area of quadrant in first quadrant - area of triangle.

$= \int_0^2 \sqrt{4-x^2}dx - \int_0^2 (2-x)dx$

$= \int_0^2 \sqrt{(2)^2 - x^2}dx - \int_0^2 (2-x)dx$

$= \left[\frac{x}{2}\sqrt{(2)^2 - x^2} + \frac{(2)^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \left[\frac{2}{2}\sqrt{4-4} + \frac{4}{2}\sin^{-1}1\right] - \left[0 + \frac{4}{2}\sin^{-1}0\right] - \left[\left(4 - \frac{4}{2}\right) - (0-0)\right]$

$= \left(0 + \frac{4}{2} \times \frac{\pi}{2}\right) - (0 + 2 \times 0) - 2 + 0 = \pi - 2$

35. $\frac{vo'}{x} = \cot\alpha$

$vo' = x\cot\alpha$

$oo' = h - x\cot\alpha$

$V = \pi x^2 . (h - x\cot\alpha)$

$V = \pi x^2 h - \pi x^3 \cot\alpha$

$\frac{dV}{dx} = 2\pi x h - 3\pi x^2 \cot\alpha$

for maximum/minimum

$\frac{dV}{dx} = 0$

$2\pi x h - 3\pi x^2 \cot\alpha = 0$

$x = \frac{2h}{3}\tan\alpha$

$\frac{d^2V}{dx^2} = 2\pi h - 6\pi x\cot\alpha$

$\left.\frac{d^2V}{dx^2}\right]_{x=\frac{2h}{3}\tan\alpha} = \pi(2h - 4h)$=-2πh<0

Therefore, V is maximum

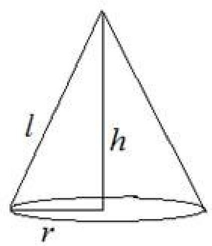
$V = \pi x^2 (h - x\cot\alpha)$

$= \pi\left(\frac{2h}{3}\tan\alpha\right)^2 \left[h - \frac{2h}{3}\tan\alpha\cot\alpha\right]$

$= \pi\frac{4h}{9}^2\tan^2\alpha . \frac{h}{3}$

$V = \frac{4}{27}\pi h^3 \tan^2\alpha$

OR



$v = \frac{1}{3} . \pi r^2 h$

Let $r^2h = k$

Where k is constant

$r^2h = k$

$h = \frac{k}{r^2}$

$s = \pi rl$

$s^2 = \pi^2 r^2 l^2$

$= \pi^2 . r^2 (r^2 + h^2)$

$= \pi^2 . r^2 \left[r^2 + \frac{k^2}{r^4}\right] \left[\because h = \frac{k}{r^2}\right]$

$z = \pi^2 r^4 + \pi^2 k^2 r^{-2}$

$\frac{dz}{dr} = 4\pi^2 r^3 - 2\pi^2 k^2 r^{-3}$

$\frac{d^2z}{dr^2} = 12\pi^2 r^2 + 6\pi^2 k^2 r^{-4}$

$\frac{dz}{dr} = 0$

$4\pi^2 r^3 - \frac{2\pi^2 k^2}{r^3} = 0$

$4\pi^2 r^6 - 2\pi^2 k^2 = 0$

$2\pi^2 k^2 = 4\pi^2 r^6$

$k^2 = 2r^6$

Putting $k = r^2h$ we get

$r^4 h^2 = 2r^6$

$h^2 = 2r^2$

$h = \sqrt{2}r$

$\left(\frac{d^2z}{dr^2}\right)_{h=\sqrt{2}r}$ is negative

z is maximum at $h = \sqrt{2}r$

36. Given equations of planes are

2x + y - z - 3 = 0 ....(i)

and 5x - 3y + 4z + 9 = 0 ....(ii)

Let the required equation of plane which passes through the line of intersection of planes (i) and (ii) be

$(2x + y - z - 3) + \lambda(5x - 3y + 4z + 9) = 0$.......................(iii)

$\Rightarrow x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + (-3 + 9\lambda) = 0$ ....(iv)

Here, DR's of plane are

$(2+5\lambda, 1-3\lambda, -1+4\lambda)$Also, given that the plane (iv) is parallel to the line, whose equation is

$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

DR's of the line are (2, 4, 5).

Since, the plane is parallel to the line.

Hence, normal to the plane is perpendicular to the line,

i.e. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, $a_1 = 2+5\lambda, b_1 = 1-3\lambda, c_1 = -1+4\lambda$

and $a_2 = 2$, $b_2 = 4$, $c_2 = 5$

$\therefore \quad (2+5\lambda)2 + (1-3\lambda)4 + (-1+4\lambda)5 = 0$

$\Rightarrow \quad 4+10\lambda+4-12\lambda-5+20\lambda = 0$

$\Rightarrow \quad 18\lambda+3 = 0 \Rightarrow \lambda = -\frac{3}{18} = -\frac{1}{6}$

On putting $\lambda = -\frac{1}{6}$ in Eq. (iii), we get the required equation of plane as

$(2x+y-z-3) - \frac{1}{6}(5x-3y+4z+9) = 0$

12 + 6y - 6z - 1 8 - 5x + 3y - 4z- 9 = 0

7x + 9y - 10z - 27 = 0