CBSE Class 12 - Mathematics

Sample Paper 06

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. The number of solutions of 2x + y = 4, x – 2y = 2, 3x + 5y = 6 is

   a. Two solution

   b. Infinitely many solution

   c. One solution

   d. No solution

2. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} =$ k, then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} =$

a. 6k

b. $\frac{6}{k}$

c. 2k

d. 3k

3. If both f and g are defined in a nhd of 0; f(0) = 0 = g(0) and f‘(0) = 8 = g’(0), then $\underset{x \to 0}{Lt} \frac{f(x)}{g(x)}$ is equal to

a. None of these

b. 0

c. 1

d. 16

4. A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by

a. $P(X = r) = C_r^n p^r q^{n-r-2}$

b. $P(X = r) = C_r^n p^r q^{n-r}$

c. $P(X = r) = C_r^n p^{2r} q^{n-r}$

d. $P(X = r) = C_{r-2}^n p^r q^{n-r}$

5. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable ?

a. X = 2, 3, 4; no

b. X = 2, 3, 5; yes

c. X = 2, 1, 3; yes

d. X = 0, 1, 2; yes

6. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. What number of rackets and bats must be made if the factory is to work at full capacity?

   a. 4 tennis rackets and 12 cricket bats

   b. 4 tennis rackets and 14 cricket bats

   c. 3 tennis rackets and 13 cricket bats

   d. 5 tennis rackets and 15 cricket bats

7. The principal value of $\sin^{-1}(\sin\ \frac{3\pi}{4}\ ) = \ldots\ldots$

   a. $\frac{\pi}{4}$

   b. $\frac{3\pi}{4}$

   c. $\frac{5\pi}{4}$

   d. $\frac{-\pi}{4}$

8. $\int\limits_1^e \frac{1+\ln x}{2x} dx$ is equal to

   a. e

   b. $\frac{1}{4}$

   c. $\frac{3}{4}$

   d. $\frac{1}{e}$

9. If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}\ and\ \frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$ are the equations of the two lines, then express acute angle between the two lines in terms of the direction cosines of the two lines.

a. $\cot\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

b. $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

c. $\tan\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

d. $\sin\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

10. If λ is a real number $\lambda\vec{a}$ is a

a. vector

b. unit vector

c. scalar

d. inner product

11. Fill in the blanks:

A function f : X $\rightarrow$ Y is said to be a ________ function, if it is both one-one and onto.

12. Fill in the blanks:

For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is ________.

13. Fill in the blanks:

If A and B are square matrices of the same order, then [k(A - B)]' = ________ where k is any scalar.

14. Fill in the blanks:

The vector equation of a plane which is at a distance p from the origin, where $\hat{n}$ is the unit vector normal to the plane is ________.

OR

Fill in the blanks:

A straight line which is perpendicular to every line lying on a plane is called a

________ to the plane.

15. Fill in the blanks:

The position vector of the point which divides the join of points with position vectors $\vec{a}+\vec{b}$ and $2\vec{a}-\vec{b}$ in the ratio 1 : 2 is _______.

OR

Fill in the blanks:

The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, $\vec{a}.\vec{b}$ = $2\sqrt{3}$ is _______.

16. Evaluate : $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$.

17. Find $\int \frac{\sin^2 x-\cos^2 x}{\sin x \cos x}$ dx.

OR

Evaluate: $\int \sqrt{\frac{1+\cos 2x}{2}}dx$

18. Evaluate the following integral $\int (2x-3\cos x+e^x)\,dx$

19. Find the length of the longest interval in which the function 3sin x-4sin^{3}x is increasing.

20. Find the order and degree of $\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$

Section B

21. Find the principal value of sec$^{-1}(-\sqrt{2})$.

OR

Show that the relation 'is congruent to' on the set of all triangles in a plane is an equivalence relation.

22. The radius of the circle is increasing uniformly at the rate of 3 cm per second. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

23. If y = $(\tan^{-1}x)^2$ show that $(x_2 + 1)^2$y2 + $2x(x^2 + 1)y_1 = 2$

24. Find angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 1, \left|\vec{b}\right| = 2, \vec{a}.\vec{b} = 1$

OR

If $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} + 2\hat{k}$ find $\left|\vec{b} \times 2\vec{a}\right|$

25. Find the Cartesian equation of the plane $\vec{r}.\left[(5 - 2t)\hat{i} + (3 - t)\hat{j} + (25 + t)\hat{k}\right] = 15$

26. Four cards are successively drawn without replacement from a deck of 52 playing cards. What is the probability that all the four cards are king?

Section C

27. Let * be the binary operation on N defined by a*b = H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on N?

28. If y = $\tan^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} - \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)$, $x^2 \leq 1$, then find $\frac{dy}{dx}$.

OR

Find the value of k, for which f(x) = $\begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, \text{ if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, \quad \text{if } 0 \leq x < 1 \end{cases}$ is continuous at x = 0.

29. Solve the following differential equation $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$.

30. Evaluate $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$.

31. A random variable X has following probability distributions:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) P(X < 3) (iii) P(X > 6) (iv) P (0 < X < 3).

OR

Consider the probability distribution of a random variable X:

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15

Calculate:

i. $V\left(\frac{X}{2}\right)$
ii. Variance of X.

32. Minimise Z = 3x + 5y subject to the constraints:

$x + 2y \geqslant 10$
$x + y \geqslant 6$
$3x + y \geqslant 8$
$x, y \geqslant 0$

Section D

33. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

OR

Using properties of determinants, show that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

34. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first

quadrant, using integration.

35. AB is the diameter of a circle and C is any point on the circle. Show that the area of $\triangle$ABC is maximum, when it is an isosceles triangle.

OR

If the length of three sides of a trapezium other than the base are each equal to 10 cm, then find the area of the trapezium, when it is maximum.

36. Find the equation of the plane that contains the point (1,-1,2) and is $\perp$ to each of the plane $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

Solution
Section A

1. (c) One solution
   Explanation:

   For Unique solution , $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

   for given system of equations we have: $\frac{2}{1} \neq \frac{1}{-2}, \frac{1}{3} \neq \frac{-2}{5}, \frac{3}{2} \neq \frac{5}{1}$

2. (a) 6k
   Explanation:

   $$\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix} = 3\begin{vmatrix} 2a & 2b & 2c \\ m & n & p \\ x & y & z \end{vmatrix} = 6\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = 6k$$

3. (c) 1
   Explanation:

   $\lim\limits_{x \to 0} \frac{f(x)}{g(x)} = \lim\limits_{x \to 0} \frac{f'(x)}{g'(x)} = \frac{f'(0)}{g'(0)} = 1$ (by using L'Hospital Rule)

4. (b) $P(X = r) = C_r^n p^r q^{n-r}$
   Explanation:

   A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by :
   $P(X = r) = C_r^n p^r q^{n-r}$.

5. (d) X = 0, 1, 2; yes
   Explanation:

   An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls.therefore the possible values of X = 0 , 1 ,2 . Yes , X is a random variable.

6. (a) 4 tennis rackets and 12 cricket bats
   Explanation:

   Let number of rackets made = x
   And number of bats made = y
   Therefore , the above L.P.P. is given as :
   Maximise , Z = x +y , subject to the constraints : 1.5x +3y ≤ 42 and. 3x +y ≤ 24, i.e.0.5x + y ≤ 14 i.e. x +2y ≤ 28 and 3x +y ≤ 24 , x, y ≥ 0.

Corner points	Z = x + y
O(0 , 0)	0
D(0,14)	14
A(8,0)	8
B(4,12)	16....................(Max.)

   Here Z = 16 is maximum. i.e Maximum number of rackets = 4 and number of bats = 12.

7. (a) $\frac{\pi}{4}$
   Explanation:

$$\sin^{-1}(\sin\frac{3\pi}{4}\ ) \ = \sin^{-1}(\sin\left(\pi - \frac{\pi}{4}\right)$$

$$= \sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$$

8. (c) $\frac{3}{4}$
   Explanation:

$$\frac{1}{2}\int_1^e \frac{1}{x}dx + \int_1^e \frac{\ln x}{x}dx = \frac{1}{2}\left[\log x + \frac{(logx)^2}{2}\right]_1^e$$

9. (b) $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ \ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$
   Explanation:

If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$ are the equations of the two lines, then the acute angle between the two lines is given by :
$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$. and $l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

10. (a) vector

    Explanation:

    If a vector is multiplied by any scalar then , the result is always a vector.

11. bijective

12. -1

13. k(A' - B')

14. $\vec{r}.\hat{n} = p$

OR

normal

15. $\frac{4\vec{a}+\vec{b}}{3}$

OR

$\frac{\pi}{3}$

16. Let $\Delta = \begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$
$= \sqrt{6} \times \sqrt{24} - \sqrt{5} \times \sqrt{20}$
$= \sqrt{6} \times \sqrt{6} \times \sqrt{4} - \sqrt{5} \times \sqrt{5} \times \sqrt{4}$
$= 6 \times 2 - 5 \times 2 = 12 - 10 = 2$

17. Let I = $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x} dx$
$= \int \left[\frac{\sin^2 x}{\sin x \cdot \cos x} - \frac{\cos^2 x}{\sin x \cdot \cos x}\right] dx$
$= \int \left[\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right] dx$
$= \int (\tan x - \cot x) dx = \int \tan x dx - \int \cot x dx$
= Iog|secx| - (-log|cosecx|)+ C

= log [sec x| + log | cosec x| + C

= log [sec x·cosec x| +C

OR

$\int \sqrt{\frac{1+\cos 2x}{2}} dx = \int \sqrt{\frac{2 \cos^2 x}{2}} dx$ [∵ $\cos^2 x = 2\cos^2 - 1$]

= $\int \cos x dx$

= sinx + C

18. $\int (2x - 3\cos x + e^x)\, dx$

$= \int 2x dx - \int 3\cos x dx + \int e^x dx$

$= 2\int x dx - 3\int \cos x dx + \int e^x dx$

$= 2\frac{x^2}{2} - 3\sin x + e^x + c$

= x^2 - 3 sin x + e^x + c

19. We have f(x)=3sin x-4$\sin^3$x

f(x) = sin 3x

The longest interval in which sin x is increasing is of length π. since sine function is increasing from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ which is the principle value branch. It is true for every interval with the length of π .

So,the length of the largest interval in which f(x)=sin 3x is increasing is $\frac{\pi}{3}$, since the period is tripled.

20. Order = 2

Degree = 1

Section B

21. We know that, for x $\in$ R, $\sec^{-1}$x represents an angle in $[0, \pi] - \{\frac{\pi}{2}\}$.
$\sec^{-1}(-\sqrt{2})$ = An angle in $[0, \pi] - \{\frac{\pi}{2}\}$ whose secant is $(-\sqrt{2})$
$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$
$\sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$

OR

Let S be the set of all triangles in a plane and let R be the relation on S defined by $(\triangle_1, \triangle_2) \in R \Leftrightarrow$ triangle $\triangle_1$ is congruent to triangle $\triangle_2$.
We observe the following properties of relation R:
Reflexivity: For each triangle $\triangle \in S$, we have
$\triangle \cong \triangle \Rightarrow (\triangle, \triangle) \in R$ for all $\triangle \in S \Rightarrow R$ is reflexive on S
Symmetry: Let $\triangle_1, \triangle_2 \in S$ such that $(\triangle_1, \triangle_2) \in R$. Then,
$(\triangle_1, \triangle_2) \in R \Rightarrow \triangle_1 \cong \triangle_2 \Rightarrow \triangle_2 \cong \triangle_1 \Rightarrow (\triangle_2, \triangle_1) \in R$
So, R is symmetric on s
Transitivity: Let $\triangle_1, \triangle_2, \triangle_3 \in S$ such that $(\triangle_1, \triangle_2) \in R$ and $(\triangle_2, \triangle_3) \in R$. Then,
$(\triangle_1, \triangle_2) \in R$ and $(\triangle_2, \triangle_3) \in R \Rightarrow \triangle_1 \cong \triangle_2$ and $\triangle_2 \cong \triangle_3 \Rightarrow \triangle_1 \cong \triangle_3$
$\Rightarrow (\triangle_1, \triangle_3) \in R$
So, R is transitive on S.
Hence, R being reflexive, symmetric and transitive, is an equivalence relation on S.

22. Let r cm be the radius and A be the area of the circle at any time t.

Rate of increase of radius of circle = 3 cm/sec

$\Rightarrow \frac{dr}{dt}$ is positive and = 3 cm/sec

$\frac{dr}{dt} = 3$....(i)

Now, $A = \pi r^2$

$\therefore$ Rate of change of area of circle $= \frac{dA}{dt} = \pi 2r \frac{dr}{dt}$

$\Rightarrow \pi 2r\,(3) = 6\pi r cm^2/$ sec (from (i))

putting r = 10 cm (given),

Since $\frac{dA}{dt}$ is positive, therefore surface area is increasing at the rate of $60\pi cm^2/$ sec

23. Given: y = $(\tan^{-1}x)^2$ ....(i)

$\therefore y_1 = 2\left(\tan^{-1}x\right)\frac{d}{dx}\tan^{-1}x \left[\because \frac{d}{dx}\{f(x)\}^n = n\{f(x)\}^{n-1}\frac{d}{dx}f(x)\right]$

And $y_1 = 2\left(\tan^{-1}x\right)\frac{1}{1+x^2}$

$= \frac{2\tan^{-1}x}{1+x^2}$

$\Rightarrow \left(1+x^2\right)y_1 = 2\tan^{-1}x$

Again differentiating both sides w.r.t. x,

$\left(1+x^2\right)\frac{d}{dx}y_1 + y_1\frac{d}{dx}\left(1+x^2\right) = 2.\frac{1}{1+x^2}$

$\Rightarrow \left(1+x^2\right)y_2 + y_1.2x = \frac{2}{1+x^2}$

$\Rightarrow \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$ .Hence proved.

24. $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta = \frac{1}{(1)(2)} = \frac{1}{2}$

$\cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$

OR

$2\vec{a} = 8\hat{i} + 6\hat{j} + 4\hat{k}$

$\vec{b}\times 2\vec{a} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 0 & 2\\ 8 & 6 & 4\end{vmatrix}$

$= \hat{i}(0-12) - \hat{j}(12-16) + \hat{k}(18-0)$

$= -12\hat{i} + 4\hat{j} + 18\hat{k}$

$\left|\vec{b}\times 2\vec{a}\right| = \sqrt{(-12)^2 + (4)^2 + (18)^2}$

$= \sqrt{484}$

= 22

25. Given equation of plane is $\vec{r}.\left[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}\right]=15$

$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}\right]=15$

$(5-2t)x+(3-t)y+(25+t)z=15$

26. Let E_1, E_2, $E_3$and E_4 are the events that the first, second, third and fourth card is king,respectively.

$\therefore P(E_1\cap E_2\cap E_3\cap E_4)= P(E_1)\cdot P\left(\frac{E_2}{E_1}\right)\cdot P\left(\frac{E_3}{E_1\cap E_2}\right)\cdot P\left[\frac{E_4}{E_1\cap E_2\cap E_3\cap E_4}\right]$
$=\frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}\cdot\frac{1}{49}=\frac{24}{52\cdot 51\cdot 50\cdot 49}$
$=\frac{1}{13\cdot 17\cdot 25\cdot 49}=\frac{1}{270725}$

Section C

27. a*b = H.C.F. of a and b.

i. a*b = H.C.F. of a and b = H.C.F. of b and a = b*a
Therefore, operation * is commutative.

ii. (a*b)*c = (H.C.F. of a and b)*c = H.C.F. of (H.C.F. of a and b) and c
= H.C.F. of a, b and c = a*b(b*c)
Therefore, the operation is associative.
1*a = a*1 $\neq$ a
Therefore, there does not exist any identity element.

28. Given, y= $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Put x^2 = sin$\theta \Rightarrow \theta=\sin^{-1}x^2$

$\therefore \quad y=\tan^{-1}\left(\frac{\sqrt{1+\sin\theta}+\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}-\sqrt{1-\sin\theta}}\right)$

= $\tan^{-1}\left(\frac{\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}+\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}{\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}-\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}\right)$

= $\tan^{-1}\left[\frac{\sqrt{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}+\sqrt{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}}{\sqrt{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}-\sqrt{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}}\right]$

= $\tan^{-1}\left[\frac{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)+\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)-\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)}\right]$

$= \tan^{-1}\left(\frac{2\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\cot\frac{\theta}{2}\right)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]$

$= \frac{\pi}{2}-\frac{\theta}{2}$

$\Rightarrow y = \frac{\pi}{2}-\frac{1}{2}\sin^{-1}x^2$

Therefore,on differentiating both sides w.r.t x, we get,

$\frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1-(x^2)^2}}(2x)$

$= \frac{-x}{\sqrt{1-x^4}}$

OR

According to the question,we are given that, f(x) =

$\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, \text{ if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, \quad \text{if } 0 \leq x < 1\end{cases}$ is continuous at x = 0

and we have to find the value of k.

Now, f(0) = $\frac{2(0)+1}{0-1} = \frac{1}{-1}$ =-1

and LHL = $\lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}$

$= \lim_{h\to 0}\frac{\sqrt{1-kh}-\sqrt{1+kh}}{-h}\times\frac{(\sqrt{1-kh}+\sqrt{1+kh})}{(\sqrt{1-kh}+\sqrt{1+kh})}$

$= \lim_{h\to 0}\frac{(1-kh)-(1+kh)}{-h(\sqrt{1-kh}+\sqrt{1+kh})}$ [(a + b)(a - b) = a^2 - b^2]

$= \lim_{h\to 0}\frac{-2kh}{-h(\sqrt{1-kh}+\sqrt{1+kh})}$

$= \lim_{h\to 0}\frac{2k}{\sqrt{1-kh}+\sqrt{1+kh}} = \frac{2k}{1+1} = \frac{2k}{2}$ = 2

Since, f(x) is continuous at x = 0.

$\therefore$ f(0) = LHL $\Rightarrow$ - 1 = k

$\Rightarrow$ k = -1

29. According to the question,

Given differential equation is,

$2x^2\frac{dy}{dx} - 2xy + y^2 = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{2x^2}$

which is a homogeneous differential equation as $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$.

put $y = vx$,

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{v^2}{2} \Rightarrow x\frac{dv}{dx} = \frac{-v^2}{2} \Rightarrow \frac{2dv}{v^2} = -\frac{1}{x}dx$

On integrating both sides, we get

$2\int v^{-2}dv = -\log|x| + C$

$\Rightarrow \quad \frac{2v^{-1}}{-1} = -\log|x| + C$

$\Rightarrow \quad \frac{-2}{v} = -\log|x| + C$

$\Rightarrow \quad \frac{-2x}{y} = -\log|x| + C \left[\text{ put } v = \frac{y}{x}\right]$

$\Rightarrow \quad -2x = y(-\log|x| + C)$

$\therefore \quad y = \frac{-2x}{-\log|x| + C}$

which is the required solution of differential equation.

30. Given, $I = \int\frac{x^2+x+1}{(x^2+1)(x+2)}dx$

By using partial fractions ,

$\Rightarrow \frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$...(i)

$\Rightarrow x^2 + x + 1 = A(x^2+1) + (Bx+C)(x+2)$

$\Rightarrow x^2 + x + 1 = x^2(A+B) + x(2B+C) + (A+2C)$

Comparing the coefficients of x^2, x and constant terms on both sides,

$\Rightarrow 1 = A + B$.....(ii)

$\Rightarrow 1 = 2B + C$...(iii)

$\Rightarrow 1 = A + 2C$...(iv)

On solving Equations (ii) and (iii) we get ,

$1 = 2A - C$ ...(v)

On solving Equations (iv) and (v) we get ,
$C = \frac{1}{5}$ and $A = \frac{3}{5}$
From Eq. (ii), we get B = $1 - \frac{3}{5} = \frac{2}{5}$
Thus, from Eq. (i), we have
$\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{3}{5} \cdot \frac{1}{(x+2)} + \frac{1}{5} \frac{(2x+1)}{(x^2+1)}$
Integrating both sides w.r.t to x ,we get :
$\Rightarrow I = \int \frac{x^2+x+1}{(x^2+1)(x+2)} dx = \frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{(2x+1)}{x^2+1} dx$
$= \frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{2x}{x^2+1} dx + \frac{1}{5} \int \frac{dx}{x^2+1}$
$= \frac{3}{5} \log|x+2| + \frac{1}{5} \log|x^2+1| + \frac{1}{5} \tan^{-1} x + C$
$\left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \quad and \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C \right]$

$\therefore I = \frac{3 \log|x+2| + \log|x^2+1| + tan^{-1} x}{5} + C$

31. Firstly, use the result that sum of all the probabilities of a random experiment is one .we will find k and then find other values by using this value of k.

    i. We know that, the sum of a probability distribution of random variable is one, i.e. $\sum P(X) = 1$

       0+ k+ 2k+ 2k+ 3k+ k^2 + $2k^2$ + $7k^2$+ k=1

       $10k^2$ + 9k - 1 = 0

       $10k^2$ + 10k - k-1 = 0

       10k(k + 1) - 1 (k + 1) = 0

       (10k - 1) (k + 1) = 0

       $k = \frac{1}{10}$ or -1
    ii. P(X < 3 = P(X = 0) + P(X = 1) + P(X = 2)

       =0+k+2k=3k

       $= 3\left(\frac{1}{10}\right) \quad \left[\because k = \frac{1}{10}\right]$

       = $\frac{3}{10}$
    iii. P(X> 6) = P(X = 7) = $7k^2$ + k

$$= 7\left(\frac{1}{10}\right)^2 + \frac{1}{10} \quad \left[\because k = \frac{1}{10}\right]$$
$$= \frac{7}{100} + \frac{1}{10}$$
$$= \frac{7+10}{100} = \frac{17}{100}$$

iv. P(O < X < 3) = P(X = 1) + P(X = 2)

= k + 2k = 3k

$$= 3\left(\frac{1}{10}\right) \quad \left[\because k = \frac{1}{10}\right]$$
$$= \frac{3}{10}$$

OR

We have,

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15
XP(X)	0	0.25	0.6	0.6	0.60
$X^2P(X)$	0	0.25	1.2	1.8	2.40

$$Var(X) = E\left(X^2\right) - [E(X)]^2$$

Where, $E(X) = \mu = \sum_{i=1}^{n} x_i P_i(x_i)$

And $E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$

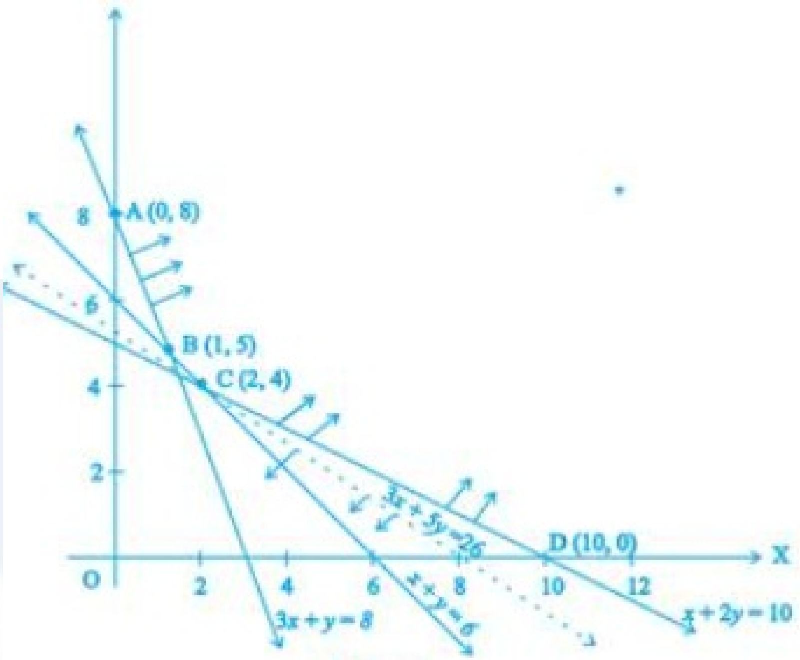
$\therefore$ E(X) = 0 + 0.25 + 0.6 + 0.6 + 0.60 = 2.05

$E(X^2)$ = 0 + 0.25 + 1.2 + 1.8 + 2.40 = 5.65

i. $V\left(\frac{X}{2}\right) = \frac{1}{4}V(X) = \frac{1}{4}[5.65 - (2.05)^2]$
   $\frac{1}{4}[5.65 - 4.2025] = \frac{1}{4} \times 1.4475 = 0.361875$
ii. V(X) = 1.44475

32. We first draw the graphs of x + 2y = 10, x + y = 6, 3x + y = 8. The shaded region ABCD is the feasible region (R) determined by the above constraints. The feasible region is unbounded. Therefore, minimum of Z may or may not occur. If it occurs, it will be on the corner point.

Corner Point	Value of Z
A(0, 8)	40
B(1, 5)	28
C(2, 4)	26 (smallest)
D(10, 0)	30



Let us draw the graph of 3x + 5y < 26 as shown in Fig by dotted line.

We see that the open half plane determined by 3x + 5y < 26 and R do not have a point in common. Thus, 26 is the minimum value of Z.

Section D

33. For n = 1

$$A' = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^{\circ} & 3^{\circ} & 3^{\circ} \\ 3^{\circ} & 3^{\circ} & 3^{\circ} \\ 3^{\circ} & 3^{\circ} & 3^{\circ} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Result is true for n = 1

Let it be true for n = k

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Therefore $A^{k+1} = A. A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Thus result is true for n = k+1

Whenever it is true for n = k.

Hence proved.

OR

Multiplying R_1, R_2 and R_3 by a, b, c respectively

$$\text{L.H.S} = \frac{1}{abc} \begin{vmatrix} a^3 + a & a^2 b & a^2 c \\ ab^2 & b^3 + b & b^2 c \\ c^2 a & c^2 b & c^3 + c \end{vmatrix}$$

Taking a, b, c, common from c_1, c_2, and c_3

$$= \frac{abc}{abc} \begin{vmatrix} a^2 + 1 & a^2 & a^2 \\ b^2 & b^2 + 1 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix}$$

$$R_1 \to R_1 + R_2 + R_3$$

$$= \begin{vmatrix} 1 + a^2 + b^2 + c^2 & 1 + a^2 + b^2 + c^2 & 1 + a^2 + b^2 + c^2 \\ b^2 & b^2 + 1 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix}$$

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + 1 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix}$$

$C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b^2 \\ -1 & -1 & c^2 + 1 \end{vmatrix}$$

Expanding along R_1

$$= (1 + a^2 + b^2 + c^2)[1(0 + 1)]$$

$$= 1^2 + a^2 + b^2 + c^2$$

L.H.S = R.H.S

34. According to the question ,

Given equation of circle is $x^2 + y^2 = 16$ ...(i)

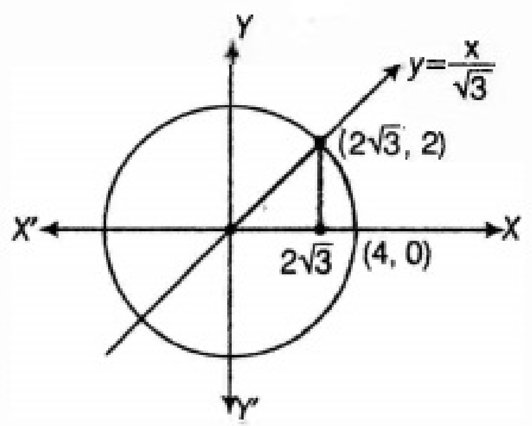
Equation of line given is ,

$\sqrt{3}y = x$ ...(ii)
$\Rightarrow y = \frac{1}{\sqrt{3}}x$ represents a line passing through the origin.
To find the point of intersection of circle and line ,

substitute eq. (ii) in eq.(i) , we get
$x^2 + \frac{x^2}{3} = 16$
$\frac{3x^2 + x^2}{3} = 16$
$\Rightarrow 4x^2 = 48$

$\Rightarrow x^2 = 12$

$\Rightarrow$ x= $\pm 2\sqrt{3}$
When x= $2\sqrt{3}$, then $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$



Required area (In first quadrant) = (Area under the line y = $\frac{1}{\sqrt{3}}$ x from x = 0 to $2\sqrt{3}$) + (Area under the circle from x = $2\sqrt{3}$ to x=4)

$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}} x dx + \int_{2\sqrt{3}}^{4} \sqrt{16-x^2} dx$

$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16-x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^{4}$

$= \frac{1}{2\sqrt{3}}\left[(2\sqrt{3})^2 - 0\right] + \left[0 + 8\sin^{-1}(1) - \frac{2\sqrt{3}}{2}\sqrt{16-12} - 8\sin^{-1}\left(\frac{2\sqrt{3}}{4}\right)\right]$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - \frac{2\sqrt{3}}{2} \times 2 - 8\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - 8\left(\frac{\pi}{3}\right)$

$= 4\pi - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3}$

$= \frac{4\pi}{3}$ sq units.

35. Let AC = x, BC = y and r be the radius of circle.

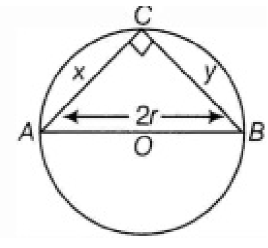
Also, $\angle C = 90^\circ$ [∵ angle made in semi-circle is 90°]

In ΔABC, we have

$(AB)^2 = (AC)^2 + (BC)^2$

$(2r)^2 = (x)^2 + (y)^2$

$4r^2 = x^2 + y^2$ ...(I)



We know that,

Area of ΔABC, $(A) = \frac{1}{2}\ x \cdot y$

On squaring both sides, we get

$A^2 = \frac{1}{4}x^2y^2$

Let $A^2 = S$

Then, $S = \frac{1}{4}x^2y^2$

$\Rightarrow \quad S = \frac{1}{4}x^2\left(4r^2 - x^2\right)$ [from Eq. (i)]

$\Rightarrow \quad S = \frac{1}{4}\left(4x^2r^2 - x^4\right)$

On differentiating both sides w.r.t.x, we get

$\frac{dS}{dx} = \frac{1}{4}\left(8r^2x - 4x^3\right)$

For maxima or minima, put $\frac{dS}{dx} = 0$

$\therefore \quad \frac{1}{4}\left(8r^2x - 4x^3\right) = 0$

$8r^2x = 4x^3$

$8r^2 = 4x^2$

$x^2 = 2r^2$

x = $\sqrt{2}r$

From Eq. (i), we get,

$y^2 = 4r^2 - 2r^2 = 2r^2 \Rightarrow y = \sqrt{2}r$

Here, x = y so triangle is an isosceles.

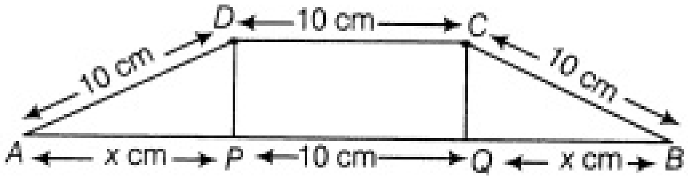
Also, $\frac{d^2S}{dx^2} = \frac{d}{dx}\left[\frac{1}{4}\left(8r^2x - 4x^3\right)\right] = \frac{1}{4}\left(8r^2 - 12x^2\right)$

$= 2r^2 - 3x^2$ m

At $x = \sqrt{2}r, \frac{d^2S}{dx^2} = 2r^2 - 3(2r^2) = -4r^2 < 0$

Therefore, Area of the triangle is maximum when it is an isosceles triangle.

OR



Let ABCD be the given trapezium in which AD = BC = CD = 10 cm.

Draw a perpendicular DP and CQ on AB. Let AP = x cm

In ΔAPD & ΔBQC,

$\angle APD = \angle BQC$ [each =90°]

AD = BC [Both 10 cm]

DP = CQ [Perpendicular between parallel lines are equal in length]

$\therefore \quad \Delta APD \cong \Delta BQC$ [RHS Congruency]

$\therefore QB = AP$ [CPCT]

$\Rightarrow QB = x\ cm$

$DP = \sqrt{10^2 - x^2}$ [by Pythagoras theorem]

Now, area of trapezium,

$A = \frac{1}{2} \times (\text{ sum of parallel sides }) \times height$

$= \frac{1}{2} \times (2x + 10 + 10) \times \sqrt{100 - x^2}$

$= (x + 10)\sqrt{100 - x^2}$.....(i)

We need to find the area of trapezium when it is maximum i.e. we need to maximize area.

On differentiating both sides of eq(i) w.r.t.x, we get

$\frac{dA}{dx} = (x + 10)\frac{(-2x)}{2\sqrt{100 - x^2}} + \sqrt{100 - x^2}$

$= \frac{-x^2 - 10x + 100 - x^2}{\sqrt{100 - x^2}}$

$= \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}}$....(ii)

For maxima or minima, put $\frac{dA}{dx} = 0$

$\Rightarrow \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}} = 0$

= -2 (x^2 + 5x - 50) =0

= -2 (x+10) (x - 5) =0

x = 5 or -10

Since, x represents distance, so it cannot be negative.

Therefore, we take x = 5.

On diffentiating both sides of eq.(ii) w.r.t.x, we get

$\frac{d^2 A}{dx^2} = \frac{\sqrt{100 - x^2} \cdot \frac{d}{dx}(-2x^2 - 10x + 100) - (-2x^2 - 10x + 100)\frac{d}{dx}(\sqrt{100 - x^2})}{(\sqrt{100 - x^2})^2}$ [by using the quotient rule of derivative]

$$= \frac{\sqrt{100-x^2}\cdot(-4x-10)-(-2x^2-10x+100)\left(\frac{-2x}{2\sqrt{100-x^2}}\right)}{(\sqrt{100-x^2})^2}$$

$$= \frac{\sqrt{100-x^2}\cdot(-4x-10)+\frac{x(-2x^2-10x+100)}{\sqrt{100-x^2}}}{100-x^2}$$

$$= \frac{(100-x^2)\cdot(-4x-10)+x(-2x^2-10x+100)}{(100-x^2)^{\frac{3}{2}}}$$

$$= \frac{-400x+4x^3-1000+10x^2-2x^3-10x^2+100x}{(100-x^2)^{\frac{3}{2}}}$$

$$\therefore \frac{d^2A}{dx^2} = \frac{2x^3-300x-1000}{(100-x^2)^{3/2}}$$

When x = 5,

$$\frac{d^2A}{dx^2} = \frac{2(5)^3-300(5)-1000}{\left[100-(5)^2\right]^{3/2}}$$

$$= \frac{250-1500-1000}{(100-25)^{3/2}} = \frac{-2250}{75\sqrt{75}} < 0$$

$\therefore$ It is maximum when x = 5

Thus, area of trapezium is maximum at x = 5 and maximum area is

$A_{\max} = (5+10)\sqrt{100-(5)^2}$ [put x = 5 in Eq. (i)]

$= 15\sqrt{100-25} = 15\sqrt{75} = 75\sqrt{3}\ \text{cm}^2$

36. The equation of the plane containing the given point is

A(x - 1) + B(y + 1) + C(z - 2) = 0....[i]

Condition of $\perp$ to the plane given in (i) with the planes

2x + 3y - 2z = 5, x + 2y - 3z = 8 implies,

2A + 3B - 2C = 0

A + 2B - 3C = 0

On solving these equations we get

A = -5, B = 4, C = 1

Required equation of the plane is:-

5x - 4y - z = 7