# CBSE Class 12 - Mathematics
## Sample Paper 05

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**Maximum Marks: 80**
**Time Allowed: 3 hours**

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**General Instructions:**

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.



### Section A

1. Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ -1 & 6 & 1 \end{bmatrix}$, then adj (A) is

   a. $\begin{bmatrix} 2 & -5 & 32 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix}$

   b. $\begin{bmatrix} 2 & -25 & -32 \\ 0 & 2 & -36 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 0 \\ -25 & 2 & 0 \\ -32 & 36 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ 32 & -6 & 2 \end{bmatrix}$

2. The value of the determinant of a skew symmetric matrix of even order is

   a. A non zero perfect square

   b. None of these

   c. 0

   d. Negative

3. $\frac{d}{dx}$(log $|x|$)is equal to $(x \neq 0)$

   a. $\pm\frac{1}{x}$

   b. $\frac{1}{x}$ or $-\frac{1}{x}$

   c. $\frac{1}{|x|}$

   d. $\frac{1}{x}$

4. If x + |y| = 2y, then y as a function of x is

   a. differentiable for all x

   b. not continuous at x = 0

   c. such that $\frac{dy}{dx} = \frac{1}{3}$ for x < 0

   d. not defined for all real x

5. General solution of $\left(1+x^2\right)dy + 2xy\ dx = \cot x\ dx\ (x \neq 0)$ is

   a. $y(1+x^2) = log|sinx| + c$

b. $y = (1+x)^{-1} \log|\sin x| - C(1+x^2)^{-1}$

c. $y = (1+x)^{-1} \log|\sin x| + C(1-x^2)^{-1}$

d. $y = (1+x)^{-1} \log|\sin x| - C(1-x^2)^{-1}$

6. If $\cot^{-1}(\sqrt{\cos\alpha}) + \tan^{-1}(\sqrt{\cos\alpha}) = \mu$, then sinμ is equal to

   a. $\tan^2\alpha$

   b. $tan\ 2\alpha$

   c. $\cot^2\left(\frac{\alpha}{2}\right)$

   d. 1

7. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

   a. $\frac{1}{5}$

   b. $\frac{1}{3}$

   c. $\frac{2}{3}$

   d. $\frac{1}{4}$

8. If f (x) = f (a –x), then , $\int_0^a x f(x) dx$ is equal to

   a. $\frac{a}{2}\int_0^a f(x) dx$

   b. $\int_0^a f(x) dx$

   c. $\frac{a^2}{2}\int_0^a f(x) dx$

d. $-\frac{a^2}{2}\int_0^a f(x)dx$

9. The vector and cartesian equations of the planes that passes through the point (1, 0, –2) and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}$

a. $\left[\vec{r}-\left(\hat{i}+2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0; x+y-z=7$

b. $\left[\vec{r}-\left(\hat{i}-5\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0; x-y-z=3$

c. $\left[\vec{r}+\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0; x+y-z=5$

d. $\left[\vec{r}-\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0x+y-z=3$

10. If $\theta$ is the angle between vectors $\vec{a}\ and\ \vec{b}$ then the cross product $\vec{a}\times\vec{b}=$

a. $2\left|a\right|\left|b\right|\sin\theta\hat{n}$

b. $\left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right|\sin\theta\hat{n}$

c. $\left|a\right|\left|b\right|\sin\theta$

d. $\left|a\right|\left|b\right|\cos\theta$

11. Fill in the blanks:

A relation R defined on a set A is said to be ________, if (x, y) ∈ R and (y, z) ∈ R ⇒ (x, z) ∈ R, where x, y, z ∈ A

12. Fill in the blanks:

When two coins are tossed simultaneously, the chances of getting atleast one tail is ________.

13. Fill in the blanks:

Matrix multiplication is ________ over addition.

14. Fill in the blanks:

The value of integral $\int \frac{1+\cos x}{x+\sin x} dx$.

**OR**

Fill in the blanks:

The intergral value of $\int\limits_{-a}^{a} f(x) dx = 0$ .

15. Fill in the blanks:

    A feasible region of a system of linear inequalities is said to be ________ , if it can be enclosed within line(s).

**OR**

Fill in the blanks:

In a LPP, the linear function which has to be maximised or minimised is called a linear ________ function.

16. If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix}$ = 3, then find the value of x.

17. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

18. Write the value of $\int_0^1 \frac{e^x}{1+e^{2x}} dx$.

**OR**

Evaluate the following integral $\int (2x - 3\cos x + e^x)\, dx$

19. Find the equation of the tangent to the curve $y = x + \frac{4}{x^2}$ which is parallel to the X-axis.

20. Evaluate the product $\left(2\vec{a} - 5\vec{b}\right) . \left(2\vec{a} + 7\vec{b}\right)$

**Section B**

21. Find gof and fog, if f : R$\rightarrow$ R and g : R$\rightarrow$R are given by f (x)= $|x|$ and g(x) = $|5x - 2|$

22. Verify Rolle's theorem for the function f(x) = $x^2$ -5 x + 6 on the interval [2,3].

**OR**

Find $\frac{dy}{dx}$, if $x^3+x^2y+xy^2+y^3=81$

23. $\vec{a}$ Is unit vector and $(\vec{x}-\vec{a})(\vec{x}+\vec{a})=8$. Then find $|\vec{x}|$

24. Find the values of x for which the function ,

$f(x)=kx^3-9x^2+9x+3$ is increasing in R

**OR**

The length x of a rectangle is decreasing at the rate of 3 cm/ min and the width y is increasing at the rate of 2cm/min. when x = 10cm and y = 6cm, find the ratio of change of (a) the perimeter (b) the area of the rectangle.

25. Find the equation of a plane trough the points (2,3,1) and (4,-5,3) and parallel to x-axis.

26. A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (one by one with replacement) at random, what is the probability that there will be at least one defective watch?

**Section C**

27. Prove that $\tan^{-1}\frac{yz}{xr}+\tan^{-1}\frac{zx}{yr}+\tan^{-1}\frac{xy}{zr}=\frac{\pi}{2}$, where x , y , z > 0 such that $x^2+y^2+z^2=r^2$

28. If y = $(\sin x-\cos x)^{(\sin x-\cos x)}$, $\frac{\pi}{4}<x<\frac{3\pi}{4}$, then find $\frac{dy}{dx}$.

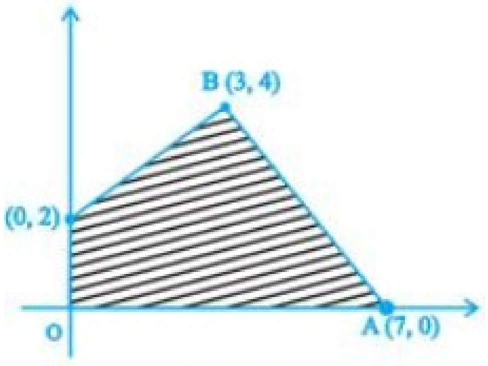
**OR**

Differentiate $\sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$

29. In a hockey match, both teams A and B scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared

the winner. If the captain of team A was asked to start, then find their respective probabilities of winning the match and state whether the decision of the referee was fair or not.

30. Feasible region (shaded) for a LPP is shown in Fig. Maximise Z = 5x + 7y.




31. Find the solution of diff . equation. $(1 + x^2)dy + 2xy\,dx = \cot x\,dx$

**OR**

Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose diff eq. is sinx cosy dx + cosx.siny dy = 0.

32. Evaluate $\int \frac{\sin 2x}{a^2+b^2 \sin^2 x} dx$

## Section D

33. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third number we get double of the second number. Find the numbers using matrix method.

**OR**

Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

34. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, with its vertex at one end of the major axis.

35. Find the area bounded by the curve $y -x|x|$, x-axis and the ordinates $x = -3$ and $x = 3$. Find the equation of the curve?

**OR**

Find the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

36. Find the distance between the point (7, 2, 4) and the plane determined by the points A(2, 5, -3), B(-2, -3, 5) and C(5, 3, -3).

## Solution
## Section A

1. (d)

$$\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ 32 & -6 & 2 \end{bmatrix}$$

**Explanation:**

adj. A = $\begin{bmatrix} 2 & -5 & 32 \\ 0 & 1 & -6 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 0 \\ 32 & -6 & 2 \end{bmatrix}$

2. (a) A non zero perfect square

**Explanation:**

The determinant of a skew symmetric matrix of even order is A non zero perfect square and odd order is equal to 0.

3. (d)

$\frac{1}{x}$

**Explanation:**

$\frac{d}{dx}(\log|x|) = \frac{1}{|x|}\frac{x}{|x|} = \frac{x}{|x|^2} = \frac{x}{x^2} = \frac{1}{x}$

4. (c)

such that $\frac{dy}{dx} = \frac{1}{3}$ for x < 0

**Explanation:**

$x + |y| = 2y \Rightarrow x = 2y - |y|$ when

$y \geqslant 0\ then\ x = 2y - y \Rightarrow x = y\ for\ y \geqslant 0\ i.e. y = x\ for\ x \geqslant 0$ and when $y < 0, then\ x = 2y - (-y) \Rightarrow x = 3y\ for\ y < 0\ i.e.\ y = 1/3\ x\ for\ \ x < 0$. We have , $y = \begin{cases} x & , x \geqslant 0 \\ \frac{1}{3}x & , x < 0 \end{cases}$

5. (a)

   $y(1+x^2) = log|sinx| + c$

   **Explanation:**

   $(1+x^2)dy = (cotx - 2xy)dx$

   $\frac{dy}{dx} = \frac{cotx-2xy}{1+x^2}$

   $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{cotx}{1+x^2}$

   It is a linear differential equation in y.

   Therefore, Solution is

   $ye^{\int \frac{2xdx}{1+x^2}} = \int \frac{cotx}{1+x^2}e^{\int \frac{2xdx}{1+x^2}}dx + c$

   $y(1+x^2) = \int \frac{cotx}{1+x^2}(1+x^2)dx + c$

   $y(1+x^2) = \int cotxdx + c$

   $y(1+x^2) = log|sinx| + c$

6. (d) 1

   **Explanation:**

   $\cot^{-1}(\sqrt{\cos\alpha}) + \tan^{-1}(\sqrt{\cos\alpha}) = \mu$

   Let $\sqrt{\cos\alpha} = \theta$

   $\cot^{-1}\theta + \tan^{-1}\theta = \mu \implies \frac{\pi}{2} = \mu$

   $\therefore sin\mu = \sin\frac{\pi}{2} = 1.$

7. (c)

$\frac{2}{3}$

**Explanation:**

Let $E_1$ and $E_2$ are events of selection of the first and second bag respectively

$\therefore P(E_1) = P(E_2) = \frac{1}{2}$

Let A = event of getting a red ball.

$\therefore P\left(\mathrm{A}/E_1\right) = \frac{4}{8} = \frac{1}{2},$

$P\left(\mathrm{A}/\mathrm{E}_2\right) = \frac{2}{8} = \frac{1}{4}$

$\Rightarrow P\left(E_1/\mathrm{A}\right)$

$= \frac{P(E_1)P(\mathrm{A}/E_1)}{P(E_1)P(\mathrm{A}/E_1)+P(E_2)P(\mathrm{A}/E_2)}$

$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}} = \frac{2}{3}$

8. (a)

$\frac{a}{2}\int\limits_0^a f(x)dx$

**Explanation:**

$I = \int\limits_0^a xf(x)dx$....(1)

$\Rightarrow I = \int\limits_0^a (a-x)f(a-x)dx \Rightarrow \int\limits_0^a f(a-x)dx$....(2)

Adding (1) and (2),

$2I = \int\limits_0^a af(x)dx \Rightarrow I = \frac{a}{2}\int\limits_0^a f(x)dx.$

9. (d)

$$\left[\vec{r}-\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0x+y-z=3$$

**Explanation:**

Let $\overrightarrow{a}$ be the position vector of the point ( 1, 0 , - 2 ) $\therefore$ $\overrightarrow{a} = \hat{i}+0\hat{j}-2\hat{k}$, here, $\therefore$ $\overrightarrow{n} = \hat{i}+\hat{j}-\hat{k}$ . Therefore, the required vector equation of the plane is:
$\overrightarrow{r}\,.\,\overrightarrow{n} = \overrightarrow{a}\,.\,\overrightarrow{n}$

$\Rightarrow \overrightarrow{r}.(\hat{i}+\hat{j}-\hat{k})$

$=(\hat{i}+0\hat{j}-2\hat{k}).(\hat{i}+\hat{j}-\hat{k})$

$\Rightarrow \overrightarrow{r}.(\hat{i}+\hat{j}-\hat{k})=3$

Putting $\overrightarrow{r} = x\hat{i}+y\hat{j}+z\hat{k}$, we get

$(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}-\hat{k})=3$

$\Rightarrow x+y-z=3$

10. (b)

$$\left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right|\sin\theta\hat{n}$$

**Explanation:**

If $\theta$ is the angle between vectors $\vec{a}\ and\ \vec{b}$ then, the cross product : $\vec{a}\times\vec{b}=$ $\left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right|\sin\theta\hat{n}$ .

11. transitive

12. $\frac{3}{4}$

13. distributive

14. $\log|x+\sin x|+C$

**OR**

True

15. bounded

**OR**

objective

16. We have,

$\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix}$ = 3

$\Rightarrow$ 2x(x - 2) - (- 3) $\times$3x = 3

$\Rightarrow$ 2x(x - 2) + 9x = 3

$\Rightarrow$ $2x^2$ - 4x + 9x = 3

$\Rightarrow$ $2x^2$ + 5x - 3 = 0

$\Rightarrow$ (2x - 1)(x + 3) = 0

$\Rightarrow$ 2x - 1 = 0 or, x + 3 = 0 $\Rightarrow$ x = $\frac{1}{2}$, -3

17. According to the question, equation of line is

$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

It can be rewritten as

$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$

Here, Direction ratios of the line are (-2, 6, -3).

:. Direction cosines of the line are

$\frac{-2}{\sqrt{(-2)^2+6^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+6^2+(-3)^2}}$

$\frac{-3}{\sqrt{(-2)^2+6^2+(-3)^2}}$ i.e. $\frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}$ and $\frac{-3}{\sqrt{49}}$

Thus, Direction cosines of line are $\left(-\frac{2}{7}, \frac{6}{7}, -\frac{3}{7}\right)$

18. Let $I = \int_0^1 \frac{e^x}{1+e^{2x}} dx = \int_0^1 \frac{e^x}{1+(e^x)^2} dx$

Put $e^x = t \Rightarrow e^x dx = dt$

Also, when x = 0, then t = 1 and when x = 1, then t = e

Now, $I=\int_1^e \frac{dt}{1+t^2}=\left(\tan^{-1}t\right)_1^e$
$=\tan^{-1}e-\tan^{-1}1=\tan^{-1}\left(\frac{e-1}{1+e}\right)$

**OR**

$\int(2x-3\cos x+e^x)\,dx$

$=\int 2xdx-\int 3\cos xdx+\int e^x dx$

$=2\int xdx-3\int \cos xdx+\int e^x dx$

$=2\frac{x^2}{2}-3\sin x+e^x+c$

= $x^2$ - 3 sin x + $e^x$ + c

19. We have, $y=x+\frac{4}{x^2}$ .........(1)

$\Rightarrow y = x + 4x^{-2}$

On differentiating w.r.t x, we get,

$\frac{dy}{dx} = 1 + 4\times(-2x^{-3}) = 1 - 8x^{-3}=1-\frac{8}{x^3}$
$\frac{dy}{dx}=1-\frac{8}{x^3}$

Since the tangent is parallel to X-axis, therefore

$\frac{dy}{dx}=0$

$\Rightarrow 1-\frac{8}{x^3}=0$

$\Rightarrow x^3=8$

$\Rightarrow x=2$

From (1), when x = 2 , we get , y=$2+\frac{4}{4}=2+1=3$

Therefore, y=3 is required equation.

20. $\left(2\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$

$4\vec{a}.\vec{a}+4\vec{a}.\vec{b}-35\vec{b}.\vec{b}$

$4|\vec{a}|^2+4\vec{a}.\vec{b}-35\left|\vec{b}\right|^2\left[\because \vec{a}\vec{a}=|\vec{a}|^2, \vec{b}.\vec{b}=\left|\vec{b}\right|, \vec{a}.\vec{b}=\vec{b}.\vec{a}\right]$

**Section B**

21. Clearly,

gof (x ) = g (f(x)) = $g(|x|)=|5|x|-2|=\begin{cases} |5x-2|, \text{ if } x\geq 0 \\ |-5x-2|, \text{ if } x<0 \end{cases}$

and, fog (x) = f(g(x)) = $f(|5x-2|)=||5x-2||=|5x-2|$

22. Since* a polynomial function is everywhere differentiable and so continuous also. Therefore, f(x) is continuous on [2,3] and differentiable on (2,3).

Also, f(2) = $2^2$ - 5 $\times$ 2 + 6 = 0 and f(3) = $3^2$ - 5 $\times$ 3 + 6 = 0

$\therefore$ f(2) = f(3)

Thus, all the conditions of Rolle's theorem are satisfied. Now, we have to show that there exists some c $\in$ (2, 3) such that f'(c) = 0.

For this, we proceed as follows.

We have,

f(x) = $x^2$ - 5x + 6 $\Rightarrow$ f'(x) = 2x - 5

$\therefore$ f'(x) = 0 $\Rightarrow$ 2x - 5 = 0 $\Rightarrow$ x = 2.5

Thus, c = 2.5 $\in$ (2, 3) such that f'(c) = 0. Hence, Rolle's theorem is verified.

OR

Given: $x^3+x^2y+xy^2+y^3=81$

$\Rightarrow \frac{d}{dx}x^3+\frac{d}{dx}x^2y+\frac{d}{dx}xy^2+\frac{d}{dx}y^3=\frac{d}{dx}81$

$\Rightarrow 3x^2+\left(x^2\frac{dy}{dx}+y.\frac{d}{dx}x^2\right)+x\frac{d}{dx}y^2+y^2\frac{d}{dx}x+3y^2\frac{dy}{dx}=0$

$\Rightarrow 3x^2+x^2\frac{dy}{dx}+y.2x+x.2y\frac{dy}{dx}+y^2.1+3y^2\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}\left(x^2+2xy+3y^2\right)=-3x^2-2xy-y^2$

$\Rightarrow \frac{dy}{dx}=\frac{-\left(3x^2+2xy+y^2\right)}{x^2+2xy+3y^2}$

23. $|\vec{a}| = 1$

$(\vec{x} - \vec{a}) . (\vec{x} + \vec{a}) = 8$

$|\vec{x}|^2 - |\vec{a}|^2 = 8$

$|\vec{x}|^2 - 1 = 8$

$|\vec{x}|^2 = 9$

$|\vec{x}| = 3$

24. we have, f(x)=$kx^3 - 9x^2 + 9x + 3$

$\Rightarrow f'(x) = 3kx^2 - 18x + 9$

Since f(x) is increasing on R, therefore , $f'(x) > 0 \, \forall \, x \in R$

$\Rightarrow \; 3kx^2 - 18x + 9 > 0, \forall \, x \in \; R$
$\Rightarrow \; kx^2 - 6x + 3 > 0, \forall \, x \in \; R$

$\Rightarrow$ k>0 and 36-12k<0
$[\because \; ax^2 + bx + c > 0, \forall \, x \in \; R \Rightarrow \; a > 0 \; and \; discriminant < 0]$

$\Rightarrow \; k > 3$

Hence,f(x) is increasing on R, if k>3.

**OR**

Given $\frac{dx}{dt} = -3cm/\min$, $\frac{dy}{dt} = 2cm/\min$

a. Let P be the perimeter
$P = 2(x + y)$
$\frac{dp}{dx} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$
$= 2(-3 + 2)$
$= -2cm/\min$ (i.e perimeter is decreasing)
b. Now area of rectangle A = xy
$\frac{dy}{dt} = x\frac{dy}{dx} + y.\frac{dx}{dt}$

$$= 10\,(2) + 6\,(-3)$$
$$= 20 - 18$$
$$= 2cm^2/\text{min}$$

25. Any plane parallel to x-axis is by+cz+d=0.

    If it passes through (2,3,1) and (4,-5,3), then

    3b+c+d=0 and -5b+3c+d=0,

    i.e , $\frac{b}{1-3} = \frac{c}{-5-3} = \frac{d}{9+5}$

    i.e, $\frac{b}{1} = \frac{c}{4} = \frac{d}{-7}$

    Hence, the plane parallel to x-axis is y+4z-7=0.

26. Probability of defective watch from a lot of 100 watches $= \frac{10}{100} = \frac{1}{10}$
    $\therefore p = \frac{1}{10}, q = \frac{9}{10}, n = 8$ and $r \geqslant 1$
    $\therefore P(r \geqslant 1) = 1 - P(r = 0) = 1 - {}^8C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{8-0}$
    $= 1 - \frac{8!}{0!8!} \cdot \left(\frac{9}{10}\right)^8 = 1 - \left(\frac{9}{10}\right)^8$

**Section C**

27. We have, $x^2 + y^2 + z^2 = r^2$
    Also, $\frac{yz}{xr} \times \frac{zx}{yr} = \frac{z^2}{r^2} = \frac{z^2}{x^2+y^2+z^2} < 1$
    $\therefore \tan^{-1}\frac{yz}{xr} + \tan^{-1}\frac{xz}{yr} + \tan^{-1}\frac{xy}{zr}$
    $= \tan^{-1}\left\{\frac{\frac{yz}{xr} + \frac{xz}{yr}}{1 - \frac{yz}{xr} \times \frac{zx}{yr}}\right\} + \tan^{-1}\frac{xy}{zr}$
    $= \tan^{-1}\left\{\frac{\frac{z\left(x^2+y^2\right)}{xyr}}{1 - \frac{z^2}{r^2}}\right\} + \tan^{-1}\left(\frac{xy}{zr}\right) = \tan^{-1}\left\{\frac{z\left(x^2+y^2\right)}{xyr} \times \frac{r^2}{x^2+y^2}\right\} + \tan^{-1}\left(\frac{xy}{zr}\right)$
    $= \tan^{-1}\left(\frac{zr}{xy}\right) + \tan^{-1}\left(\frac{xy}{zr}\right) = \cot^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{xy}{zr}\right) = \frac{\pi}{2}$

28. We have, $y = (\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$,

    Therefore, on taking logarithm both sides, we get, log y = log (sin x - cos x)$^{(\text{sin x - cos x})}$,

$\Rightarrow$ log y= (sin x - cos x). log(sin x - cos x)

Therefore,on differentiating both sides w.r.t x, we get,

$\frac{1}{y}\cdot\frac{dy}{dx} = (\sin x - \cos x)\times\frac{d}{dx}\log(\sin x - \cos x)$ + log(sin x - cos x) $\times\frac{d}{dx}(\sin x - \cos x)$ [By using product rule of derivative]

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx}$ = (sin x - cos x) $\frac{1}{(\sin x - \cos x)}\ \frac{d}{dx}(\sin x - \cos x)$ + log(sin x - cos x).(cos x + sin x)

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\sin x - \cos x)\frac{1}{(\sin x - \cos x)}$ (cos x + sin x) + log(sin x - cos x).(cos x + sin x)

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)$+(cos x + sin x) + log(sin x - cos x)

$\Rightarrow \quad \frac{dy}{dx}$ = y(cos x + sin x)[ 1+log(sin x - cos x)]

$\therefore \quad \frac{dy}{dx}$ = (sin x - cos x)$^{(\text{sin x - cos x})}$(cos x + sin x)[ 1+log(sin x - cos x)]

**OR**

$$y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$$

$$= \sin^{-1}\left[\frac{2^x.2}{1+(2^x)^2}\right]$$

Put $2^x = \tan\theta$

$$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$y = 2.\tan^{-1}2^x$$

$$\frac{dy}{dx} = 2.\frac{1}{1+(2^x)^2}\cdot\frac{d}{dx}(2^x)$$

$$= \frac{2}{1+4^x}.2^x.\log 2$$

$$= \frac{2^{x+1}\log 2}{1+4^x}$$

29. Let $E_1$ = Event of A getting six.

and $E_2$=Event of B getting six

In throwing a die,

Total number of elements in sample space,

n(s) = 6

$\therefore \quad P(E_1) = P(E_2) = \frac{1}{6}$

and probability of not getting a six

$P\left(\overline{E_1}\right) = P\overline{E_2}) = 1 - \frac{1}{6} = \frac{5}{6}$

Since, the referee gives first chance to captain A for throwing a die.

$\therefore$ Probability of A winning

$= P(E_1) + P\left(\overline{E}_1 \cap \overline{E}_2 \cap E_1\right) + \ldots$

$= P(E_1) + P\left(\overline{E_1}\right) P\overline{(E_2)} P(E_1) + \ldots$

[$\because$ $E_1$ and $E_2$ are independent events, then their complements are also independent]

$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \ldots\right]$

$= \frac{1}{6}\left(\frac{1}{1-\frac{25}{36}}\right)$ [$\because$ sum of an infinite GP

$S_\infty = \frac{a}{1-r}$ Here, a = 1 and $r = \frac{25}{36}$]

$= \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$

Probability of B winning

$= P\left(\overline{E}_1 \cap E_2\right) + P\left(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_1 \cap E_2\right) + \ldots\ldots$

$= P\left(\overline{E}_1\right) P(E_2) + P\left(\overline{E}_1\right) \quad P\left(\overline{E}_2\right) \cdot P\left(\overline{E}_1\right) P(E_2) + \ldots$

$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots$

$= \frac{5}{36}\left[1 + \left(\frac{5}{6}\right)^2 + \ldots\right]$ [ .:series is an infinite GP ]

$= \frac{5}{36}\left(\frac{1}{1-\frac{25}{36}}\right) = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$

Here, we see that P(A) > P(B).

Hence, team A has more chance of winning the match.

As the referee first give a chance to team A, so it is not a fair decision

30. The Shaded region is bounded and has coordinate of corner points as (0, 0), (7, 0), (3,

4) and (0, 2). Also, Z = 5x + 7y.

| Corner Points | Corresponding value of Z |
|---|---|
| (0, 0) | 0 |
| (7, 0) | 35 |
| (3, 4) | 43 (Maximum) |
| (0, 2) | 14 |

Hence, the maximum value of Z is 43 at (3, 4).

31. $(1 + x^2)dy + 2xy\,dx = \cot x\,dx$

$\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{\cot x}{1+x^2}$

Given diff. eq is of the form

$\frac{dy}{dx} + Py = Q$

$P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$

$I.F = e^{\int pdx}$

$= e^{\int \frac{2x}{1+x^2}}$

$= e^{\log\left(1+x^2\right)}$

$= 1 + x^2$

Solution is,

$y \times \left(1 + x^2\right) = \int \frac{\cot x}{1+x^2} \times \left(1 + x^2\right) dx + c$

$y\left(1 + x^2\right) = \log(\sin x) + c$

$y = \frac{\log(x)}{1+x^2} + \frac{c}{1+x^2}$

**OR**

Given diff eq. is sinx cosy dx + cosx.siny dy = 0.

sinx cosy dx = - cosx.siny dy

$\Rightarrow \int \frac{sinx}{cosx} dx = -\int \frac{siny}{cosy} dy$

$\Rightarrow \int \tan x dx = -\int \tan y dy$

$\Rightarrow$ log(sec x) = -log(sec y) + log c

$\Rightarrow$ log(sec x.sec y) = log c

secx.secy = c .......(1)

when x = 0, $y = \frac{x}{4}$ , therefore we get,

$c = \sqrt{2}$

put the value of c in (1), we get,

$\sec x . \sec y = \sqrt{2}$

32. Let $I = \int \frac{\sin 2x}{a^2+b^2 \sin^2 x} dx$ ...(i)

Let $a^2 + b^2 \sin^2 x = t$ then,

$d(a^2 + b^2 \sin^2 x) = dt$

$\Rightarrow b^2 (2 \sin x \cos x)\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{b^2(2 \sin x \cos x)}$

$= \frac{dt}{b^2 \sin 2x}$

Putting $a^2 + b^2 \sin^2 x = t$ and $dx = \frac{dt}{b^2 \sin 2x}$ in equation (i), we get

$I = \int \frac{\sin 2x}{t} \times \frac{dt}{b^2 \sin 2x}$

$= \frac{1}{b^2} \int \frac{dt}{t}$

$= \frac{1}{b^2} \log|t| + c$

$= \frac{1}{b^2} \log |a^2 + b^2 \sin^2 x| + c$

**Section D**

33. Let the three numbers be x,y and z.Then,

x + y + z = 6

y + 3z = 11

x + z = 2y

This system can be written as AX = B whose

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$|A| = 9 \neq 0$$

$$A_{11} = 7, A_{12} = 3, A_{13} = -1$$

$$A_{21} = -3, A_{22} = 0, A_{23} = 3$$

$$A_{31} = 2, A_{32} = -3, A_{33} = 1$$

$$adjA = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x=1, y=2, z=3

**OR**

We have, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times2} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a+6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

⇒ -6a - 3c + 10b + 5d = 1 ....(i)

⇒ 4a + 2c - 6b - 3d = 0 ....(ii)

⇒ -9a - 6c + 15b + 10d = 0 ....(iii)

⇒ 6a + 4c - 9b - 6d = 1 .......(iv)

On adding Eqs. (i) and (iv), we get

c + b - d = 2 ⇒ d = c + b - 2 ....(v)

On adding Eqs. (ii) and (iii), we get

-5a - 4c + 9b + 7d = 0 ....(vi)

On adding Eqs. (vi) and (iv), we get

a + 0 + 0 + d = 1 ⇒ d = 1 - a ....(vii)

From Eqs. (v) and (vii)

⇒ c + b - 2 = 1 - a ⇒ a + b + c = 3 ....(viii)

⇒ a = 3 - b - c

Now, using the values of a and d in Eq. (iii) , we get

-9(3 - b - c) - 6c + 15b + 10(-2 + b + c) = 0

⇒ -27 + 9b + 9c - 6c + 15b - 20 + 10b + 10c = 0

⇒ 34b + 13c = 47

Now, using the values of a and d in Eq. (ii), we get

4(3 - b - c) + 2c - 6b - 3(b + c - 2) = 0

⇒ 12 - 4b - 4c + 2c - 6b - 3b - 3c + 6 = 0

⇒ -13b + 5c = 18 ....(x)

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$\begin{array}{r} -169b - 65c = -234 \\ 170b + 65c = 235 \\ \hline b = 1 \end{array}$$

$\Rightarrow$ -13 $\times$ 1 - 5c = -18 [from Eq. (x)]

$\Rightarrow$ -5c = -18 + 13 = -5 $\Rightarrow$ c = 1

a = 3 - 1 - 1 = 1 and d = 1 - 1 = 0

$$A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

34. Given equation of ellipse is

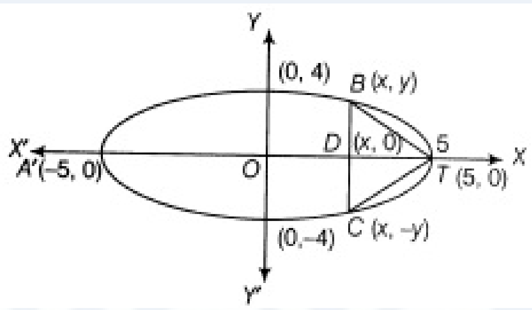
$\frac{x^2}{25}+\frac{y^2}{16}=1$.

Here, a = 5, b = 4

$\therefore$ a > b

So, major axis is along X-axis.

Let $\triangle$BTC be the isosceles triangle which is inscribed in the ellipse and OD = x, BC = 2y and TD = 5 - x.




Let A denotes the area of triangle. Then, we have

$A=\frac{1}{2}\times$ base $\times$ height $=\frac{1}{2}\times BC\times TD$

$\Rightarrow \quad A=\frac{1}{2}2y(5-x)\Rightarrow A=y(5-x)$

Therefore,on squaring both sides, we get,

$A^2=y^2(5-x)^2\ldots\ldots\ldots(i)$

Now, $\frac{x^2}{25}+\frac{y^2}{16}=1$

$\Rightarrow \quad \frac{y^2}{16}=1-\frac{x^2}{25}$

$\Rightarrow \quad y^2=\frac{16}{25}\left(25-x^2\right)$

On putting value of $y^2$ in Eq. (i), we get

$A^2=\frac{16}{25}\left(25-x^2\right)(5-x)^2$

Let $A^2$ = Z

Then, $Z=\frac{16}{25}\left(25-x^2\right)(5-x)^2$

Therefore,on differentiating both sides w.r.t x, we get,

$\frac{dZ}{dx}=\frac{16}{25}\left[\left(25-x^2\right)2(5-x)(-1)+(5-x)^2(-2x)\right]$ [by using product rule of derivative]

$= \frac{16}{25}(-2)(5-x)^2(2x+5)$

$= \frac{-32}{25}(5-x)^2(2x+5)$

For maxima or minima, put $\frac{dZ}{dx} = 0$

$\Rightarrow -\frac{32}{25}(5-x)^2(2x+5) = 0 \Rightarrow x = 5, -\frac{5}{2}$

Now, when x = 5, then

$Z = \frac{16}{25}(25-25)(5-5)^2 = 0$

Which  is not possible.

So, x = 5 is rejected.

$\therefore \quad x = -\frac{5}{2}$

Now, $\frac{d^2Z}{dx^2} = \frac{d}{dx}\left[-\frac{32}{25}(5-x)^2(2x+5)\right]$

$= \frac{32}{25}\left[(5-x)^2 \cdot 2 - (2x+5) \cdot 2(5-x)\right]$

$= -\frac{64}{25}(5-x)(-3x) = \frac{192x}{25}(5-x)$

$\therefore$ At $x = \frac{-5}{2}$, $\left(\frac{d^2Z}{dx^2}\right)_{x=-\frac{5}{2}} < 0$

$\Rightarrow$ Z is maximum.

$\therefore$ Area A is maximum, when $x = -\frac{5}{2}$ and y = 12

Clearly,

$Z = A^2 = \frac{16}{25}\left(25 - \frac{25}{4}\right)\left[5 + \frac{5}{2}\right]^2$

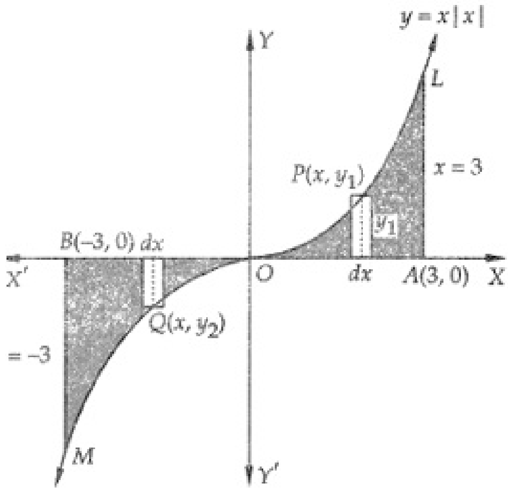
$= \frac{16}{25} \times \frac{75}{4} \times \frac{225}{4} = 3 \times 225$

$\therefore$ The maximum area, $A = \sqrt{3 \times 225} = 15\sqrt{3}$ sq units.

35. The equation of the curve is

y = $x|x|$ = $\begin{cases} x^2, x \geq 0 \\ -x^2, x < 0 \end{cases}$

The graph of y = x$|x|$ is shown in Fig. and the region bounded by y = x$|x|$, x-axis and the ordinates x = - 3 and x = 3 is shaded in the given Fig.




Clearly, y = x|x|, being an odd function is symmetric in opposite quadrants. Therefore, the required area is twice the area of the shaded region in the first quadrant.

Let us slice the region in first quadrant into vertical strips. The approximating rectangle shown in Fig. has length = $|y_1|$dx. As it can move between x = 0 and x = 3, therefore, area of the shaded region in first quadrant

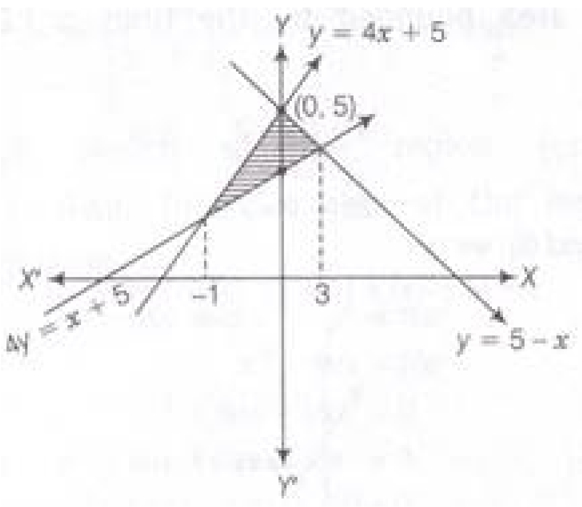
A = $\int_0^3 |y_1| \, dx$ = $\int_0^3 y_1 \, dx$ .......[$\because$ $y_1 \geq 0$ $\therefore$ $|y_1|$ = $y_1$]

$\Rightarrow$ A = $\int_0^3 x^2 dx$ ........[$\because$ P($x_1$, $y_1$) lies on y = $x^2$ $\therefore$ $y_1$ = $x^2$]

$\Rightarrow$ A = $\left[\frac{x^3}{3}\right]_0^3$ = 9 sq. units.

Hence required area = 2A = 2 $\times$ 9 = 18 sq. units.

**OR**




Given equations of lines are

y = 4x + 5 ...(i)

y = 5 - x ...(ii) and

4y = x + 5 ...(iii)

On solving Eqs. (i) and (ii), we get

4x + 5 = 5 - x

$\Rightarrow x = 0$

On solving Eqs. (i) and (iii)

4(4x + 5) = x + 5

$\Rightarrow 16x + 20 = x + 5$

$\Rightarrow 15x = -15$

$\Rightarrow x = -1$

On solving Eqs. (ii) and (iii), we get

4(5 - x) = x + 5

$\Rightarrow 20 - 4x = x + 5$

$\Rightarrow x = 3$

$\therefore$ Required area $= \int_{-1}^{0} (4x+5)\,dx + \int_{0}^{3} (5-x)\,dx - \frac{1}{4}\int_{-1}^{3} (x+5)\,dx$

$= \left[\frac{4x^2}{2} + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$

$= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0\right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2} + 5\right]$

$= 3 + \frac{21}{2} - \frac{1}{4}.24$

$= -3 + \frac{21}{2} = \frac{15}{2}$ sq units

36. According to the question, the points are

$A(2,5,-3), B(-2,-3,5)\ and\ C(5,3,-3).$

Consider $(x_1, y_1, z_1) = A(2,5,-3)$

$(x_2, y_2, z_2) = B(-2,-3,5)$

and $(x_3, y_3, z_3) = C(5,3,-3)$

Equation of plane passing through three non-collinear points is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

On putting the values of three points, we get

$$\begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -2 - 2 & -3 - 5 & 5 + 3 \\ 5 - 2 & 3 - 5 & -3 + 3 \end{vmatrix} = 0$$

$$\Rightarrow\begin{vmatrix} x_1-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$$

$\Rightarrow (x-2)(0+16)-(y-5)(0-24)+(z+3)(8+24)=0$

$\Rightarrow 16x-32+24y-120+32z+96=0$

$\Rightarrow 16x+24y+32z-56=0$

$\Rightarrow 2x+3y+4z-7=0$

[ divide by 8]……(i)

Now, distance between the plane (i) and the point (7,2, 4) is

$$d=\left|\frac{2(7)+3(2)+4(4)-7}{\sqrt{(2)^2+(3)^2+(4)^2}}\right|$$

$$\begin{bmatrix} \because \text{ distance between the plane} \\ ax+by+cz+d=0 \text{ and the point} \\ (x_1,y_1z_1) \text{ is } \left|\frac{ax_1+by_1+a_1-d}{\sqrt{a^2+b^2+c^2}}\right| \end{bmatrix}$$

$$=\left|\frac{14+6+16-7}{\sqrt{4+9+16}}\right|=\frac{29}{\sqrt{29}}=\sqrt{29}\,units$$