CBSE Class 12 - Mathematics
Sample Paper 03

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. The system of equations, x + 2y = 5 ,4x + 8y = 20 has
   a. no solution
   b. none of these
   c. a unique solution
   d. infinitely many solutions
2. If A and B are invertible matrices of order 3, then det(adj A) =
   a. $|A|^2$

b. None of these

c. (det A)^2

d. 1

3. $\underset{x \to \pi}{Lt} \frac{1+\cos^3 x}{(x-\pi)^2}$ is equal to

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{3}{2}$

d. None of these

4. Differential coefficient of a function f(g(x)) w.r.t. the function g (x) is

a. f‘(g (x))

b. None of these

c. $\frac{f'(g(x))}{g'(x)}$

d. f ‘(g (x)) g’ (x)

5. General solution of $\frac{dy}{dx}+y=1\ (y\ \neq 1)$ is

a. $y\ =\ B\ +\ Ae^{-x}$

b. $y\ =\ 1\ +\ Ae^{-x}$

c. $y\ =\ 1\ +\ Ae^{-3x}$

d. $y\ =\ 1\ +\ Ae^{x}$

6. $\cos\left(\cos^{-1}\left(\frac{7}{25}\right)\right) =$

a. $\frac{25}{7}$

b. None of these

c. $\frac{25}{24}$

d. $\frac{24}{25}$

7. If A and B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, Find P(not A and not B).

a. $\frac{1}{8}$

b. $\frac{2}{5}$

c. $\frac{3}{5}$

d. $\frac{3}{8}$

8. $\int\limits_{0}^{\pi/2}$ log (tan x) dx is equal to

a. $\int\limits_{0}^{\pi/2}$ log (cot x) dx

b. 1

c. $-\frac{\pi}{2}\log 2$

d. $\frac{\pi}{2}\log 2$

9. Find the coordinates of the foot of the perpendicular drawn from the origin to 3y + 4z – 6 = 0

a. $\left(1, \frac{18}{25}, \frac{24}{25}\right)$

b. $\left(0, \frac{18}{25}, \frac{27}{25}\right)$

c. $\left(0, \frac{18}{25}, \frac{24}{25}\right)$

d. $\left(0, \frac{19}{25}, \frac{24}{25}\right)$

10. If θ is the angle between two vectors $\vec{a}\ and\ \vec{b}$, then $\vec{a}.\vec{b} \geqslant 0$ only when

   a. $0 < \theta < \frac{\pi}{2}$

   b. $0 \leqslant \theta \leqslant \pi$

   c. $0 < \theta < \pi$

   d. $0 \leqslant \theta \leqslant \frac{\pi}{2}$

11. Fill in the blanks:

    A function f : X $\rightarrow$ Y is said to be an ________ function, if every element of Y is image of some element of set X under f.

12. Fill in the blanks:

    The probabiltiy of drawing two clubs from a well shuffled pack of 52 cards is ________.

13. Fill in the blanks:

    If A is symmetric matrix, then B'AB is ________ matrix.

14. Fill in the blanks:

    The function A(x) denotes the ________ function and is given by A (x) = $\int\limits_a^x f(x)dx$.

OR

Fill in the blanks:

The value of integral is $\int_0^1 \frac{dx}{e^x+e^{-x}}$ is ________.

15. Fill in the blanks:

    Any point in the feasible region that gives the optimal value of the objective function is called an ________ solution.

OR

Fill in the blanks:

The process of obtaining the optimal solution of the linear programming problem is called ________.

16. Evaluate $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

17. If a line has the direction ratios -18, 12, -4 then what are its direction cosines

18. Evaluate $\int_0^{\pi/4} \tan x dx$.

OR

$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

19. Find the length of the longest interval in which the function 3sin x-4sin^{3}x is increasing.

20. Find the magnitude of the vector $\vec{a}$ = 2$\hat{i}$ + 3$\hat{j}$ - 6$\hat{k}$.

Section B

21. Find the value of parameter α for which the function f (x) = 1 + αx, $\alpha \neq 0$ is the inverse of itself.

22. Find $\frac{dy}{dx}$, if $x^3 + x^2y + xy^2 + y^3 = 81$

OR

Find the second order derivative of the function e^{x}sin 5x

23. Find sine of the angle between the vectors. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

24. Find point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents is parallel to x-axis.

OR

Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$

25. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

26. A Box of oranges is inspected by examining three randomly selected oranges drown without replacement. If all the three oranges are good, the box is approved for sale, other wise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approve for sale?

Section C

27. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

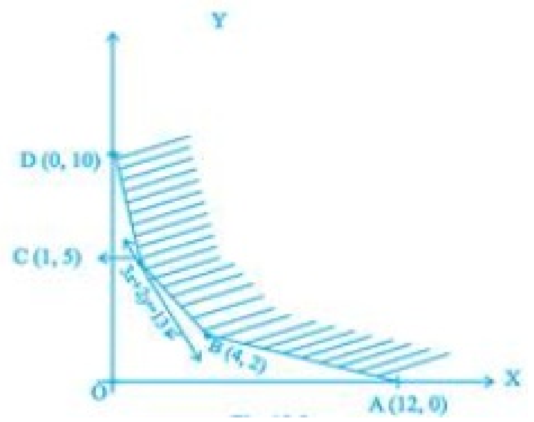
28. Find the relationship between a and b, so that the function f defined by f(x)= $\begin{cases} ax+1, \text{ if } x \leq 3 \\ bx+3, \text{ if } x > 3 \end{cases}$ is continuous at x = 3.

OR

Prove that the greatest integer function [x] is continuous at all points except at integer points.

29. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of a number of defective bulbs?

30. Determine the minimum value of Z = 3x + 2y (if any), if the feasible region for an LPP is shown in Fig.



31. If y = sin(sin x), prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0.$

OR

Verify that the given function (explicit or implicit) is a solution of the corresponding differential equation: y - cos y = x : y sin y + cos y + xy' = y

32. Evaluate $\int_0^1 \frac{\log|1+x|}{1+x^2}dx$.

Section D

33. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

OR

Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that A_2 - 4A + 7I = 0. Using this result calculate A^5 also.

34. An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

35. Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

OR

Find the area bounded by the lines y = 4x + 5, y = 5 - x and 4y = x + 5.

36. Find the coordinate where the line thorough (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7.

Solution
Section A

1. (d) infinitely many solutions

   Explanation:

   For Infinitely many solutions , $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, for given system of equations we have : $\frac{1}{4}=\frac{2}{8}=\frac{5}{20}$.

2. (a) $|A|^2$

   Explanation:

   Let A be a non singular square matrix of order n then, det(adjA)=$|A|^{n-1}$

   Here order is 3 so det(adj A)=$|A|^{3-1}$=$|A|^2$

3. (c)

   $\frac{3}{2}$

   Explanation:

   $\lim\limits_{x\to\pi}\frac{1+\cos^3 x}{(x-\pi)^2}=\lim\limits_{h\to 0}\frac{1+\cos^3(\pi+h)}{(h)^2}$

   $=\lim\limits_{h\to 0}\frac{1-\cos^3 h}{(h)^2}$

   $=\lim\limits_{h\to 0}\frac{1-\cosh}{h^2}.\lim\limits_{h\to 0}(1+\cosh+\cos^2 h)$

   $=\frac{1}{2}(1+1+1)=\frac{3}{2}$

4. (a) f‘(g (x))

   Explanation:

   $\frac{d}{d(g(x))}(f(g(x))=f'(g(x))$

5. (b)

$y = 1 + Ae^{-x}$

Explanation:

$\frac{dy}{dx} + y = 1$

$\frac{dy}{dx} + Py = Q$

Here, $P = 1, Q = 1$ It is of the form of linear differential equation.hence the solution is y X IF =$\int Q(x) * IF\ dx + c$

$\Rightarrow I.F. = e^{\int P.dx} = e^{\int 1.dx} = e^x$

$\Rightarrow y.e^x = \int 1.e^x.dx$

$\Rightarrow y = 1 + Ae^{-x}$

6. (b) None of these

Explanation:

We know that $cos : [0,1] \to [-1,1]$ is bijective function

$\Rightarrow cos^{-1} : [-1,1] \to [0,1]$ is inverse of cos function.

.$\Rightarrow \cos(\cos^{-1}x) = x$ when $x \in [-1,1]$

here, $cos(\cos^{-1}\frac{7}{25}) = \frac{7}{25}, \quad \frac{7}{25} \in [-1,1]$

7. (d)

$\frac{3}{8}$

Explanation:

Since A and B are independent events, not A and not B are also independent events .
$P(\overline{A}\overline{B}) = P(\overline{A})P(\overline{B}) = (1 - \frac{1}{4})(1 - \frac{1}{2}) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

8. (a)

$\int_0^{\pi/2} \log(\cot x)\, dx$

Explanation:

$\int_0^{\pi/2} \log(\tan x) dx = \int_0^{\pi/2} \log(\tan(\frac{\pi}{2} - x)) dx. \implies \int_0^{\pi/2} \log(\cot x)\ dx$

9. (c)

$\left(0, \frac{18}{25}, \frac{24}{25}\right)$

Explanation:

D.R.'s of the line are < 0 , 3 , 4 > .

Therefore , equation of the line is : $\frac{x-0}{0} = \frac{y-0}{3} = \frac{z-0}{4} = \lambda$

Thus , the coordinates of any point P on the above line are P (0 , 3λ ,4λ) .

But , this point P also lies on the given plane :

0 + 3(3λ) +4(4λ) – 6 = 0.$\Rightarrow 25\lambda = 6 \Rightarrow \lambda = \frac{6}{25}$

Therefore , the coordinates of the foot of perpendicular are given by :

$\left(0, 3 \times \frac{6}{25}, 4 \times \frac{6}{25}\right) i.e. \left(0, \frac{18}{25}, \frac{24}{25}\right)$

10. (d)

$0 \leqslant \theta \leqslant \frac{\pi}{2}$

Explanation:

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta,$

Also, $\vec{a}.\vec{b} \geqslant 0$

$\Rightarrow |\vec{a}||\vec{b}|cos\theta \Rightarrow cos\theta \leqslant 0 \Rightarrow 0 \leqslant \theta \leqslant \frac{\pi}{2}$

11. onto

12. $\frac{1}{17}$

13. symmetric

14. area

OR

$\tan^{-1} e - \frac{\pi}{4}$

15. optimal

OR

optimisation technique

16. Let A = $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

$|A|$ = x (5x + 1) + 7 $\times$ x

= $5x^2$ + x + 7x

= $5x^2$ + 8x

Hence $|A|$ = $5x^2$ + 8x

17. a = -18, b = 12, c = - 4

$a^2 + b^2 + c^2 = (-18)^2 + (12)^2 + (-4)^2$

= 484

Therefore, direction cosines are,

$\left(-\frac{18}{484}, \frac{12}{484}, -\frac{4}{484}\right)$

18. $\int_0^{\pi/4} \tan x dx = [\log|\sec x|]_0^{\pi/4}$

$= \log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|$

$= \log|\sqrt{2}| - \log|1|$

$= \frac{1}{2}\log 2$

OR

$I = \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

$= \int \frac{1-2\sin^2 x+2\sin^2 x}{\cos^2 x} dx$

$= \int \frac{1}{\cos^2 x} dx$

$= \int \sec^2 x dx$

= tan x + c

19. We have f(x)=3sin x-4sin^3 x

    f(x) = sin 3x

    The longest interval in which sin x is increasing is of length π. since sine function is increasing from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ which is the principle value branch. It is true for every interval with the length of π .

    So,the length of the largest interval in which f(x)=sin 3x is increasing is $\frac{\pi}{3}$, since the period is tripled.

20. Magnitude of a vector x$\hat{i}$ + y$\hat{j}$ + z$\hat{k}$ is given by $\sqrt{x^2+y^2+z^2}$
    So,
    $|\vec{a}|$ = $\sqrt{(2)^2+(3)^2+(-6)^2}$
    = $\sqrt{4+9+36}$
    = $\sqrt{49}$
    = 7

Section B

21. Clearly, f (x) is a bijection from R to itself.
    Now,
    fof $^{-1}$ (x) = x $\Rightarrow$f (f $^{-1}$(x)) = x $\Rightarrow$ 1 + α f $^{-1}$ (x) = x $\Rightarrow$ f $^{-1}$ $(x) = \frac{x-1}{\alpha}$
    It is given that
    f (x) = f $^{-1}$ (x) for all x $\in$R
    $\Rightarrow \quad 1+\alpha x = \frac{x-1}{\alpha}$ for all x $\in$R
    $\Rightarrow \quad \alpha x+1 = \left(\frac{1}{\alpha}\right) x + \left(\frac{-1}{\alpha}\right)$ for all x $\in$ R
    $\Rightarrow \quad \alpha = \frac{1}{\alpha}$ and $1 = -\frac{1}{\alpha}$ $\Rightarrow \alpha^2 = 1$ and α = -1 $\Rightarrow$ α = -1.

22. Given: $x^3 + x^2y + xy^2 + y^3 = 81$

$\Rightarrow \frac{d}{dx}x^3 + \frac{d}{dx}x^2y + \frac{d}{dx}xy^2 + \frac{d}{dx}y^3 = \frac{d}{dx}81$

$\Rightarrow 3x^2 + \left(x^2\frac{dy}{dx} + y.\frac{d}{dx}x^2\right) + x\frac{d}{dx}y^2 + y^2\frac{d}{dx}x + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + y.2x + x.2y\frac{dy}{dx} + y^2.1 + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}\left(x^2 + 2xy + 3y^2\right) = -3x^2 - 2xy - y^2$

$\Rightarrow \frac{dy}{dx} = \frac{-\left(3x^2+2xy+y^2\right)}{x^2+2xy+3y^2}$

OR

Let y = $e^x \sin 5x$

$\therefore \frac{dy}{dx} = e^x\frac{d}{dx}\sin 5x + \sin 5x\frac{d}{dx}e^x$

$= e^x \cos 5x\frac{d}{dx}5x + \sin 5x.e^x = e^x \cos 5x \times 5 + e^x \sin 5x$

= e^x(5 cos 5x + sin 5x)

$\Rightarrow \frac{d^2y}{dx^2} = e^x\frac{d}{dx}(5\cos 5x + \sin 5x) + (5\cos 5x + \sin 5x)\frac{d}{dx}e^x$

$= e^x\left[5(-\sin 5x) \times 5 + (\cos 5x) \times 5\right]$ + (5 cos 5x + sin 5x)e^x

= e^x(-25 sin 5x + 5 cos 5x + 5 cos 5x + sin 5x)

= e^x(10 cos 5x - 24 sin 5x)

= $2e^x$(5 cos 5x - 12 sin 5x)

23. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$

$= -11\hat{i} - \hat{j} + 7\hat{k}$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{(-11)^2 + (-1)^2 + (7)^2}$$

$$= \sqrt{171} = 3\sqrt{19}$$

$$\sin\theta = \frac{\left|\vec{a}\times\vec{b}\right|}{|\vec{a}|\left|\vec{b}\right|} = \frac{3\sqrt{19}}{\sqrt{14}.\sqrt{14}} = \frac{3}{14}\sqrt{19}$$

24. $\frac{x^2}{4} + \frac{y^2}{25} = 1$ ...(1)

Differentiate side w.r.t. to x

$$\frac{2x}{4} + \frac{2y}{25}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-25}{4}\cdot\frac{x}{y}$$

For tangent | | to x – axis the slope of tangent is zero

$$\frac{0}{1} = \frac{-25x}{4y}$$

x = 0

Put x = 0 in equation (1)

$$y = \pm 5$$

Points are (0, 5) and (0, -5) at which tangent is | | is to x – axis

OR

$$\frac{dx}{dt} = 3a\sin^2 t.\cos t dt$$

$$\frac{dy}{dt} = -3b\cos^2 t\sin t dt$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3bcos^2\ tsin\ t}{3asin^2\ tcos\ t} = \frac{-b}{a}\cot t$$

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{2}} = \frac{-b}{a}\times cot\ \frac{\pi}{2} = \frac{-b}{a}\times 0 = 0$$

When $t = \frac{\pi}{2}, x = a$ and y = 0

Therefore ,equation of tangent is,

$y-y_1=\frac{dy}{dx}(x-x_1),y-0=0(x-a)$i.e y=0

25. Equation of one line $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$

$\therefore$ Direction ratios of this line are $7,-5,1=a_1,b_1,c_1$

$\Rightarrow \vec{b}_1=7\hat{i}-5\hat{j}+\hat{k}$

Again equation of another line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

$\therefore$ Direction ratios of this line are 1, 2, 3 $=a_2,b_2,c_2$

$\Rightarrow \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Now $\vec{b}_1.\vec{b}_2=a_1a_2+b_1b_2=c_1c_2=7\times 1+(-5)\times 2+1\times 3=7-10+3=0$

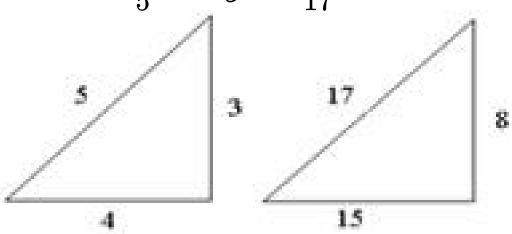
Hence, the given two lines are perpendicular to each other.

26. Total oranges are 15 in which 12 are good ones and 3 are bad ones.Let E_1 be the event of drawing first orange (good ones), E_2 be the event of drawing second orange (good ones) & E_3 be the event of drawing third orange (good ones).

Then,$P(E_1)=\frac{12}{15},P(E_2/E_1)=\frac{11}{14},P(E_3/E_2/E_1)=\frac{10}{13}$
Required Probability$=P(E_1\cap E_2\cap E_3)=\frac{12}{15}\times\frac{11}{14}\times\frac{10}{13}=\frac{44}{91}$

Section C

27. Let $\sin^{-1}\frac{3}{5}=x,\sin^{-1}\frac{8}{17}=y$
$\sin x=\frac{3}{5}\ \sin y=\frac{8}{17}$



$\cos(x-y)=\cos x.\cos y+\sin x.\sin y$
$\cos(x-y)=\frac{4}{5}\times\frac{15}{17}+\frac{3}{5}\times\frac{8}{17}$

$= \frac{60}{85} + \frac{24}{85}$
$= \frac{60+24}{85} = \frac{84}{85}$
$x - y = \cos^{-1}\left(\frac{84}{85}\right)$
$\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\left(\frac{84}{85}\right)$

28. According to the question,we have to find the relationship between a and b, so that the function f defined by f(x)= $\begin{cases} ax+1, \text{ if } x \leq 3 \\ bx+3, \text{ if } x > 3 \end{cases}$ is continuous at x = 3.

Therefore , LHL = RHL = f(3).........(i)

Now, LHL = $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} (ax+1)$

$= \lim\limits_{h \to 0}[a(3-h)+1]$

$\Rightarrow$ LHL = 3a + 1

and RHL = $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (bx+3)$

$= \lim\limits_{h \to 0}[b(3+h+3]$

$= \lim\limits_{h \to 0}(3b+bh+3)$

$\Rightarrow$ RHL = 3b + 3

From Eq. (i), we have

LHL = RHL$\Rightarrow$ 3a + 1 = 3b + 3

Therefore, 3a - 3b = 2, which is the required relation between a and b.

OR

Let f (x) = [x] be the greatest integer function and let k be any integer. Then,

f (x) = [x] = $\begin{cases} k-1, & \text{if } k-1 \leq x < k \\ k & , \text{if } k \leq x < k+1 \end{cases}$ [By definition of [x]]

Now, (LHL at x = k) = $\lim\limits_{x \to k^-}$ f(x) = $\lim\limits_{h \to 0}$ f(k - h) = $\lim\limits_{h \to 0}$ [k - h]

= $\lim\limits_{h \to 0}$ (k - 1) = k - 1 [$\because$ k - 1 $\leq$ k - h < k $\therefore$ [k - h] = k - 1]

and, (RHL at x = k) = $\lim\limits_{x \to k^-}$ f(x) = $\lim\limits_{h \to 0}$ f (k + h) = $\lim\limits_{h \to 0}$ [k + h]

= $\lim\limits_{h \to 0}$ k = k [$\because$ k $\leq$ k + h < k + 1 $\therefore$ [k + h] = k]

$\therefore \lim\limits_{x \to k^-}$ f(x) $\neq \lim\limits_{x \to k^+}$ f(x)

So, f(X) is not continuous at x = k.

Since k is an arbitrary integer. Therefore, f(x) is not continuous at integer points.

Let a be any real number other than an integer. Then, there exists an integer k such that k-1 < a < k.

Now, (LHL at x = a) = $\lim_{x \to a^-}$ f(x) = $\lim_{h \to 0}$ f(a - h) = $\lim_{h \to 0}$ [a - h]

= $\lim_{h \to 0}$ k - 1 = k - 1 [$\because$ k - 1 < a - h < k $\therefore$ [a - h] = k - 1]

(RHL at x = a) = $\lim_{x \to a^+}$ f(x) = $\lim_{h \to 0}$ f (a + h)

= $\lim_{h \to 0}$ [a + h] = $\lim_{h \to 0}$ (k - 1) = k - 1 [$\because$ k - 1 < a + h < k $\therefore$ [a + h] = k - 1]

and, f(a) = [a] = k - 1 [$\because$ k - 1 < a < k $\therefore$[a] = k - 1]

Thus, $\lim_{x \to a^-}$ f(x) = $\lim_{x \to a^+}$ f(x) = f(a)

So, f(x) is continuous at x = a. Since a is an arbitrary real number, other than an integer.

Therefore, f(x) is continuous at all real points except integer points.

29. There are 3 defective bulbs and 7 non-defective bulbs. Let X denote the random variable of "the no. of defective bulb'. Then X can take values 0, 1, 2 since bulbs are replaced.

p = P(D) = $\frac{3}{10}$ and q = P($\overline{D}$) = 1 - $\frac{3}{10}$ = $\frac{7}{10}$

We have,

P(X = 0) = $\frac{{}^7C_2 \times {}^3C_0}{{}^{10}C_2}$ = $\frac{7 \times 6}{10 \times 9}$ = $\frac{7}{15}$

P(X = 1) = $\frac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2}$ = $\frac{7 \times 3 \times 2}{10 \times 9}$ = $\frac{7}{15}$

P(X = 2) = $\frac{{}^7C_0 \times {}^3C_2}{{}^{10}C_2}$ = $\frac{1 \times 3 \times 2}{10 \times 9}$ = $\frac{1}{15}$

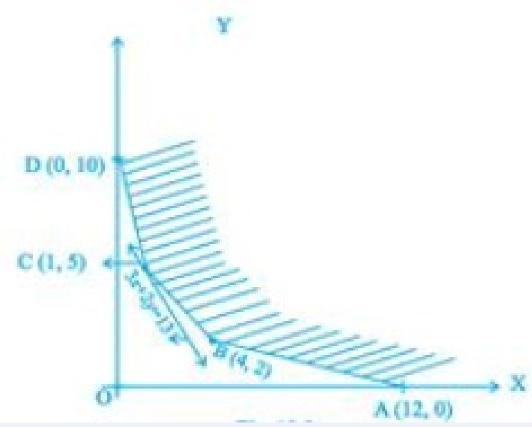
$\therefore$ Required probability distribution is

X	0	1	2
P(x)	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

30. The feasible region (R) is unbounded. Therefore, a minimum of Z may or may not exist. If it exists, it will be at the corner point Fig.

Corner Point	**Value of Z**
A (12, 0)	3(12) + 2(0) = 36

B(4, 2)	3(4) + 2(2) = 16
C(1, 5)	3(1) + 2(5) = 13 (smallest)
D(0, 10)	3(0) + 2(10) = 20



Let us graph 3x + 2y < 13. We see that the open half plane determined by 3x + 2y < 13 and R do not have a common point. So, the smallest value 13 is the minimum value of Z.

31. According to the question, $y = sin(sinx)$ ....(i)

On differentiating both sides w.r.t x, we get

$\frac{dy}{dx} = \cos(\sin x).\cos x$ ....(ii)

On differentiating both sides w.r.t x, we get

$\frac{d^2y}{dx^2} = cos(sinx).(-sinx) + cosx([-sin(sinx)]cosx)$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{\cos x}\left(\frac{dy}{dx}\right).(-\sin x) - y\cos^2 x$ [Using Eqs.(i) and (ii)]

$\Rightarrow \frac{d^2y}{dx^2} = -\tan x\frac{dy}{dx} - y\cos^2 x$

$\Rightarrow \frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$

Hence Proved.

OR

Given: y - cos y = x ....(i)

To prove: y given by eq. (i) is a solution of differential equation

(y sin y + cosy + x)y' = y ....(ii)

Proof: Differentiating both sides of eq. (i) w.r.t x, we have

y' + (sin y)y' = 1

$\Rightarrow y'(1+\sin y)=1$

$\Rightarrow y'=\frac{1}{1+\sin y}$ ....(iii)

Putting the value of x from eq. (i) and value of y' from eq. (iii) in L.H.S. of eq. (ii),

(y sin y + cos y + x)y'

$\Rightarrow (y\sin y+\cos y+y-\cos y)\frac{1}{1+\sin y}$

$\Rightarrow (y\sin y+y)\frac{1}{1+\sin y}$

$\Rightarrow y(\sin y+1)\frac{1}{1+\sin y}=y$ = R.H.S. of (ii)

Hence, Function given by eq. (i) is a solution of (y sin y + cos y + x)y' = y.

32. According to the question, $\int_0^1 \frac{\log|1+x|}{1+x^2}dx$

Let , x = tan θ

$\Rightarrow \quad dx=\sec^2\theta d\theta$

Lower limit, when x = 0, then $\theta=tan^{-1}0=0$

Upper limit, when x = 1, then $\theta=tan^{-1}= \ \frac{\pi}{4}$.

$\therefore \quad I=\int_0^{\pi/4}\frac{\log|1+\tan\theta|}{1+\tan^2\theta}\sec^2\theta d\theta$

$\Rightarrow \quad I=\int_0^{\pi/4}\frac{\log|1+\tan\theta|\sec^2\theta}{\sec^2\theta}d\theta$

$\Rightarrow \quad I=\int_0^{\pi/4}\log|1+\tan\theta|d\theta$...(i)

$=\int_0^{\pi/4}\log\left|1+\tan\left(\frac{\pi}{4}-\theta\right)\right|d\theta\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$=\int_0^{\pi/4}\log\left|1+\frac{1-\tan\theta}{1+\tan\theta}\right|d\theta$

$=\int_0^{\pi/4}\log\left|\frac{2}{1+\tan\theta}\right|d\theta$

$=\int_0^{\pi/4}\log 2d\theta-\int_0^{\pi/4}\log|1+\tan\theta|d\theta$

$\Rightarrow \quad I=\log 2\cdot[\theta]_0^{\pi/4}-I$ [from Equation (i)]

$\Rightarrow \quad 2I=\frac{\pi}{4}\log 2$

$\therefore \quad I=\frac{\pi}{8}\log 2$

Section D

33. $L.H.S = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

Multiplying C_1, C_2, C_3 by a,b,c respectively and then dividing the determinant by abc,

$$= \frac{1}{abc}\begin{vmatrix} a\left(a^2+1\right) & ab^2 & ac^2 \\ a^2b & b\left(b^2+1\right) & bc^2 \\ a^2c & b^2c & c\left(c^2+1\right) \end{vmatrix}$$

Taking a,b, and c common from R_1, R_2 and R_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

$[C_1 \to C_1 + C_2 + C_3]$

$$= \frac{abc}{abc}\begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

Taking $1+a^2+b^2+c^2$ common from C_1

$$= \left(1+a^2+b^2+c^2\right)\begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

$[R_2 \to R_2 - R_1 \ and \ R_3 \to R_3 - R_1]$

$$= \left(1+a^2+b^2+c^2\right)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along first column

$= \left(1 + a^2 + b^2 + c^2\right)(1)(1 - 0)$

$= 1 + a^2 + b^2 + c^2$ = R.H.S.

OR

We have $A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$

$-4A = \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix}$ and $7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

Therefore, $A^2 - 4A + 7I = \begin{bmatrix} 1-8+7 & 12-12+0 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$\Rightarrow$ $A^2 = 4A - 7I$

Thus $A^3 = AA^2 = A(4A - 7I) = 4(4A - 7I) - 7A$

$= 16A - 28I - 7A = 9A - 28I$

And so $A^5 = A^3A^2$

$= (9A - 28I)(4A - 7I)$

$= 36A^2 - 63A - 112A + 196I$

$= 36(4A - 7I) - 175A + 196I$

$= -31A - 56I$

$= -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$

34. Let the dimensions of the box be x and y. Also, let V denotes its volume and S denotes its total surface area.

Now, $S = x^2 + 4xy$

Given, $x^2 + 4xy = C^2$

$\Rightarrow \quad y = \frac{C^2 - x^2}{4x}$ ....(i)

Also, volume of the box is given by

$V = x^2y$

$\Rightarrow \quad V = x^2\left(\frac{C^2 - x^2}{4x}\right)$ [From Eq.(i)]

$\Rightarrow \quad V = \frac{xC^2 - x^3}{4}$

On differentiating both sides w.r.t. x, we get

$\frac{dV}{dx} = \frac{C^2 - 3x^2}{4}$

For maxima or minima, put $\frac{dV}{dx}$ = 0

$\Rightarrow \frac{C^2 - 3x^2}{4} = 0$

$\Rightarrow C^2 = 3x^2$

$\therefore \quad x = C/\sqrt{3}$

Also, $\frac{d^2V}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = \frac{d}{dx}\left(\frac{C^2 - 3x^2}{4}\right)$

$= \frac{-6x}{4} = \frac{-3x}{2}$

$\therefore \quad \left.\frac{d^2V}{dx^2}\right|_{at x = C/\sqrt{3}} < 0$

This implies V is maximum.

Now, maximum volume at x = $\frac{C}{\sqrt{3}}$ is

$V = \frac{xC^2 - x^3}{4}$

$= \frac{1}{4}\left[\frac{C}{\sqrt{3}} \cdot C^2 - \left(\frac{C}{\sqrt{3}}\right)^3\right] \left[\text{put } x = \frac{C}{\sqrt{3}}\right]$

$= \frac{1}{4}\left[\frac{C^3}{\sqrt{3}} - \frac{C^3}{3\sqrt{3}}\right] = \frac{1}{4}\left[\frac{3C^3 - C^3}{3\sqrt{3}}\right]$

$= \frac{1}{4} \times \frac{2C^3}{3\sqrt{3}} = \frac{C^3}{6\sqrt{3}}$

Hence, the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

35. Given circles are $x^2 + y^2 = 4$...(i)

$(x-2)^2 + y^2 = 4$...(ii)

Eq. (i) is a circle with centre origin and

Radius = 2.

Eq. (ii) is a circle with centre C (2, 0) and

Radius = 2.

On solving Eqs. (i) and (ii), we get

$(x-2)^2 + y^2 = x^2 + y^2$

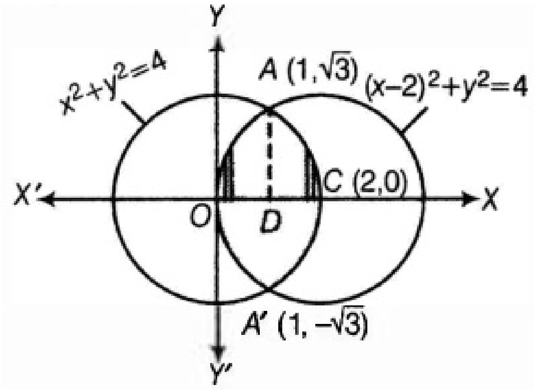
$\Rightarrow x^2 - 4x + 4 + y^2 = x^2 + y^2$

$\Rightarrow x = 1$

On putting x = 1 in Eq. (i), we get

$y = \pm\sqrt{3}$

Thus, the points of intersection of the given circles are A (1, $\sqrt{3}$) and A'(1,-$\sqrt{3}$).



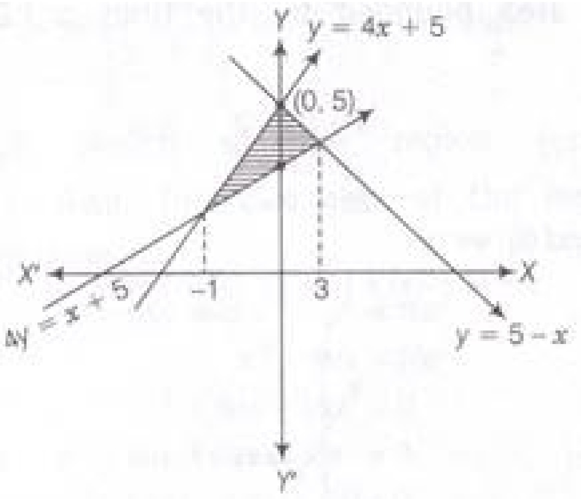
Clearly, required area= Area of the enclosed region OACA'O between circles

= 2 [Area of the region ODCAO]

=2 [Area of the region ODAO + Area of the region DCAD]

$$= 2\left[\int_0^1 y_2\,dx + \int_1^2 y_1\,dx\right]$$
$$= 2\left[\int_0^1 \sqrt{4-(x-2)^2}dx + \int_1^2 \sqrt{4-x^2}dx\right]$$
$$= 2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2} + \frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1$$
$$+2\left[\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$$
$$= \left[(x-2)\sqrt{4-(x-2)^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1 + \left[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}\right]_1^2$$
$$= \left[\left\{-\sqrt{3} + 4\sin^{-1}\left(\frac{-1}{2}\right)\right\} - 0 - 4\sin^{-1}(-1)\right]$$
$$+\left[0 + 4\sin^{-1}1 - \sqrt{3} - 4\sin^{-1}\frac{1}{2}\right]$$
$$= \left[\left(-\sqrt{3} - 4\times\frac{\pi}{6}\right) + 4\times\frac{\pi}{2}\right] + \left[4\times\frac{\pi}{2} - \sqrt{3} - 4\times\frac{\pi}{6}\right]$$
$$= \left(-\sqrt{3} - \frac{2\pi}{3} + 2\pi\right) + \left(2\pi - \sqrt{3} - \frac{2\pi}{3}\right)$$
$$= \frac{8\pi}{3} - 2\sqrt{3} \text{ sq units.}$$

OR



Given equations of lines are

y = 4x + 5 ...(i)

y = 5 - x ...(ii) and

4y = x + 5 ...(iii)

On solving Eqs. (i) and (ii), we get

4x + 5 = 5 - x

$\Rightarrow x = 0$

On solving Eqs. (i) and (iii)

4(4x + 5) = x + 5

$\Rightarrow 16x + 20 = x + 5$

$\Rightarrow 15x = -15$

$\Rightarrow x = -1$

On solving Eqs. (ii) and (iii), we get

4(5 - x) = x + 5

$\Rightarrow 20 - 4x = x + 5$

$\Rightarrow x = 3$

$\therefore$ Required area $= \int_{-1}^{0} (4x + 5)\,dx + \int_{0}^{3} (5 - x)\,dx - \frac{1}{4}\int_{-1}^{3} (x + 5)\,dx$

$= \left[\frac{4x^2}{2} + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$

$= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0\right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2} + 5\right]$

$= 3 + \frac{21}{2} - \frac{1}{4}.24$

$= -3 + \frac{21}{2} = \frac{15}{2}$ sq units

36. Given points are A (3, -4, -5), B (2, -3, 1)

Direction ratios of line AB are (3 - 2, -4 + 3, -5 - 1)

i.e (1, -1, -6)

Eq. of line AB is,

$\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{-6} = \lambda$

$\Rightarrow x = \lambda + 2, y = -\lambda - 3, z = -6\lambda + 1$

Thus, any point on the line is $(\lambda + 2, -\lambda - 3, -6\lambda + 1)$

This point lies on the plane 2x + y + z = 7t

Therefore $2(\lambda + 2) - \lambda - 3 - 6\lambda + 1 = 7$

$\Rightarrow -5\lambda + 2 = 7$

$\Rightarrow -5\lambda = 5$

$\Rightarrow \lambda = -1$

Required point of intersection of line and plane is

(-1 + 2, 1 - 3, 6 + 1)
(1, - 2, 7)