# CBSE Class 12 - Mathematics
## Sample Paper 02

**Maximum Marks: 80**
**Time Allowed: 3 hours**

**General Instructions:**

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.



**Section A**

1. The system of linear equations x + y + z = 2, 2x + y - z = 3, 3x + 2y - kz = 4 has a unique solution if ,
   a. k = 0
   b. -1 < k < 1
   c. -2 < k < 2
   d. $k \neq 0$
2. If $\omega$ is non real cube root of unity, then $\begin{vmatrix} 2 & 2\omega & -\omega^2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$ is equal to
   a. –1
   b. 0

c. None of these

d. 1

3. $\frac{d}{dx}(\tan^{-1}(\sec x + \tan x)$ is equal to

a. $-\frac{1}{2}$

b. $\frac{1}{2}$

c. $\frac{1}{2\sec x(\sec x + \tan x)}$

d. None of these

4. If x sin (a + y) = sin y, then $\frac{dy}{dx}$ is equal to

a. $\frac{\sin a}{\sin(a+y)}$

b. $\frac{\sin^2(a+y)}{\sin a}$

c. $\frac{\sin a}{\sin^2(a+y)}$

d. $\frac{\sin(a+y)}{\sin a}$

5. General solution of $\frac{dy}{dx} = \left(1 + x^2\right)\left(1 + y^2\right)$ is

a. $tan^{-1}y$ = x + $\frac{x^3}{3} + C$

b. $cos^{-1}y$ = x + $\frac{x^3}{3} + C$

c. $cot^{-1}y$ = x + $\frac{x^3}{3} + C$

d. $sin^{-1}y$ = x + $\frac{x^3}{3} + C$

6. The period of the function f(x) = cos4x + tan3x is

a. $\frac{\pi}{3}$

b. $\pi$

c. None of these

d. $\frac{\pi}{2}$

7. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two-headed coin?

a. $\frac{4}{9}$

b. $\;\frac{5}{9}$

c. $\frac{1}{9}$

d. $\frac{2}{9}$

8. $\int \log\left(\frac{1}{x}-1\right)$ dx is equal to

a. log 2x + C

b. $\frac{1}{2}\log(2-x)+C$

c. $(x-1)\log(1-x)+1-x\ln x+C$

d. $(3x+1)\log(2+x)+1-x\ln 2x+C$

9. If $\theta$ is the acute angle between $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ $and$ $\vec{r}=\overrightarrow{a_2}+\lambda\overrightarrow{b_2}$ $,\lambda\in R$ express $cos\theta$ .

a. $\tan\theta=\left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$

b. $\cot\theta=\left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$

c. $\sin\theta=\left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$

d. $\cos\theta=\left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$

10. Position vectors of points A, B, C, etc., with respect to the origin O are generally denoted by

a. $\vec{a}, \vec{b}, \vec{c}$

b. $\vec{A}, \vec{B}, \vec{C}$

c. $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$

d. $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$

11. Fill in the blanks:

    A relation R from a set X to a set Y is defined as a ________ of the cartesian product X $\times$ Y.

12. Fill in the blanks:

    If X follows binomial distribution with parameters n = 5, p and P(X = 2) = 9, P(X = 3), then p = ________.

13. Fill in the blanks:

    The negative of a matrix is obtained by multiplying it by ________.

14. Fill in the blanks:

    The value of $\int \frac{dx}{\sqrt{16-9x^2}}$

**OR**

Fill in the blanks:

$\int f(x)dx$ =F(x)+c, these type of integrals are called ________ integrals.

15. Fill in the blanks:

    If the feasible region for a LPP is ________, then the optimal value of the objective function Z = ax + by may or may not exist.

**OR**

Fill in the blanks:

The feasible region for an LPP is always a ________ polygon.

16. Find the area of the triangle with vertices A(5, 4), B(-2, 4) and C(2, -6).

17. If the line has direction ratios 2, -1, -2 determine its direction Cosines.

18. Evaluate $\int (ax+b)^3 dx$.

**OR**

Evaluate $\int \frac{(x+1)(x+\log x)^2}{x} dx$

19. Find the value of k for which the function f(x) = kx - cos x is monotonically increasing for all x $\in R$.

20. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

**Section B**

21. Let f : R $\rightarrow$ R and g : R$\rightarrow$ R be defined by f (x) = x + 1 and g(x) = x - 1. Show that fog = gof = $I_R$.

22. Differentiate the following function with respect to x: $\sin\left(\tan^{-1} e^{-x}\right)$

**OR**

Verify L.M.V theorem for the following function f(x) = x2 + 2x + 3, for [4, 6]

23. Find the area of the $\Delta$ with vertices A (1, 1, 2) B (2, 3, 4) and C (1, 5, 5).

24. Find the approximate value of $(0.0037)^{\frac{1}{2}}$

**OR**

An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

25. Find distance of a point (2,5,-3) from the plane $\vec{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$

26. A die is thrown 6 times. If ‘getting an odd number’ is a success, what is the probability of:

   i. 5 successes?
   ii. at least 5 successes?
   iii. at most 5 successes?

**Section C**

27. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$.

28. If y = $\left(x+\sqrt{1+x^2}\right)^n$, then show that $\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=n^2y.$

OR

If x = sint and y = sin pt prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$

29. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ & $\frac{1}{3}$ respectively If both try to solve the problem independently, find the probability that

   i. The problem is solved
   ii. Exactly one of them solves the problem.

30. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs 100 and that on a bracelet is Rs 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximise the profit? It is being given that at least one of each must be produced.

31. Find the particular solution of the following differential equation $xy\frac{dy}{dx}=(x+2)(y+2);y=-1$when x = 1.

**OR**

Solve the following differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$.

32. $\int\tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

## Section D

33. Show that system of linear equations has an infinite number of solutions and solve:

x - y + 3z = 6

x + 3y - 3z = -4

5x + 3y + 3z = 10

**OR**

If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, prove $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

34. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

35. Using integration, find the area of the region {(x, y): $x^2 + y^2 \leq 16$, $x^2 \leq 6y$}.

**OR**

Using integration, find the area of the following region. {(x, y): Ix - 1I$\leq$ y $\leq \sqrt{5 - x^2}$}.

36. Find the equation of plane passing through point P(1, 1, 1 ) and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$. Also, show that plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$.

## Solution

## Section A

1. (d)

   $k \neq 0$

   Explanation:

   The given system of equation has a unique solution if :
   $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -k \end{vmatrix} \neq 0 \Rightarrow 1(-k+2)\ -1(-2k+3)+1(4-3) \neq 0 \Rightarrow k \neq 0$

2. (b) 0

   **Explanation:**

   Expanding along $R_3$

   $= 1(2\omega+\omega^2)+1(2+\omega^2) = (2+2\omega+2\omega^2 = 2(1+\omega+\omega^2) = 2(0) = 0$

3. (b)

   $\frac{1}{2}$

   **Explanation:**

   $\frac{d}{dx}(\tan^{-1}(\sec x+\tan x)) = \frac{\sec x \tan x+\sec^2 x}{1+(\sec x+\tan x)^2} = \frac{\sec x(\sec x+\tan x)}{2\sec x(\sec x+\tan x)} = \frac{1}{2}$

4. (b)

   $\frac{\sin^2(a+y)}{\sin a}$

   **Explanation:**

   $x\sin(a+y) = \sin y \Rightarrow x = \frac{\sin y}{\sin(a+y)}$

$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$

$= \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\text{sina}}{\sin^2(a+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

5. (a)

   $tan^{-1}y$ = x + $\frac{x^3}{3} + \ C$

   **Explanation:**

   $\frac{dy}{dx} = \left(\ 1 + x^2\right)\left(1 + y^2\right)$

   $\int \frac{1}{1+y^2} dy = \int (1 + x^2)dx$ Since$\int \frac{dx}{1+x^2} \ = tan^{-1}x + c$

   $\tan^{-1}y = x + \frac{x^3}{3} + C$

6. (b)

   $\pi$

   **Explanation:**

   $f\left(\pi\right) = \left(cos\ 4\pi + tan3\pi\ \right)$gives the same value as f (0). Therefore, the period of the function is $\pi$

7. (a)

   $\frac{4}{9}$

   **Explanation:**

   Let
   $E_1, E_2$ and $E_3$ and are events of selection of a two headed coin , biased coin and unbiased coin respectively.
   $\therefore P(E_1) = P(E_2) = P(E_2) = \frac{1}{3}.$
   Let A = event of getting head.

$P(A/E_1) = 1, P(A/E_2) = \frac{3}{4}, P(A/E_3) = \frac{1}{2}.$

$P(E_1/A) = \frac{P(A/E_1).P(E_1)}{P(A/E_1).P(E_1)+P(A/E_2).P(E_2)+P(A/E_3).P(E_3)}$

$= \frac{\frac{1}{3}.1}{\frac{1}{3}.1+\frac{1}{3}.\frac{3}{4}+\frac{1}{3}\frac{1}{2}}$

$= \frac{4}{9}$

8. (c)

$(x-1)\log(1-x)+1-x\ln x+C$

**Explanation:**

$\int log(1-x)dx - \int \log x\ dx \Rightarrow (x-1)\log(1-x)+1-x\ln x+C$

9. (d)

$\cos\theta = \left|\frac{\vec{b_1}.\vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right|$

**Explanation:**

If $\theta$ is the acute angle between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ *and* $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ , then cosine of the angle between these two lines is given by : $\cos\theta = \left|\frac{\vec{b_1}.\vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right|$

Because angle between two lines is the angle between their parallels

10. (d)

$\overrightarrow{OA},\ \overrightarrow{OB}, \overrightarrow{OC}$

**Explanation:**

Position vectors of any point in space are usually calculated from the origin, so we write $\overrightarrow{OA},\ \overrightarrow{OB}, \overrightarrow{OC}$ for three points A ,B,C in space to represent the origin as the initial point.

11. subset

12. $\frac{1}{10}$

13. -1

14. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$

**OR**

Indefinite

15. unbounded

OR

convex

16. The area $\Delta$ of triangle ABC is given by

$\Delta = \frac{1}{2}\begin{vmatrix} 5 & 4 & 1 \\ -2 & 4 & 1 \\ 2 & -6 & 1 \end{vmatrix}$

$\Rightarrow \Delta = \frac{1}{2}\begin{vmatrix} 5 & 4 & 1 \\ -7 & 0 & 0 \\ -3 & -10 & 0 \end{vmatrix}$ [Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$]

$\Rightarrow \Delta = \frac{1}{2} \times 1 \times \begin{vmatrix} -7 & 0 \\ -3 & -10 \end{vmatrix}$ [Ecpanding along $C_3$]

$\Rightarrow \Delta = \frac{1}{2}(70 - 0) = 35$ sq. units.

17. a = 2, b = -1, c = -2

$$\sqrt{a^2+b^2+c^2}=\sqrt{2^2+(-1)^2+(-2)^2}=\sqrt{4+1+4}=3$$

Therefore, direction cosines are, $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

18. Let I = $\int(ax+b)^3dx$

Put t = ax + b

$\Rightarrow \quad \frac{dt}{dx} = a \Rightarrow \frac{dt}{a} = dx$

$\therefore \quad I = \int \frac{t^3}{a} dt = \frac{1}{a} \cdot \frac{t^4}{4} + C$

$= \frac{(ax+b)^4}{4a} + C$ [put t = ax + b]

**OR**

Let I=$\int \frac{(x+1)(x+\log x)^2}{x} dx$

Put $x + \log x = t$

$\left(1 + \frac{1}{x}\right) dx = dt$

$\left(\frac{x+1}{x}\right) dx = dt$

I= $\int t^2 dt$

$= \frac{t^3}{3} + c$

$= \frac{(x+\log x)^3}{3} + c$

19. f(x) = kx - cos x

f'(x) = k + sin x

For monotonically increasing function f(x), f'(x) $\geq 0$

$\Rightarrow \; k + sin\ x \geq 0$

sin $x \geq -k$

$we\ know\ that\ -1 \ \leq \sin\ x \leq 1$

sin x to be greater than -1 , which gives the condition that k is greater than or equal to 1.

Thus, $k \geq 1$ is the condition for monotonically increasing f(x).

20. Here, we are given that $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$. Therefore, we have

$$3\vec{a} + 2\vec{b} = 3(\hat{i} + \hat{j} - 2\hat{k}) + 2(2\hat{i} - 4\hat{j} + 5\hat{k})$$
$$= (3\hat{i} + 3\hat{j} - 6\hat{k}) + (4\hat{i} - 8\hat{j} + 10\hat{k})$$
$$= 7\hat{i} - 5\hat{j} + 4\hat{k}$$

Hence, direction ratio of vectors $3\vec{a} + 2\vec{b}$ are 7, - 5 and 4.

**Section B**

21. Let f : R $\rightarrow$R and g : R$\rightarrow$R are defined as

f(x) = x + 1 and g(x) = x - 1

Now,

fog(x) = f(g(x)) = f(x - 1) = x - 1 + 1

= x = $I_R$ ...(i)

Again,

fog(x) = f(g(x)) = g(x + 1) = x + 1 - 1

= x = $I_R$ ... (ii)

from (i) & (ii)

fog = gof = $I_R$

22. Let $y = \sin\left(\tan^{-1} e^{-x}\right)$

$$\therefore \frac{dy}{dx} = \cos\left(\tan^{-1} e^{-x}\right) \frac{d}{dx}\left(\tan^{-1} e^{-x}\right) \quad \left[\because \frac{d}{dx} \sin f(x) = \cos f(x) \frac{d}{dx} f(x)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right) \frac{1}{1+(e^{-x})^2} \frac{d}{dx} e^{-x} \quad \left[\because \frac{d}{dx} \tan^{-1} f(x) = \frac{1}{1+(f(x))^2} \frac{d}{dx} f(x)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right) \frac{1}{1+e^{-2x}} e^{-x} \frac{d}{dx}(-x)$$

$$= -\frac{e^{-x} \cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}}$$

**OR**

Since f(x) is polynomial hence f(x) is continuous in the interval [4, 6] and differentiable in (4, 6). Both conditions of L.M.V theorem are satisfied. Therefore, there exists c $\in (4, 6)$ such that,

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$2c + 2 = \frac{f(6) - f(4)}{6 - 4}$

$2c + 2 = \frac{51 - 27}{2}$

$c = 5 \in (4, 6)$

Hence,L.M.V is verified.

23. A (1, 1, 2) B(2, 3, 4) C (1, 5, 5)

$\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\hat{j} + 3\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 0 & 4 & 3 \end{vmatrix}$$

$= -2\hat{i} - 3\hat{j} + 4\hat{k}$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\sqrt{(-2)^2 + (-3)^2 + (4)^2}$

$= \frac{1}{2}\sqrt{29}$ sq unit.

24. Let $y = x^{\frac{1}{2}}$

$x = 0.0036, \Delta x = 0.0001$

Now $y+\Delta y=(x+\Delta x)^{\frac{1}{2}}$

$$\implies \Delta y=(x+\Delta x)^{\frac{1}{2}}-y$$

$$\implies \left(\frac{dy}{dx}\right).\Delta x=(x+\Delta x)^{\frac{1}{2}}-(x)^{\frac{1}{2}}$$

$$\implies \frac{1}{2\sqrt{x}}.\Delta x=(x+\Delta x)^{\frac{1}{2}}-(x)^{\frac{1}{2}}$$

$$\implies \frac{1}{2\sqrt{0.0036}}\times 0.0001=(0.0037)^{\frac{1}{2}}-(0.0036)^{\frac{1}{2}}$$

$$\implies \frac{1}{2(0.06)}\times 0.0001=(0.0037)^{\frac{1}{2}}-(0.06)$$

$$\implies \frac{1}{1200}=(0.0037)^{\frac{1}{2}}-(0.06)$$

$$\implies (0.0037)^{\frac{1}{2}}=0.06+\frac{1}{1200}=0.06083$$

**OR**

Let x cm be the edge and y be the volume of the variable cube at any time t.

Rate of increase of edge = 3 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 3 cm/sec

$\Rightarrow \frac{dx}{dt}=3cm/\sec$...(i)

$\Rightarrow y=x^3$

$\therefore$ Rate of change of volume of cube $=\frac{dy}{dt}=\frac{d}{dt}x^3$

$=3x^2\frac{dx}{dt}=3x^2(3)$ [from (1)]

$=9x^2cm^3/\sec$

Putting x = 10cm (given),

$\frac{dy}{dt}=9(10)^2=900cm^3/\sec$

Since $\frac{dy}{dt}$ is positive, therefore volume of cube is increasing at the rate of 900 $cm^3$/sec.

25. $\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

$\overrightarrow{N} = 6\hat{i} - 3\hat{j} + 2\hat{k}, d = 4$

$Required\ distance = \frac{\left|\vec{a}.\vec{N} - d\right|}{\left|\overrightarrow{N}\right|}$ $\left[\because \vec{r}.\vec{N} = d\right]$

$= \frac{\left|\left(2\hat{i}+5\hat{j}-3\hat{k}\right).\left(6\hat{i}-3\hat{j}+2\hat{k}\right)-4\right|}{\left|6\hat{i}-3\hat{j}+2\hat{k}\right|}$

$= \frac{|12-15-6-4|}{\sqrt{36+9+4}} = \frac{13}{7}$

26. We know that the repeated throws of a die are Bernoulli's trials., Then

S = {1, 2, 3, 4, 5, 6} $\Rightarrow n(S) = 6$

A = {1, 3, 5} $\Rightarrow n(A) = 3$

p = P (a success) = $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ and q = 1 - p $= 1 - \frac{1}{2} = \frac{1}{2}$

n = 6 and P (X = r) = C (n, r) $p^r q^{n-r}$

i. r = 5

   P (X = 5) = C (6, 5) $\left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) = 6 \times \frac{1}{64} = \frac{3}{32}$

ii. r = 5, 6

   P (at least 5 success) = P (X = 5) + P (X = 6) = $\frac{6}{64} + \left(\frac{1}{2}\right)^6 = \frac{6}{64} + \frac{1}{64} = \frac{7}{64}$

iii. P (at most 5 success) = 1 – P (X = 6) = $1 - \frac{1}{64} = \frac{63}{64}$

## Section C

27. We need to prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Let us consider, $\sin^{-1}\frac{8}{17} = x$ and $\sin^{-1}\frac{3}{5} = y, \quad x, y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \quad \sin x = \frac{8}{17}$ and $\sin y = \frac{3}{5}$

Now, $\cos^2 x = 1 - \sin^2 x = 1 - \frac{64}{289} = \frac{225}{289}$

$\Rightarrow \quad \cos x = \sqrt{\frac{225}{289}} = \frac{15}{17}$ [taking positive square root as x $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ]

and $\cos^2 y = 1 - \sin^2 y = 1 - \frac{9}{25} = \frac{16}{25}$

$\Rightarrow \quad \cos y = \sqrt{\frac{16}{25}} = \frac{4}{5}$ [taking positive square root as $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ]

Clearly, tan x = $\frac{\sin x}{\cos x} = \frac{8}{15}$ and $\tan y = \frac{3}{4}$

$\Rightarrow \quad x = \tan^{-1}\frac{8}{15}$ and $y = \tan^{-1}\frac{3}{4}$

Now, in general we can write $\text{LHS} = x + y = \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$

$= \tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right) = \tan^{-1}\left(\frac{32+45}{60-24}\right) = \tan^{-1}\left(\frac{77}{36}\right) = RHS$

Hence proved.

28. According to the question, y = $\left(x+\sqrt{1+x^2}\right)^n$ ..........(i)

Differentiating both sides w.r.t x,

$\Rightarrow \frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)$ [ Using chain rule of derivative]

$\Rightarrow \quad \frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left(\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{n\left(x+\sqrt{1+x^2}\right)^n}{\sqrt{1+x^2}}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{ny}{\sqrt{1+x^2}}$ [ From Equation(i)]

$\Rightarrow \sqrt{1+x^2}\frac{dy}{dx} = ny$.........(ii)

Differentiating both sides w.r.t x again,

$\Rightarrow \sqrt{1+x^2}\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{1+x^2}}\cdot\frac{dy}{dx} = n\frac{dy}{dx}$

$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\cdot\sqrt{1+x^2}\frac{dy}{dx}$ [ multiplying both sides by $\sqrt{1+x^2}$]

$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\cdot\frac{ny}{\sqrt{1+x^2}}$ [ From Equation(ii)]

$\therefore \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$

Hence Proved

**OR**

We have, x = sint and y = sin pt,

$\therefore \frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = \cos pt.p$

$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p.\cos pt}{\cos t}$ ...(i)

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{\cos t.\frac{d}{dt}(p.\cos pt)\frac{dt}{dx} - p\cos pt.\frac{d}{dt}\cos t.\frac{dt}{dx}}{\cos^2 t}$$

$$= \frac{[\cos t.p.(-\sin pt).p - p\cos pt.(-\sin t)]\frac{dt}{dx}}{\cos^2 t}$$

$$= \frac{\left[-p^2 \sin pt.\cos t + p\ sint.cospt\right].\frac{1}{\cos t}}{\cos^2 t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-p^2 \sin pt.\cos t + p\cos pt.\sin t}{\cos^3 t} \text{ ...(ii)}$$

Since, we have to prove

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$$

$$\therefore LHS = \left(1-\sin^2 t\right)\frac{\left[-p^2\ sinpt.\ cost + p\ cospt.\sin t\right]}{\cos^3 t}$$

$$-\ sint.\frac{p\cos pt}{\cos t} + p^2 \sin pt$$

$$= \frac{1}{\cos^3 t}\begin{bmatrix}\left(1-\sin^2 t\right)\left(-p^2\sin pt.\cos t + p\cos pt.\sin t\right)\\ -p\cos pt.\sin t.\cos^2 t + p^2\sin pt.\cos^3 t\end{bmatrix}$$

$$= \frac{1}{\cos^3 t}\begin{bmatrix}-p^2\sin pt.\cos^3 t + p\cos pt.\sin t.\cos^2 t\\ -p\cos pt.\sin t.\cos^2 t + p^2\sin pt.\cos^3 t\end{bmatrix}\left[\because 1-\sin^2 t = \cos^2 t\right]$$

$$= \frac{1}{\cos^3 t}.0$$

= 0 Hence proved.

29. $E_1$ : A solves the problem

$E_2$ : B solves the problem

i. P (the problem is solved)

= 1 - P (the problem is not solved)

=1 - P(A not solve the problem and B not solve the problem)

$= 1 - P(\bar{E_1})P(\bar{E_2}) = 1 - (1 - P(E_1))(1 - P(E_2)$

$= 1 - \left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\right]$

$= 1 - \frac{1}{3} = \frac{2}{3}$

ii. P (Exactly one of them solves the problem)

= P(A solve the problem and B not) + (B solve the problem and A not)

$= P(E_1)(1 - P(E_2) + P(E_2)\left(1 - P(E_1)\right)$

$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{3}\left(1 - \frac{1}{2}\right) = \frac{1}{2}$

30. Let number of necklaces and bracelets produced by firm per day be x and y, respectively.Given that the maximum number of both necklaces and bracelets that firm can handle per day is almost 24.

which means that the firm can produce maximum 24 items which includes both necklaces and bracelets per day. Hence the inequality related to the number constraint is given as
$\therefore$ x + y $\leq$ 24
Also given that It takes one hour per day to make a bracelet and half an hour per day to make a necklace and maximum number of hours available per day is 16.

Hence the inequality representing the hour constraint is given as
$\therefore$ x + $\frac{1}{2}$y $\leq$ 16 $\Rightarrow$ ( when multiplying throughout the inequality by 2 we get )

2x + y $\leq$ 32

Also the non negative constraints which restricts the feasible region of the problem within the first quadrant is given as x$\geq$0, y$\geq$0, since the given situations are real world connected and cannot have the solution as negative which means that the values of the variables x and y are non negative.
Let z be the objective function which represents the total maximum profit.Hence the equation of the profit function Z is given as
$\therefore$ z = 100x + 300y, which is to be maximised
subject to the constraints,
x + y $\leq$ 24
2x + y $\leq$32
x, y $\geq$0

31. We have, $xy\frac{dy}{dx} = (x+2)(y+2)$
On separating the variables, we get
$\frac{ydy}{y+2} = \frac{x+2}{x}dx$
$\Rightarrow \left(\frac{y+2-2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$
$\Rightarrow \left(1 - \frac{2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$

On integrating both sides, we get

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$$

$\Rightarrow \quad y-2\log|y+2|=x+2\log|x|+C$ ...(i)

Given that y = -1, when x = 1

On putting x = 1 and y = -1 in Eq. (i), we get

-1 - 2log(1) = 1 + 2log|1| + C

$\Rightarrow \quad -1=1+C\Rightarrow C=-2$

On putting C = -2 in Eq. (i), we get

y - 2log|y + 2| = x + 2log|x| - 2

which is required particular solution.

**OR**

According to the question, We have to solve

$y^2dx + (x^2 - xy + y^2)dy = 0$

The given equation can be rewritten as,

$\frac{dy}{dx}=\frac{-y^2}{x^2-xy+y^2}$ ...(i)

This is homogeneous differential equation.

Now, on putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ in Eq. (i),we get,

$$v+x\frac{dv}{dx}=\frac{-v^2x^2}{x^2-vx^2+v^2x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{-v^2}{1-v+v^2}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{-v^2}{1-v+v^2}-v\Rightarrow x\frac{dv}{dx}=\frac{-v-v^3}{1-v+v^2}$$

$$\therefore \frac{1-v+v^2}{v(1+v^2)}dv=-\frac{1}{x}dx$$

On integrating both sides, we get

$$\int\frac{1+v^2}{v(1+v^2)}dv-\int\frac{v}{v(1+v^2)}dv=-\int\frac{1}{x}dx$$

$$\Rightarrow \quad \int\frac{1}{v}dv-\int\frac{1}{1+v^2}dv=-\int\frac{1}{x}dx$$

$$\Rightarrow \quad \log|v|-\tan^{-1}v=-\log|x|+\log C$$

$$\Rightarrow \quad \log\left|\frac{vx}{C}\right|=\tan^{-1}v\Rightarrow\left|\frac{vx}{C}\right|=e^{\tan^{-1}v}$$

$$\Rightarrow \quad \left|\frac{y}{C}\right|=e^{\tan^{-1}(y/x)}[\because vx=y]$$

$\therefore |y|=Ce^{\tan^{-1}(y/x)}$, which is the required solution

32. $I=\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

Put $x=\cos\theta$

$dx=-\sin\theta d\theta$

$\therefore I=\int \tan^{-1}\sqrt{\left(\frac{1-\cos\theta}{1+\cos\theta}\right)}\times-\sin\theta d\theta$

$=\int \tan^{-1}\sqrt{\left(\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)}\times-\sin\theta d\theta$

$=\int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(-\sin\theta)\,d\theta$

$=-\int \frac{\theta}{2}\sin\theta d\theta=\frac{-1}{2}\int \underset{I}{\theta}.\underset{II}{\sin}\theta d\theta$

$=\frac{-1}{2}[\theta.(-\cos\theta)-\int 1\times(-\cos\theta)d\theta]$

$=\frac{-1}{2}[-\theta.\cos\theta+\sin\theta]+c$

$=\frac{-1}{2}\left[-\theta.\cos\theta+\sqrt{1-\cos^2\theta}\right]+c$

$=\frac{-1}{2}\left[-x.\cos^{-1}x+\sqrt{1-x^2}\right]+c$

**Section D**

33. Here,

D = $\begin{vmatrix}1 & -1 & 3\\ 1 & 3 & -3\\ 5 & 3 & 3\end{vmatrix}$ = 1(9 + 9) + 1(3 + 15) + 3(3 - 15) = 18 + 18 + 3(-12) = 0

$D_1$ = $\begin{vmatrix}6 & -1 & 3\\ -4 & 3 & -3\\ 10 & 3 & 3\end{vmatrix}$

= $\begin{vmatrix}2 & 2 & 0\\ -4 & 3 & -3\\ 6 & 6 & 0\end{vmatrix}$ ($R_1 \to R_1 + R_2$, $R_3 \to R_3 + R_2$)

$= 3 \begin{vmatrix} 2 & 2 & 0 \\ -4 & 3 & -3 \\ 2 & 2 & 0 \end{vmatrix}$

= 0

$D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 1 & -4 & -3 \\ 5 & 10 & 3 \end{vmatrix}$

$= \begin{vmatrix} 2 & 2 & 0 \\ 1 & -4 & -3 \\ 6 & 6 & 0 \end{vmatrix}$ $(R_1 \to R_1 + R_2, R_3 \to R_3 + R_2)$

$= 3 \begin{vmatrix} 2 & 2 & 0 \\ 1 & -4 & -3 \\ 2 & 2 & 0 \end{vmatrix}$

= 0

$D_3 = \begin{vmatrix} 1 & -1 & 6 \\ 1 & 3 & -4 \\ 5 & 3 & 10 \end{vmatrix}$

$= \begin{vmatrix} 1 & -1 & 6 \\ 0 & 4 & -10 \\ 0 & 8 & -20 \end{vmatrix}$ $(R_2 \to R_2 + R_1, R_3 \to R_3 + 5R_1)$

= 1(-80 + 80) + 0 + 0 = 0

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinnite solution.

Consider the first two equations, writtten as

x - y = 6 - 3z

x + 3y = -4 + 3z

Solving by Cramer's rule. Here,

$D = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix}$ = 3 + 1 = 4

$D_1 = \begin{vmatrix} 6 - 3z & -1 \\ -4 + 3z & 3 \end{vmatrix}$ = 3 (6 - 3z) + (-4 + 3z) = 14 - 6z

$D_2 = \begin{vmatrix} 1 & 6 - 3z \\ 1 & -4 + 3z \end{vmatrix}$ = (-4 + 3z) - (6 - 3z) = -10 + 6z

$\therefore$ x = $\frac{D_1}{D} = \frac{14-6z}{4} = \frac{7-3z}{2}$

y = $\frac{D_2}{D} = \frac{6z-10}{4} = \frac{3z-5}{2}$

Let z = k, then

x = $\frac{7-3k}{2}$ , y = $\frac{3k-5}{2}$ , z = k are the infinite solutions of the given system of equations.

**OR**

Put $\tan\frac{\alpha}{2} = t$

$$A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

$$\text{L.H.S} = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= (I - A)\begin{bmatrix} \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+\tan^2\frac{\alpha}{2}} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{t.2t}{1+t^2} & \frac{-2t}{1+t^2} + t\left(\frac{1-t^2}{1+t^2}\right) \\ -t\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} & -t\left(\frac{-2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t^3-t}{1+t^2} \\ \frac{t^3+t}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$

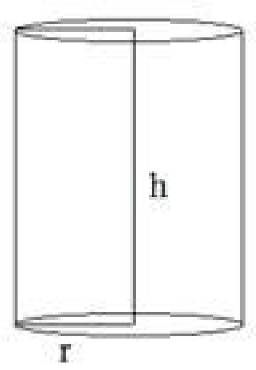
$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

Thus, L.H.S = R.H.S

Hence proved

34. Let radius of the cylinder = r

Height of the cylinder = h |




$s = 2\pi rh + 2\pi r^2 \quad (1)$

$\Rightarrow \frac{s-2\pi r^2}{2\pi r} = h$

Now volume of cylinder is, $v = \pi r^2 h$

$\Rightarrow v = \pi . r^2 \left(\frac{s-2\pi r^2}{2\pi r}\right)$

$\Rightarrow v = \frac{1}{2}\left[sr - 2\pi r^3\right]$

Now, $\frac{dv}{dr} = \frac{1}{2}\left[s - 6\pi r^2\right]$

$\Rightarrow \frac{d^2v}{dr^2} = \frac{1}{2}[0 - 12\pi r]$

For maximum/minimum

$\frac{dv}{dr} = 0$

$\Rightarrow s = 6\pi r^2$

From equ (1)

$$\Rightarrow 2\pi rh + 2\pi r^2 = 6\pi r^2$$
$$\Rightarrow r = \frac{h}{2}$$
$$\left[\frac{d^2v}{dr^2}\right]_{r=\frac{h}{2}} = \frac{1}{2}\left[0 - 12\pi \times \frac{h}{2}\right]$$
$$= -3\pi h < 0$$

$\Rightarrow$ s is maximum at $r = \frac{h}{2}$

Hence h = 2r

35. According to the question , given region is (x, y) : $x^2 + y^2 \leq 16, x^2 \leq 6y$

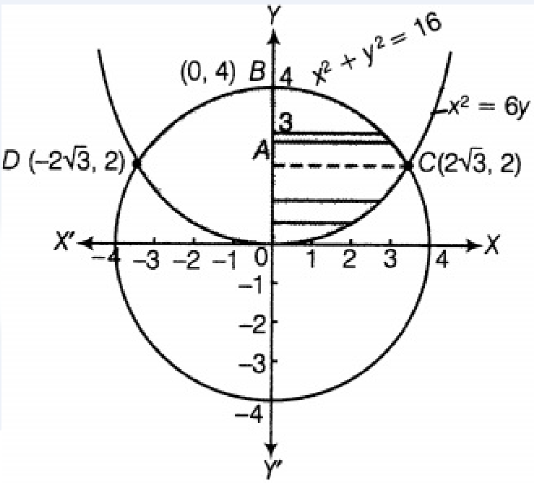
Above region consists a parabola whose vertex is (0, 0) and

Axis of parabola is along Y-axis.

Above region also consists a circle $x^2 + y^2 = 16$ whose centre is (0, 0) and

Radius of circle is = 4.

First, let us sketch the region, as shown below:




For finding the points of intersection of two curves, we have

$x^2 + y^2 = 16$ .......(i)

and $x^2 = 6y$ ....... (ii)

On putting $x^2 = 6y$ from Eq.(ii) in Eq. (i), we get

$y^2 + 6y - 16 = 0$

$y^2 + 8y - 2y - 16 = 0$

y(y + 8) - 2 (y + 8) = 0

(y - 2) (y + 8) = 0

y = 2 or - 8

When y = 2, then from Eq. (ii), we get

x = ±$\sqrt{12}$ = ± 2$\sqrt{3}$

When y = - 8, then from Eq. (ii), we get

$x^2$ = - 48 which is not possible [∵ square root of negative terms does not exist.]

So, y = -8 is rejected.

Here, we consider only one value of y i.e. 2

Thus, the two curves meet at points C(2$\sqrt{3}$, 2) and D(-2$\sqrt{3}$, 2).

Now,

Required area = Area of shaded region OCBDO

= 2 [ Area of region OACO + Area of region ABCA]

$$= 2\left[\int_0^2 x_{\text{ (parabola)}} dy + \int_2^4 x_{\text{ (circle)}} dy\right]$$

$$= 2\left[\int_0^2 \sqrt{6y} dy + \int_2^4 \sqrt{16 - y^2} dy\right]$$

$$= 2\left[\sqrt{6}\int_0^2 \sqrt{y} dy + \int_2^4 \sqrt{16 - y^2} dy\right]$$

$$= 2\left(\left[\sqrt{6} \cdot y^{3/2} \cdot \frac{2}{3}\right]_0^2 + \left[\frac{y}{2}\sqrt{16 - y^2} + \frac{16}{2}\sin^{-1}\frac{y}{4}\right]_2^4\right)$$

$$= 2\left\{\left[\frac{2\sqrt{6}}{3} y^{3/2}\right]_0^2 + \left[0 + 8\sin^{-1} 1 - \sqrt{12} - 8\sin^{-1}\frac{1}{2}\right]\right\}$$

$$= 2\left[\frac{2\times\sqrt{2}\times\sqrt{3}}{3} \times \left[(\sqrt{2})^2\right]^{3/2} + 8\sin^{-1}\left(\sin\frac{\pi}{2}\right) - 2\sqrt{3} - 8\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right]$$

$$= 2\left[\frac{2\sqrt{2}\times\sqrt{3}}{3} \times 2\sqrt{2} + 4\pi - 2\sqrt{3} - \frac{8\pi}{6}\right]$$

$$= 2\left[\frac{8\sqrt{3}}{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right]$$

$$= 2\left[\frac{2\sqrt{3}}{3} + \frac{8\pi}{3}\right]$$

$$= \left(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3}\right)$$

$\therefore I = \frac{4\sqrt{3}}{3} + \frac{16\pi}{3}$ sq units.

**OR**

According to the question ,

Given region is $(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}$

Above region has two equations

$y = |x - 1|$ and

y = $\sqrt{5 - x^2}$

Now ,$y = |x-1| = \begin{cases} x-1, \text{ if } x-1 \geq 0 \\ -(x-1), \text{ if } x-1 < 0 \end{cases}$

$\therefore \quad y = \begin{cases} x-1, & \text{if } x \geq 1 \\ 1-x, & \text{if } x < 1 \end{cases}$

Also, other curve is

y = $\sqrt{5-x^2}$

Squaring on both sides, we get

$y^2 = 5 - x^2$

$\Rightarrow x^2 + y^2 = 5$ which represents equation of circle

centre of the circle is (0, 0)

radius of circle is = $\sqrt{5}$

But the actual equation of curve is y =$\sqrt{5-x^2}$ which represents a semi-circle whose centre is ( 0, 0) and radius r = $\sqrt{5}$

For finding the points of intersection of the curves, we have

$y = 1 - x$......(i)

$y = x - 1$......(ii)

and

y = $\sqrt{5-x^2}$.....(iii)

On putting y = 1 - x from Eq. (i) in Eq. (iii), we get

1 - x = $\sqrt{5-x^2}$

Squaring on both sides ,

$\Rightarrow x^2 + (1 - x)^2 = 5$

$\Rightarrow x^2 + 1 + x^2 - 2 = 5$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x - 2) + 1 (x - 2) = 0$

$\Rightarrow (x + 1) (x - 2) = 0$

$\therefore$ x = - 1 or 2

When x = -1, then y = $\sqrt{5-x^2} = \sqrt{5-1} = \sqrt{4} \Rightarrow$y = 2

When x = 2, then y = $\sqrt{5-x^2} = \sqrt{5-4} = \sqrt{1} \Rightarrow$y = 1

So, points of intersection of Eqs. (i) ad (iii) are (-1, 2) and (2, 1).

Similarly, on solving Eq. (ii) and Eq. (iii), we get

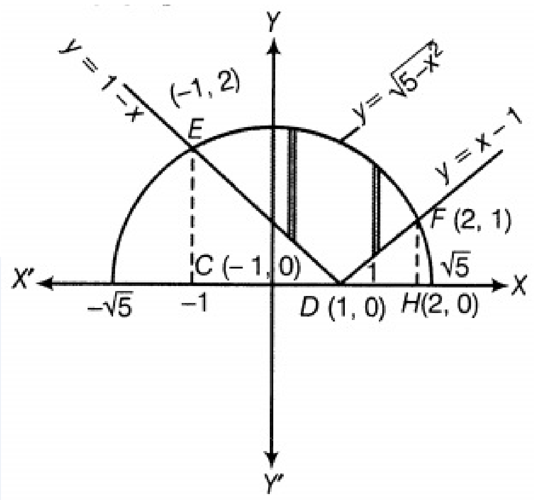
x = -1 or 2

When x = -1 , then y = 2

When x = 2, then y = 1.

Hence, the two curves intersect at (-1, 2) and (2, 1).

On drawing the rough sketch, we get the following graph:




Now, required area

$= \int_{-1}^{2} y_{(\text{ circle })} dx - \int_{-1}^{1} y_{(\text{ for line } DE)} dx$

$= \int_{-1}^{2} \sqrt{5-x^2} dx - \int_{-1}^{1} (1-x) dx - \int_{1}^{2} (x-1) dx$

$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$

$= \left[\left(1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) - \left\{-1 + \frac{5}{2}\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right)\right\}\right]$

$- \left[\left(1 - \frac{1}{2}\right) - \left(-1 - \frac{1}{2}\right)\right] - \left[\left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right]$

= 1 + $-\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right) - \left(\frac{1}{2} + \frac{3}{2}\right) - \left(0 + \frac{1}{2}\right)$

= 1 + $\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}} - 2 - \frac{1}{2}$

$= -\frac{1}{2} + \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right)$

= $\left[\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2}\right]$ sq units.

36. According to the question,we are required to find the equation of plane passing through point P(1, 1, 1 ) and containing the line

$\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}-5\hat{k})$. Moreover, we need to show that plane contains the line $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$.

We know that Equation of plane passing through point P(1, 1, 1) is given by

a (x - 1) + b (y - 1) + c (z - 1) = 0 ............(i)

[since equation of plane passing through ($x_1$, $y_1$, $z_1$) is given as a (x - $x_1$) + b (y - $y_1$) + c (z - $z_1$) = 0]

Given equation of line is

$\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}-5\hat{k})$.....(ii)

Direction ratios of the line are (3, -1, -5) and the line passes though point (-3, 1, 5).

Now, as the plane (i) contains line (ii), therefore,

a (-3-1) + b (1- 1) + c (5 - 1) = 0

[as plane contains a line, it means point of line lies on a plane.]

-4a + 4c =0

4a = 4c

a = c .....(iii)

Since, plane contains a line, therefore normal to the plane is perpendicular to the line.So,

3a - b - 5c = 0 ............(iv)

[∵ $aa_1 + bb_1 + cc_1 = 0$]

Therefore on putting a= c in Eq. (iv), we get,

3c - b -5c =0

-b - 2c =0

b= -2c

Therefore,on putting a = c and b = -2c in Eq. (i), we get the required equation of plane as

c (x - 1) - 2c (y - 1) + c (z - 1) = 0

On dividing both sides by c, we get

x - 1 - 2y + 2 + z - 1 = 0

x - 2y + z = 0 ................(v)

Now, we have to show that the above plane (v) contains the line

$\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$.....(vi)

Vector equation of plane (v) is

$\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$ ..........(vii)

The plane (vi) will contains line (v), if it satisfies the following  two conditions:

(i) it passes through $-\hat{i} + 2\hat{j} + 5\hat{k}$

(ii) it is parallel to the line

Now,we have, $(-\hat{i} + 2\hat{j} + 5\hat{k}).(\hat{i} - 2\hat{j} + \hat{k})$

= -1 -4 + 5=0 $[\because \vec{a} \cdot \vec{n} = d]$

Therefore, the plane passes through the point $-\hat{i} + 2\hat{j} + 5\hat{k}$

Now,  plane will be parallel to line if ,

$(\hat{i} - 2\hat{j} - 5\hat{k}) \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$

(1) (1) - 2 (-2) - 5 (1) = 0

1 + 4 - 5 = 0

=>  0=0 ,which is  always true.

Hence it follows that the plane contains the given line.