CBSE Class 12 - Mathematics

Sample Paper 10

Maximum Marks:

Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. Let for any matrix M, M^{-1} exist. Which of the following is not true.

   a. none of these

   b. $(M^{-1})^{-1} = M$

   c. $(M^{-1})^2 = (M^2)^{-1}$

   d. $(M^{-1})^{-1} = (M^{-1})^1$

2. The value of det. $\begin{bmatrix} a & 0 & 0 & 0 \\ 2 & b & 0 & 0 \\ 4 & 6 & c & 0 \\ 6 & 8 & 10 & d \end{bmatrix}$ is

a. a + b + c + d

b. None of these

c. abcd

d. 0

3. If $y = \tan^{-1}$x and $z = \cot^{-1} x$ then $\frac{dy}{dz}$ is equal to

a. 1

b. $\frac{\pi}{2}$

c. – 1

d. None of these

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that all the five cards are spades?

a. $\frac{5}{1024}$

b. $\frac{3}{1024}$

c. $\frac{7}{1024}$

d. $\frac{1}{1024}$

5. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

a. $\frac{11}{13}$

b. $\frac{7}{13}$

c. $\frac{12}{13}$

d. $\frac{9}{13}$

6. In a LPP, the linear inequalities or restrictions on the variables are called

a. Limits

b. Inequalities

c. Linear constraints

d. Constraints

7. $\frac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0}$ is equal to

a. $\tan 37^0$

b. $\tan 53^0$

c. $\tan 82^0$

d. None of these

8. If $\int\limits_{-2}^{5} f(x)dx = 4, \int\limits_{0}^{5} (1 + f(x))dx = 7$, then the value of the integral $\int\limits_{-2}^{0} f(x)$ dx is equal to

a. –3

b. 2

c. 3

d. 5

9. $\overrightarrow{PQ}$ is a vector joining two points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2).If $\left|\overline{PQ}\right| = d$, Direction

cosines of $\overrightarrow{PQ}$ are

a. $\frac{x_2-x_1}{d}, \frac{y_2-y_1}{d}, \frac{z_2+z_1}{d}$

b. $\frac{x_2-x_1}{d}, \frac{y_2+y_1}{d}, \frac{z_2-z_1}{d}$

c. $\frac{x_2-x_1}{d}, \frac{y_2-y_1}{d}, \frac{z_2-z_1}{d}$

d. $\frac{x_2+x_1}{d}, \frac{y_2-y_1}{d}, \frac{z_2-z_1}{d}$

10. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find $\angle ABC.$ [$\angle ABC$ is the angle between the vectors $\overrightarrow{BA}\ and\ \overrightarrow{BC}$]

a. $\cos^{-1}\left(\frac{13}{\sqrt{102}}\right)$

b. $\cos^{-1}\left(\frac{11}{\sqrt{102}}\right)$

c. $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$

d. $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

11. Fill in the blanks:

A function f : X → Y is said to be an ________ function, if every element of Y is image of some element of set X under f.

12. Fill in the blanks:

If f(x) = |cos x|, then $f'\left(\frac{\pi}{4}\right)$ = ________.

13. Fill in the blanks:

A matrix which is not a square matrix is called a ________ matrix.

14. Fill in the blanks:

Equation of a plane which is at a distance p from the origin with direction cosines of the normal to the plane as l, m, n, is ________.

OR

Fill in the blanks:

Direction ratios of two _________ lines are proportional.

15. Fill in the blanks:

The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is ________.

OR

Fill in the blanks:

The magnitude of the vector $6\hat{i}$ + $2\hat{j}$ + $3\hat{k}$ is ________.

16. If A = $= \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then write A^{-1} in terms of A.

17. Evaluate $\int \frac{dx}{\sqrt{1-x^2}}$.

OR

Find the value of $\int \frac{dx}{x^2+16}$.

18. If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$, then find the value of a.

19. Find the slope of tangent to the cure y = $3x^2$ - 4x at point whose x-coordinate is 2.

20. Write the integrating factor of the following differential equation.
$\left(1+y^2\right)+(2xy-\cot y)\frac{dy}{dx}=0.$

Section B

21. $\tan^{-1}(-1)$

OR

Prove that the function f given by f(x) = x^2 - x + 1 is neither strictly increasing nor strictly decreasing on (-1, 1).

22. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

23. Differentiate $\frac{x}{\sin x}$ w.r.t. sin x.

24. Consider two point P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the positions vector of a point R which divides the line joining P and Q in the ratio 2:1

    i. internally
    ii. externally.

OR

Consider two point P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the positions vector of a point R which divides the line joining P and Q in the ratio 2:1

i. internally
ii. externally.

25. Find the equation of a plane through the points (2,3,1) and (4,-5,3) and parallel to the x-axis.

26. In a multiple-choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Section C

27. Let f : N $\rightarrow$ N be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \; if \; n \; is \; odd \\ \frac{n}{2}, \; if \; n \; is \; even \end{cases}$ for all n $\in$ N. State whether the function f is bijective. Justify your answer.

28. If y = $(\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$ ..

OR

For what values of λ is the function f(x) = $\begin{cases} \lambda\left(x^2-2x\right), \text{ if } x \leq 0 \\ 4x+1, \text{ if } x > 0 \end{cases}$ is continuous at x = 0?

29. Find a particular solution satisfying the given condition:
$\cos\left(\frac{dy}{dx}\right) = a\,(a \in R)\,; y = 1$ when x = 0

30. $\int \frac{5}{(x+1)(x^2+9)} dx$

31. From a lot of 30 bulbs which includes 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

OR

A black and a red die are rolled.

a. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
b. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

32. Maximise Z = 3x + 4y, subject to the constraints: $x + y \leqslant 1, x \geqslant 0, y \geqslant 0$.

Section D

33. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = 0$ and hence find A^{-1}.

OR

Using matrix method, solve the system of equations:
2x + y + z = 1,
x - 2y - z = $\frac{3}{2}$,
3y - 5z = 9

34. Find the area enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$.

35. Find the value of p for which the curves $x^2 = 9p(9- y)$ and $x^2 = p (y + 1)$ cut each other at right angles.

OR

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, with its vertex at one end of the major axis.

36. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

Solution

Section A

1. (d) $(M^{-1})^{-1} = (M^{-1})^{1}$

   Explanation:

   Clearly , $(M^{-1})^{-1} = (M^{-1})^{1}$ is not true.

2. (c) abcd

   Explanation:

   The determinant of a lower triangular or an upper triangular matrix is equal to the product of the diagonal elements.

3. (c) – 1

   Explanation:

   $$\frac{dy}{dz} = \frac{\frac{d}{dx}(\tan^{-1}x)}{\frac{d}{dx}(\cot^{-1}x)} = \frac{\frac{1}{1+x^2}}{-\frac{1}{1+x^2}} = -1$$

4. (d) $\frac{1}{1024}$

   Explanation:

   Here, probability of getting a spade from a deck of 52 cards = $\frac{13}{52}$ = $\frac{1}{4}$. p = $\frac{1}{4}$, q = $\frac{3}{4}$.
   let , x is the number of spades, then x has the binomial distribution with n = 5 , p = $\frac{1}{4}$, q = $\frac{3}{4}$.

   P(all 5 cards are spades)= P (x = 5) = ${}^5C_5\left(\frac{3}{4}\right)^0\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$.

5. (c) $\frac{12}{13}$

   Explanation:

   Let $E_1 and E_2$ are events that the student knows the answer and the student guesses respectively. $P(E_1) = \frac{3}{4}, P(E_2) = \frac{1}{4}$.

Let A = event that the student answers correctly.

$P(E_1) = \frac{3}{4}, P(E_2) = \frac{1}{4}$

$P(A/E_1) = 1$(as it is sure event)

$P(A/E_2) = \frac{1}{4}$

$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)} = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \frac{12}{13}$

6. (c) Linear constraints

   Explanation:

   In an LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

7. (a) $\tan 37^0$

   Explanation:

   $\frac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0}$

   $= \frac{\frac{\cos 8^0}{\cos 8^0} - \frac{\sin 8^0}{\cos 8^0}}{\frac{\cos 8^0}{\cos 8^0} + \frac{\sin 8^0}{\cos 8^0}}$

   $= \frac{1-\tan 8^0}{1+\tan 8^0} = \tan(45^0 - 8^0) = \tan 37^0$

8. (b) 2

   Explanation:

   $\because \int_0^5 (1 + f(x))dx = 7$

   $\therefore \int_0^5 dx + \int_0^5 f(x)dx = 7$

   $\Rightarrow [x]_0^5 + \int_0^5 f(x)dx = 7$

$$\Rightarrow \int_{0}^{5} f(x)dx = 7 - 5 = 2,$$

Also, $\int_{-2}^{5} f(x)dx = 4$

$$\Rightarrow \int_{-2}^{0} f(x)dx + \int_{0}^{5} f(x)dx = 4$$

$$\Rightarrow \int_{-2}^{0} f(x)dx = 2$$

9. (c) $\frac{x_2 - x_1}{d}, \frac{y_2 - y_1}{d}, \frac{z_2 - z_1}{d}$
   Explanation:

   since we know Direction cosines of a line are coefficient of i,j,k of a unit vector along that line,first find a vector $\overrightarrow{PQ} = (x2 - x1)\hat{i} + (y2 - y1)\hat{j} + (z2 - z1)\hat{k}$ then to convert it unit vector divide by its magnitute $|\overrightarrow{PQ}|$ the coefficient of this unit vector will be $\frac{x_2 - x_1}{d}, \frac{y_2 - y_1}{d}, \frac{z_2 - z_1}{d}$
10. (d) $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$
    Explanation:

    Position vectors of the points A, B and C are $\hat{i} + 2\hat{j} + 3\hat{k}, -\hat{i}, and, \hat{j} + 2\hat{k}$ respectively.
    Then;

    $$cos\theta = \frac{\overrightarrow{BA}\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$$

    $$= \frac{(2\hat{i}+2\hat{j}+3\hat{k}).(\hat{i}+\hat{j}+2\hat{k})}{\sqrt{17}\sqrt{6}}$$

    $$\Rightarrow cos\theta = \frac{10}{\sqrt{102}}$$

    $$\Rightarrow \angle ABC = cos^{-1}(\frac{10}{\sqrt{102}})$$
11. onto

12. $\frac{-1}{\sqrt{2}}$

13. rectangular

14. lx + my + nz = p

OR

Parallel

15. $\sqrt{3}$

OR

7

16. We have, $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

Clearly, adj A = $\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

and $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$

$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$\equiv \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19} A$

17. Let I = $\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{(1)^2 - x^2}}$

$= \sin^{-1} x + C \left[\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C\right]$

OR

Let I = $\int \frac{dx}{x^2+16} = \int \frac{dx}{x^2+(4)^2}$

$= \frac{1}{4} \tan^{-1} \frac{x}{4} + C \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c\right]$

18. If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$, then we have to find the value of a.

Given $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ ....(i)

Now, consider $I=\int_0^a \frac{1}{x^2+(2)^2}dx$

$\Rightarrow \quad I=\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^a \left[\because \int \frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+C\right]$

$\Rightarrow \quad I=\frac{1}{2}\tan^{-1}\frac{a}{2}-\frac{1}{2}\tan^{-1}(0)$

$\Rightarrow \quad I=\frac{1}{2}\tan^{-1}\frac{a}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{1}{2}\tan^{-1}\frac{a}{2}=\frac{\pi}{8}\Rightarrow \tan^{-1}\frac{a}{2}=\frac{\pi}{4}\Rightarrow \frac{a}{2}=\tan\frac{\pi}{4}$

$\Rightarrow \quad \frac{a}{2}=1\Rightarrow a=2 \quad \left[\because \tan\frac{\pi}{4}=1\right]$

19. We have to find the slope of tangent to the cure y = 3x^2 - 4x at point whose x-coordinate is 2.

Now, y = 3x^2 - 4x

On differentiating both sides w.r.t. x, we get,

$\frac{dy}{dx}=6x-4$

Now, slope of tangent= $\left(\frac{dy}{dx}\right)_{x=2}$ = 6(2) - 4 = 12 - 4 = 8

Hence, required slope is 8.

20. Given differential equation is

$\left(1+y^2\right)+(2xy-\cot y)\frac{dy}{dx}=0.$

The above equation can be rewritten as

$(\cot y-2xy)\frac{dy}{dx}=1+y^2$

$\Rightarrow \quad \frac{\cot y-2xy}{(1+y^2)}=\frac{dx}{dy}$

$\Rightarrow \quad \frac{dx}{dy}=\frac{\cot y}{1+y^2}-\frac{2xy}{1+y^2}$

$\Rightarrow \quad \frac{dx}{dy}+\frac{2y}{1+y^2}\cdot x=\frac{\cot y}{1+y^2}$

which is a linear differential equation of the form

$\frac{dx}{dy}+Px=Q$, here $P=\frac{2y}{1+y^2}$ and $Q=\frac{\cot y}{1+y^2}$.

Now, integrating factor $=e^{\int pdy}=e^{\int \frac{2y}{1+y^2}dy}$

Put 1 + y^2 = t

$\Rightarrow$ 2ydy = dt

$\therefore \quad \mathrm{IF}=e^{\int \frac{dt}{t}}=e^{\log|t|}=t=1+y^2$

Section B

21. Let $\tan^{-1}(-1) = y$

$\Rightarrow$ tan y = - 1

$\Rightarrow \tan y = -\tan \frac{\pi}{4}$

$\Rightarrow \tan y = \tan\left(-\frac{\pi}{4}\right)$

Since, the principal value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

OR

Given: $f(x) = x^2 - x + 1$f(x) = x^2 - x + 1

$\Rightarrow$ f'\(x) = 2x - 1

f(x) is strictly increasing if f'(x) < 0

$\Rightarrow$2x - 1 > 0

$\Rightarrow x > \frac{1}{2}$

i.e., increasing on the interval $\left(\frac{1}{2}, 1\right)$

f(x) is strictly decreasing if f'(x) < 0

$\Rightarrow$ 2x - 1 < 0

$\Rightarrow x < \frac{1}{2}$

i.e., decreasing on the interval $\left(-1, \frac{1}{2}\right)$

hence, f(x) is neither strictly increasing nor decreasing on the interval (-1, 1).

22. Let x cm be the edge and y be the volume of the variable cube at any time t.

Rate of increase of edge = 3 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 3 cm/sec

$\Rightarrow \frac{dx}{dt} = 3cm/$sec...(i)

$\Rightarrow y = x^3$

$\therefore$ Rate of change of volume of cube $= \frac{dy}{dt} = \frac{d}{dt}x^3$

$= 3x^2\frac{dx}{dt} = 3x^2(3)$ [from (1)]

$= 9x^2 cm^3/\sec$

Putting x = 10cm (given),

$\frac{dy}{dt} = 9(10)^2 = 900cm^3/\sec$

Since $\frac{dy}{dt}$ is positive, therefore volume of cube is increasing at the rate of 900 cm^3/sec.

23. Let $u = \frac{x}{\sin x}$ and $v = \sin x$

$\therefore \frac{du}{dx} = \frac{\sin x.\frac{d}{dx}x - x.\frac{d}{dx}\sin x}{(\sin x)^2}$

$= \frac{\sin x - x\cos x}{\sin^2 x}$ ...(i)

and $\frac{dv}{dx} = \frac{d}{dx}\sin x = \cos x$ ...(ii)

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\sin x - x\cos x/\sin^2 x}{\cos x}$

$= \frac{\sin x - x\cos x}{\sin^2 x\cos x} = \frac{\frac{\sin x - x\cos x}{\cos x}}{\frac{\sin^2 x\cos x}{\cos x}}$

[Dividing by cos x in both numerator and denominator]

$= \frac{\tan x - x}{\sin^2 x}$

24. i. $\overrightarrow{OR} = \frac{2\left(\vec{a}+\vec{b}\right)+1\left(3\vec{a}-2\vec{b}\right)}{2+1}$
$= \frac{5\vec{a}}{3}$

ii. $\overrightarrow{OR} = \frac{2\left(\vec{a}+\vec{b}\right)-\left(3\vec{a}-2\vec{b}\right)}{2-1}$
$= \frac{2\vec{a}+2\vec{b}-3\vec{a}+2\vec{b}}{1}$
$= 4\vec{b} - \vec{a}$

OR

i. $\overrightarrow{OR} = \frac{2\left(\vec{a}+\vec{b}\right)+1\left(3\vec{a}-2\vec{b}\right)}{2+1}$
$= \frac{5\vec{a}}{3}$

ii. $\overrightarrow{OR} = \frac{2\left(\vec{a}+\vec{b}\right)-\left(3\vec{a}-2\vec{b}\right)}{2-1}$
$= \frac{2\vec{a}+2\vec{b}-3\vec{a}+2\vec{b}}{1}$
$= 4\vec{b} - \vec{a}$

25. Any plane parallel to x-axis is by+cz+d=0.

If it passes through (2,3,1) and (4,-5,3), then

3b+c+d=0 and -5b+3c+d=0,

i.e , $\frac{b}{1-3} = \frac{c}{-5-3} = \frac{d}{9+5}$

i.e, $\frac{b}{1} = \frac{c}{4} = \frac{d}{-7}$

Hence, the plane parallel to x-axis is y+4z-7=0.

26. $p = \frac{1}{3}$ and q = 1 - p $= 1 - \frac{1}{3} = \frac{2}{3}$

n = 5, r = 4, 5 and P (X = r) = ${}^nC_r p^r q^{n-r}$
P (Four or more success) = P (X = 4) + P (X = 5)
$={}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1+{}^5C_5\left(\frac{1}{3}\right)^5 = 5 \times 2 \times (\frac{1}{3})^5 + \left(\frac{1}{3}\right)^5 = \frac{11}{243}$

Section C

27. f : N $\rightarrow$ N be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \ if\ n\ is\ odd \\ \frac{n}{2}, \ if\ n\ is\ even \end{cases}$

a. f(1) = $\frac{1+1}{2}$ = 1 and f(2) = $\frac{2}{2}$ = 1
The elements 1, 2, belonging to domain of f have the same image 1 in its co-domain.

So, f is not one-one, therefore, f is not injective.

b. Every number of co-domain has pre-image in its domain e.g., 1 has two pre-images 1 and 2.

So, f is onto, therefore, f is not bijective.

28. According to the question, $y = (\sin x)^x + \sin^{-1}\sqrt{x}$ ....(i)

Let u = $(\sin x)^x$...............................(ii)

Then, Equation.(i) becomes, y = u + $\sin^{-1}\sqrt{x}$ ....(iii)

Differentiating both sides of (iii) w.r.t x,we get,

$\frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{d}{dx}(\sqrt{x})$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$ ....(iv)

Taking log on both sides of Equation.(ii),

log u = x log sinx

Differentiating both sides w.r.t x,

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log\sin x) + \log\sin x\frac{d}{dx}(x)$[Using product rule of derivative]

$\Rightarrow \frac{du}{dx} = u\left[x \times \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x(1)\right]$

$\Rightarrow \frac{du}{dx}$ = (sin x)$^{x}\left[\frac{x}{\sin x} \times \cos x + \log\sin x\right]$ [From Eq(ii)]

From (iv),we get,

$\Rightarrow \frac{dy}{dx} = (sinx)^x[x\ cot\ x + log\ sinx] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

OR

If, f(x) = $\begin{cases} \lambda\left(x^2 - 2x\right), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$ is continuous at x = 0,

We shall use definition of continuity to find the value of λ.

Since f(x) is continuous at x=0,therefore,

$(LHL)_{x=0} = (RHL)_{x=0}$ = f(0).........(i)

$\Rightarrow$ f(0) = λ[0-0] = 0,

$LHL = \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h)$

$= \lambda\lim_{h\to0}\left[(0-h)^2 - 2(0-h)\right]$

$= \lambda \times 0 = 0$

$RHL = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$
$= \lim_{h \to 0} 4(0+h)$ + 1 = 1
$\because$ LHL $\neq$ RHL, which is contradiction to Equation (i)
$\therefore$ There is no value of λ for which f(x) is continuous at x = 0.

29. Given: Differential equation $\cos\left(\frac{dy}{dx}\right) = a\,(a \in R)\,; y = 1$ when x = 0

$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$

$\Rightarrow dy = \left(\cos^{-1} a\right) dx$

Integrating both sides,

$\Rightarrow \int 1 dy = \int \left(\cos^{-1} a\right) dx$

$\Rightarrow y = \left(\cos^{-1} a\right) \int 1 dx$

$\Rightarrow y = \left(\cos^{-1} a\right) x + c$ ...(i)

Now putting y = 1 when x = 0 in eq. (i), we get c = 1

Putting c = 1 in eq. (i), y = $(\cos^{-1}a)x + 1$

$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$

$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$

30. Let $\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9}$

$\Rightarrow 5x = A(x^2+9) + (Bx+c)(x+1)$

On comparing coefficients of x^2 and x, we get,
0 = A + B ....(i)
5 = C+B ....(ii)
Eqn. (i) - Eqn (ii), we get
A - C = -5 ....(iii)
Compairing the constant terms

9A + C = 0 ....(iv)

Eqn (iii) + Eqn(iv)

$$\Rightarrow (A - C + 5) - (9A + C) \Rightarrow 10A = -5 \Rightarrow A = -\frac{1}{2}$$

B = 1/2 [by (i)]

$C = \frac{9}{2}$ [by (ii)]

$$I = \int \frac{5x}{(x+1)(x^2+9)} = \int \frac{-1/2}{(x+1)} + \int \frac{\frac{1}{2}x+\frac{9}{2}}{x^2+9} dx$$

$$= -\frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{x}{x^2+9} dx + \frac{9}{2} \int \frac{dx}{x^2+9}$$

$$= -\frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{4} \int \frac{2x}{x^2+9} dx + \frac{9}{2} \int \frac{dx}{x^2+3^2}$$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{9}{2}. \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

31. Let X be a random variable which represents the number of defective bulbs.

Therefore,X can take values 0,1,2,3,4

Total number of bulbs=30

Number of defective bulbs=6

p = probability of getting defective bulb= $\frac{6}{30} = \frac{1}{5}$

q = probability of not getting defective bulb= $\frac{24}{30} = \frac{4}{5}$

$$P(X=0) = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{256}{625}$$

$$P(X=1) = {}^4C_1 p^1 q^3 = 4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^3 = \frac{256}{625}$$

$$P(X=2) = {}^4C_2 p^2 q^2 = 6\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$$

$$P(X=3) = {}^4C_3 p^3 q^1 = 4\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right) = \frac{16}{625}$$

$$P(X=4) = {}^4C_0 p^4 q^0 = 1\left(\frac{1}{5}\right)^4 = \frac{1}{625}$$

X	**0**	**1**	**2**	**3**	**4**
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

OR

a. n(s) = 6 $\times$ 6 = 36

Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

A = (46, 64, 55, 36, 63, 45, 54, 65, 56, 66) $\Rightarrow$ n(A) = 10

P (A) = B = (51, 52, 53, 54, 55, 56) $\Rightarrow$ n(B) = 6

P (B) = $\frac{n(B)}{n(S)} = \frac{6}{36}$

$A \cap B$ = (55, 56) $\Rightarrow n\left(A \cap B\right)$ = 2

$P(A \cap B) = \frac{2}{36}$

$P\left(A|B\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{2}{6} = \frac{1}{3}$

b. Let A denote the sum is 8

$\therefore$ A = {(2, 6). (3, 5), (4, 4), (5, 3), (6, 2)}

B = Red die results in a number less than 4, either first or second die is red

$\therefore$ B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}

P (B) = $\frac{n(B)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

$A \cap B$ = {(2, 6), (3, 5) $\Rightarrow n\left(A \cap B\right) = 2$

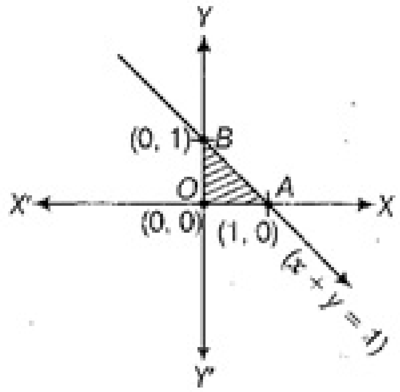
$P\left(A \cap B\right)$ = $\frac{2}{36} = \frac{1}{18}$

$P\left(A|B\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{1}{2}} = \frac{2}{18} = \frac{1}{9}$

32. Maximise Z = 3x + 4y. Subject to the constraints

$x + y \leqslant 1, x \geqslant 0, y \geqslant 0$

The Shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0, 0), (1, 0) and (0, 1), respectively.



Corner Points	Corresponding value of Z
(0, 0)	0
(1, 0)	3
(0, 1)	4 (Maximum)

Hence, the maximum value of Z is 4 at (0, 1).

Section D

33. We have, $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$

$3A=3\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}$

And $7I=7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$\therefore A^2-3A-7I=\begin{bmatrix}22 & 9\\ -3 & 1\end{bmatrix}-\begin{bmatrix}15 & 9\\ -3 & -6\end{bmatrix}-\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$

$=\begin{bmatrix}22-15-7 & 9-9-0\\ -3+3-0 & 1+6-7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

=0 Hence proved.

Since, $A^2 - 3A - 7I = 0$

$\Rightarrow A^{-1}[(A^2) - 3A - 7I] = A^{-1}0$

$\Rightarrow A^{-1}A.A - 3A^{-1}A - 7A^{-1}I = 0$ [$\because A^{-1}0 = 0$]

$\Rightarrow IA - 3I - 7A^{-1} = 0$ [$\because A^{-1}A = I$]

$\Rightarrow A - 3I - 7A^{-1} = 0$ [$\because A^{-1}I = A^{-1}$]

$\Rightarrow$ -7A^{-1} = -A + 3I

$$= \begin{bmatrix} -5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{-1}{7} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

OR

The given system of equations can be rewritten as,

AX=B,where,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}.$$

Now,

|A| = 2(10 + 3) -1(-5 - 3) + 0 = 2(13) -1(-8) = 26 + 8 + 34 $\neq$ 0

Thus, A is non-singular. Therefore, its inverse exists.

Now, A_{11} = 13, A_{12} = 5, A_{13} = 3

A_{21} = 8, A_{22} = -10, A_{23} = -6

A_{31} = 1, A_{32} = 3, A_{33} = -5

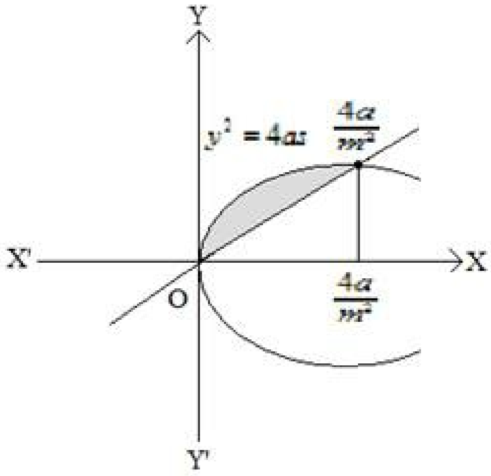
$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix}$$

$$= \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

Hence, x = 1, $y = \frac{1}{2}$, and $z = -\frac{3}{2}$.

34. 

$y^2 = 4ax$ ....(1)

$y = mx$........(2)

Using (2) in (1), we get,

$(mx)^2 = 4ax$

$\Rightarrow m^2x^2 = 4ax$

$x(m^2x - 4a) = 0$

$\Rightarrow x = 0, \frac{4a}{m^2}$

From (2),

When x = 0, y = m(0) = 0

When $x = \frac{4a}{m^2}, y = m \times \frac{4a}{m^2} = \frac{4a}{m}$

$\therefore$ points of intersection are (0, 0) and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$

Area $= \int_0^{4a/m^2} \sqrt{4ax}\,dx - \int_0^{\frac{4a}{m^2}} mx\,dx$

$= \sqrt{4a} \int_0^{\frac{4a}{m^2}} \sqrt{x}\,dx - m \int_0^{\frac{4a}{m^2}} xdx$

$= \sqrt{4a}\left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$

$= \sqrt{4a}\left[\frac{2}{3}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} - 0\right] - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2 - 0\right]$

$= \frac{2}{3m^3}(4a)^2 - \frac{1}{2m^3}(4a)^2$

$= \frac{(4a)^2}{m^3}\left[\frac{2}{3} - \frac{1}{2}\right]$

$= \frac{8a^2}{3m^3} sq\,unit.$

35. Given equations of curves are,

$x^2 = 9p(9- y)$ ....(i)

and $x^2 = p (y + 1)$ ....(ii)

As, these curves cut each other at right angle, therefore their tangents at point of intersection are perpendicular to each other.

So, let us first find the point of intersection and slope of tangents to the curves.

From Eqs. (i) and (ii), we get

9p (9 - y) = p (y + 1)

$\therefore$ 9(9 - y) = y + 1 [$\because$ p $\neq$ 0, as if p = 0, then curves becomes straight, which will be parallel]

$\Rightarrow$ 81 - 9y = y + 1 $\Rightarrow$ 80 = 10y $\Rightarrow$ y = 8

On substituting the value of y in Eq. (i),we get

x^2 = 9p $\Rightarrow$ $x = \pm 3\sqrt{p}$

Therefore, the points of intersection are $(3\sqrt{p}, 8)$ and $(-3\sqrt{p}, 8)$

Now, consider Eq.(i),we get

$\frac{x^2}{9p} = 9 - y \Rightarrow y = 9 - \frac{x^2}{9p}$

Therefore,on differentiating both sides w.r.t. x, we get,

$\frac{dy}{dx} = \frac{-2x}{9p}$..................(iii)

From Eq. (ii), we get $\frac{x^2}{p}$ = y + 1

$\Rightarrow$ $y = \frac{x^2}{p} - 1$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{2x}{p}$ ....(iv)

Now, for intersection point $(3\sqrt{p}, 8)$,we have slope of tangent to the first curve

$= \frac{-2(3\sqrt{p})}{9p} = \frac{-6\sqrt{p}}{9p}$[using Eq.(iii)]

and slope of tangent to the second curve

$= \frac{2(3\sqrt{p})}{p} = \frac{6\sqrt{p}}{p}$[using Eq.(iv)]

$\because$ Tangents are perpendicular to each other.

Then,

Slope of first curve $\times$ Slope of second curve = - 1

$\frac{-6\sqrt{p}}{9p} \times \frac{6\sqrt{p}}{p}$ = -1

$\Rightarrow \frac{4}{p} = 1 \Rightarrow p = 4$

Thus,the value of p is 4.

OR

Given equation of ellipse is

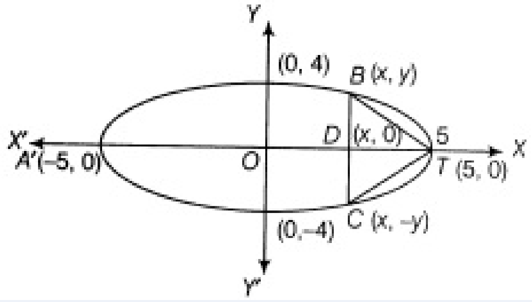
$\frac{x^2}{25}+\frac{y^2}{16}=1$.

Here, a = 5, b = 4

$\therefore$ a > b

So, major axis is along X-axis.

Let $\triangle$BTC be the isosceles triangle which is inscribed in the ellipse and OD = x, BC = 2y and TD = 5 - x.



Let A denotes the area of triangle. Then, we have

$A=\frac{1}{2}\times \text{ base } \times \text{ height } =\frac{1}{2}\times BC\times TD$

$\Rightarrow \quad A=\frac{1}{2}2y(5-x)\Rightarrow A=y(5-x)$

Therefore,on squaring both sides, we get,

$A^2=y^2(5-x)^2.........(i)$

Now, $\frac{x^2}{25}+\frac{y^2}{16}=1$

$\Rightarrow \quad \frac{y^2}{16}=1-\frac{x^2}{25}$

$\Rightarrow \quad y^2=\frac{16}{25}\left(25-x^2\right)$

On putting value of y^2 in Eq. (i), we get

$A^2=\frac{16}{25}\left(25-x^2\right)(5-x)^2$

Let A^2 = Z

Then, $Z=\frac{16}{25}\left(25-x^2\right)(5-x)^2$

Therefore,on differentiating both sides w.r.t x, we get,

$\frac{dZ}{dx}=\frac{16}{25}\left[\left(25-x^2\right)2(5-x)(-1)+(5-x)^2(-2x)\right]$ [by using product rule of derivative]

$=\frac{16}{25}(-2)(5-x)^2(2x+5)$

$=\frac{-32}{25}(5-x)^2(2x+5)$

For maxima or minima, put $\frac{dZ}{dx}=0$

$\Rightarrow -\frac{32}{25}(5-x)^2(2x+5)=0\Rightarrow x=5,-\frac{5}{2}$

Now, when x = 5, then

$Z=\frac{16}{25}(25-25)(5-5)^2=0$

Which is not possible.

So, x = 5 is rejected.

$\therefore \quad x=-\frac{5}{2}$

Now, $\frac{d^2 Z}{dx^2}=\frac{d}{dx}\left[-\frac{32}{25}(5-x)^2(2x+5)\right]$

$=\frac{32}{25}\left[(5-x)^2\cdot 2-(2x+5)\cdot 2(5-x)\right]$

$=-\frac{64}{25}(5-x)(-3x)=\frac{192x}{25}(5-x)$

$\therefore$ At $x=\frac{-5}{2},\left(\frac{d^2 Z}{dx^2}\right)_{x=-\frac{5}{2}}<0$

$\Rightarrow$ Z is maximum.

$\therefore$ Area A is maximum, when $x=-\frac{5}{2}$ and y = 12

Clearly,

$Z=A^2=\frac{16}{25}\left(25-\frac{25}{4}\right)\left[5+\frac{5}{2}\right]^2$

$=\frac{16}{25}\times\frac{75}{4}\times\frac{225}{4}=3\times 225$

$\therefore$ The maximum area, $A=\sqrt{3\times 225}=15\sqrt{3}$ sq units.

36. Let P(2 3, 4) be the given point and given equation of line be

$\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$

Let $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}=\lambda(\text{ say })$

$x=3\lambda-3, y=6\lambda+2, z=2\lambda$

Coordinates of any point T on given line are $(3\lambda-3,6\lambda+2,2\lambda)$.

Now, DR's of line PT

$=(3\lambda-3-2,6\lambda+2-3,2\lambda-4)$

$=(3\lambda-5,6\lambda-1,2\lambda-4)$

since, the line PT is parallel to the plane

3x + 2y + 2 - 5 = 0,then,

$a_1a_2+b_1b_2+c_1c_2=0$

$$\left[\begin{array}{l}\text{line is parallel to the plane, therefore normal}\\ \text{to the plane is perpendicular to the line.}\\ \text{where } a_1=3\lambda-5, b_1=6\lambda-1, c_1=2\lambda-4\\ \text{and } a_2=3, b_2=2, c_2=2\end{array}\right]$$

$\Rightarrow(3\lambda-5)3+(6\lambda-1)2+(2\lambda-4)2=0$

$\Rightarrow \quad 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$

$\Rightarrow \quad 25\lambda - 25 = 0$

$\Rightarrow \quad 25\lambda = 25 \Rightarrow \lambda = 1$

Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda)$

$= (0, 8, 2)$ [$\because$ λ= 1]

Now, the required distance between points

P(2 3, 4) and T(0, 8, 2) is given by

$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$

$[\because (x_1, y_1, z_1) = (2, 3, 4) \text{ and } (x_2, y_2, z_2) = (0, 8, 2)]$

$= \sqrt{4 + 25 + 4} = \sqrt{33}\,.units$