## CBSE Class 11 Physics <br> Sample Paper 09 (2019-20)

## Maximum Marks: 70

Time Allowed: 3 hours

## General Instructions:

1. All questions are compulsory. There are 37 questions in all.
2. This question paper has four sections: Section A, Section B, Section C and Section D.
3. Section A contains twenty questions of one mark each, Section B contains seven questions of two marks each, Section $C$ contains seven questions of three marks each, and Section $D$ contains three questions of five marks each.
4. There is no overall choice. However, internal choices have been provided in two questions of one mark each, two questions of two marks, one question of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

## Section A

1. Good and bad effects of a particular scientific/technological advance such as prenatal sex determination should in general be weighed by
a. well being and welfare of the scientists
b. well being and welfare of the society i.e. whether or not it does good for the human kind
c. well being and welfare of the originators of the advance
d. well being and welfare of the rulers
2. The speed of a projectile when it is at its greatest height is $\sqrt{\frac{2}{5}}$ times its speed at half the max height. What is the angle of projection?
a. $60^{0}$
b. $90^{\circ}$
c. $15^{0}$
d. $45^{0}$
3. A truck starts from rest and accelerates uniformly at $2.0 \mathrm{~ms}^{-2}$. At $t=10 \mathrm{~s}$, a stone is dropped by a person standing on the top of the truck ( 6 m high from the ground). What is the magnitude of velocity (in $\mathrm{ms}^{-1}$ ) of the stone at $\mathrm{t}=11 \mathrm{~s}$ ? (Neglect air resistance.)
a. $22.4 \mathrm{~m} / \mathrm{s}$
b. $19.6 \mathrm{~m} / \mathrm{s}$
c. $18.5 \mathrm{~m} / \mathrm{s}$
d. $21.0 \mathrm{~m} / \mathrm{s}$
4. if particles A and B are moving with velocities $\mathbf{v}_{A}$ and $\mathbf{v}_{B}$ (each with respect to some common frame of reference, say ground.). Then, velocity of particle A relative to that of $B$ is:
a. $\mathbf{v}_{\mathbf{A B}}=\mathbf{v}_{\mathbf{A}}+\mathbf{v}_{\mathbf{B}}$
b. $\mathbf{v}_{\mathbf{A B}}=\mathbf{v}_{\mathbf{A}}-\mathbf{v}_{\mathbf{B}}$
c. $\mathbf{v}_{\mathbf{A B}}=-\mathbf{v}_{\mathbf{A}}+\mathbf{v}_{\mathbf{B}}$
d. $\mathbf{v}_{\mathbf{A B}}=-\mathbf{v}_{\mathbf{A}}-\mathbf{v}_{\mathbf{B}}$
5. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of $7 \mathrm{~m} / \mathrm{s}$. It hits the floor of the elevator (length of the elevator $=3 \mathrm{~m}$ ) and does not rebound. What is the heat produced by the impact?
a. 9.22 J
b. 8.42 J
c. 8.82 J
d. 8.11 J
6. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.
a. $\mathrm{D}_{\text {copper }} / \mathrm{D}_{\text {iron }}=1.55$
b. $\mathrm{D}_{\text {copper }} / \mathrm{D}_{\text {iron }}=1.25$
c. $\mathrm{D}_{\text {copper }} / \mathrm{D}_{\text {iron }}=1.35$
d. $\mathrm{D}_{\text {copper }} / \mathrm{D}_{\text {iron }}=1.45$
7. A body cools from $80^{\circ} \mathrm{C}$ to $50^{\circ} \mathrm{C}$ in 5 minutes. Calculate the time it takes to cool from $60^{\circ} \mathrm{C}$ to $30^{\circ} \mathrm{C}$. The temperature of the surroundings is $20^{\circ} \mathrm{C}$.
a. 12 min
b. 9 min
c. 15 min
d. 6 min
8. A geyser heats water flowing at the rate of 3.0 liters per minute from $27^{\circ} \mathrm{C}$ to $77^{\circ} \mathrm{C}$. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is $4.0 \times 10^{4} \mathrm{~J} / \mathrm{g}$ ?
a. $16 \mathrm{~g} \mathrm{per} \min$
b. 18 g per min
c. 14 g per min
d. 12 g per min
9. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be $3 \stackrel{0}{A}$.
a. $5 \times 10^{-4}$
b. $4.5 \times 10^{-4}$
c. $3.5 \times 10^{-4}$
d. $3.8 \times 10^{-4}$
10. in the same medium transverse and longitudinal waves
a. travel changing longitudinal wave to transverse wave
b. travel with different speeds
c. travel changing transverse wave to longitudinal wave
d. travel with same speeds
11. Fill in the blanks:

Moon revolving around the earth comes under $\qquad$ dimensional motion.

## OR

Fill in the blanks:

The earth is a $\qquad$ frame of reference.
12. Fill in the blanks:
$\qquad$ deals with the study of phenomena related to light.
13. Fill in the blanks:

The sum of two vectors is $\qquad$ when both the vectors are in the same direction.
14. Fill in the blanks:

The internal restoring force acting per unit area of a deformed body is called $\qquad$ .
15. Fill in the blanks:

Latent heat of $\qquad$ is the latent heat for solid-liquid state change.
16. Can there be motion in two dimensions with an acceleration only in one dimension?
17. Under what condition is the work done by a force zero inspite of displacement of the body?
18. A boulder is thrown into a deep lake. As it sinks deeper and deeper into water, does the buoyant force changes?
19. Why is the energy of thermal radiation less than that of visible light?
20. If an explosion takes place at the bottom of lake or sea, will the shock waves in Water be longitudinal or transverse?

## OR

When two waves of almost equal frequencies $n_{1}$ and $n_{2}$ reach at a point, simultaneously. What is the time interval between successive maxima?
21. Derive $\mathrm{v}=\mathrm{u}+$ at from velocity-time graph.
22. A bullet fired at an angle of $30^{\circ}$ with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to be fixed, and neglect air resistance.
23. A 3 m long ladder weighting 20 kg leans on a frictionless wall. Its feet rest on the floor 1 m from the wall as shown in figure. Find the reaction forces $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ of the wall and the floor.

24. The mean orbital radius of the earth around the sun is $1.5 \times 10^{8} \mathrm{~km}$. Calculate mass of the sun if $\mathrm{G}=6.67 \times 10^{11} \mathrm{Nm}^{2} / \mathrm{kg}^{-2}$ ?
25. A car moving on a straight highway with speed of $126 \mathrm{~km} / \mathrm{hr}$. is brought to stop within a distance of 200 m . What is the retardation of the car (assumed uniform) and how
long does it take for the car to stop?
26. Find the temperature of $149^{\circ} \mathrm{F}$ on Kelvin scale.

## OR

What are the basic requirements of a cooking utensil in respect of specific heat, thermal conductivity and coefficient of expansion?
27. What are the assumptions of kinetic theory of gas?

## OR

A vessel contains a mixture of 1 mole of oxygen and 2 moles of nitrogen at 300 K . Find the ratio of average rotational kinetic energy per oxygen molecule to per nitrogen molecule.
28. If $l_{1}=(10.0 \pm 0.1) \mathrm{cm}$ and $\mathrm{l}_{2}=(9.0 \pm 0.1) \mathrm{cm}$, find their sum, difference and percentage error in each.
29. A particle is moving along a straight line and its position is given by the relation $x=\left(t^{3}\right.$ $\left.-6 t^{2}-15 t+40\right) m$
Find
i. The time at which velocity is zero.
ii. Position \& Displacement at this point
iii. Acceleration for the particle at the point.
30. Show that total mechanical energy of a freely falling body remains constant throughout the fall.
31. Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the midpoint of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reason for your answer.
32. If eight raindrops each of radius 1 mm are falling through air at a terminal velocity of $5 \mathrm{~cm} \mid \mathrm{s}$. If they coalesce to form a bigger drop, what is the terminal velocity of bigger drop?

## OR

The lower end of a capillary tube of diameter 2 mm is dipped 8 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at temperature of the experiments is $7.30 \times 10^{-2} \mathrm{Nm}^{-1}$, atm $=1.01 \times 10^{5} \mathrm{~Pa}$, density of water $=1000$ $\mathrm{kg} / \mathrm{m}^{3}, \mathrm{~g}=9.80 \mathrm{~ms}^{-2}$. Also, calculate the excess pressure.
33. State and explain First Law of Thermodynamics.
34. Two similar sonometer wires of the same material produce 2 beats per second. The length of one is 50 cm and that of the other is 50.1 cm . Calculate the frequencies of two wires?
35. Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light, inextensible string. A horizontal force F = 60 kgf is applied to
i. A,
ii. B along the direction of string.

What is the tension in the string in each case? [Given, $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ ]

## OR

i. State and prove impulse-momentum theorem.
ii. Figure shows the position-time graph of a body of mass 0.04 kg . Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the body? What is the magnitude of each impulse?

36. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to $10 \pi \mathrm{rad} \mathrm{s}^{-1}$. Which of the two will start to roll earlier? The co-efficient of kinetic friction is $\mu_{k}=0.2$.

## OR

i. Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $\frac{2 M R^{2}}{5}$, where M is the mass of the sphere and R is the radius of the sphere.
ii. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{M R^{2}}{4}$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.
37. The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t)=A \cos (\omega t+\phi)$ If the initial $(t=0)$ position of the particle is 1 cm and its initial velocity is $\omega \mathrm{cm} / \mathrm{s}$, then what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi \mathrm{s}^{-1}$. If instead of the cosine function, we choose the sine function to describe the $\mathrm{SHM}, \mathrm{x}=\mathrm{B} \sin (\omega \mathrm{t}+\phi)$, then what are the amplitude and initial phase of the particle with the above initial conditions?

## OR

One end of a V-tube containing mercury is connected to a suction pump and the other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of $45^{\circ}$ each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V-tube execute simple harmonic motion? Neglect capillary and viscous forces. Find the time period of oscillation.

## CBSE Class 11 Physics

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## Solution <br> Section A

1. (b) well being and welfare of the society i.e. whether or not it does good for the human kind

Explanation: The advancement of new technology has been taking place since the beginning of human history. From the invention of items like the spear and knifes made out of rocks and sticks to aid in the capturing and killing of animals for food, to items like the first printing press and the computer.

The advancement in technology has been exceptionally fast in the 20th and 21st century. With electronic technology and machines being produced and improved all the time, it was very likely that along with the positive aspects of these new advancements, people would also consider the negative aspects and look to criticise new technology.

What society is taking up from the technology advances, it should be weighed by the well being and welfare of the society.
2. (a) $60^{\circ}$ Explanation:

Time taken to reach the maximum height $=\frac{u \sin \theta}{g}$
Maximum height $=\frac{u^{2} \sin ^{2} \theta}{2 g}$
Half the maximum height $=\frac{u^{2} \sin ^{2} \theta}{4 g}$
Horizontal velocity at half the maximum height $=u \cos \theta$
Vertical velocity at half the maximum height $=\frac{u \sin \theta}{\sqrt{2}}$
Velocity at half the maximum height $=\sqrt{u^{2} \cos ^{2} \theta+\frac{u^{2} \sin ^{2} \theta}{2}}$
According to question,

$$
u \cos \theta=\sqrt{\frac{2}{5}} \sqrt{u^{2} \cos ^{2} \theta+\frac{u^{2} \sin ^{2} \theta}{2}}
$$

Squaring both sides,

$$
\begin{aligned}
& u^{2} \cos ^{2} \theta=\frac{2}{5} u^{2}\left(\cos ^{2} \theta+\frac{\sin ^{2} \theta}{2}\right) \\
& =>5-5 \sin ^{2} \theta=2\left(1-\sin ^{2} \theta+\frac{\sin ^{2} \theta}{2}\right) \\
& =>5-5 \sin ^{2} \theta=2-\sin ^{2} \theta \\
& =>4 \sin ^{2} \theta=3 \\
& =>\sin \theta=\frac{\sqrt{3}}{2} \\
& =>\theta=60^{0}
\end{aligned}
$$

3. (a) $22.4 \mathrm{~m} / \mathrm{s}$

Explanation: During first 10 , the horizontal component of the velocity is $v_{x}=u+$ at $=$ $0+2 \times 10=20 \mathrm{~m} / \mathrm{s}$

From 10 to 11 s , the vertical component of the velocity is $v_{y}=u+g t=0+10 x 1=10 \mathrm{~m} / \mathrm{s}$

Relative velocity is,

$$
\begin{aligned}
v & =\sqrt{v_{x}^{2}+v_{y}^{2}} \\
& =\sqrt{20^{2}+10^{2}} \\
& =\sqrt{500} \\
& =22.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

4. (b) $\mathbf{v}_{\mathbf{A B}}=\mathbf{v}_{\mathbf{A}}-\mathbf{v}_{\mathbf{B}}$

## Explanation:

The relative velocity of an object A with respect to another object B is the velocity that object A would appear to have to an observer situated on object B moving along with it.

In simple words relative velocity of A with respect to Be is the vector difference between the velocities of A and B .

It is represented as
$V_{A B}=V_{A}-V_{B}$
5. (c) 8.82 J

## Explanation:

Whole of the potential energy of bolt converted in to heat energy
heat produced by the impact $=\mathrm{mgh}=0.3 \times 9.8 \times 3=8.82 \mathrm{~J}$
6. (b) $\mathrm{D}_{\text {copper }} / \mathrm{D}_{\text {iron }}=1.25$ Explanation:
given condition is that tension in all three wires should be same. thus for same tension the extension $\Delta L$ in all three wires will be same.
as natural length of all three wires is same i.e. 2.0 m , strain in all three wires will be equal. SO young modulus of wires.
$y=\frac{\text { stress }}{\text { strain }}=\frac{F \times L}{A \times \Delta L}=\frac{F \times L}{\pi \mathrm{r}^{2} \times \Delta \mathrm{L}} \quad(D=2 r)$
$y \alpha \frac{1}{D^{2}}$
$\Longrightarrow \frac{y_{\text {copper }}}{y_{\text {iron }}}=\frac{D^{2} \text { iron }}{D^{2} \text { copper }}$
$\frac{D_{\text {copper }}}{D_{\text {iron }}}=\sqrt{\frac{y_{\text {iron }}}{y_{\text {copper }}}}=\sqrt{\frac{19 \times 10^{10}}{12 \times 10^{10}}}$
$\frac{D_{\text {copper }}}{D_{\text {iron }}}=1.25$
7. (b) 9 min

Explanation: According to Newton's law of cooling
$\left[\frac{\theta_{1}-\theta_{2}}{t}\right]=K\left[\frac{\theta_{1}+\theta_{2}}{2}-\theta_{0}\right]$
$\frac{30}{5}=K \times 45 \ldots \ldots \ldots \ldots .(1)$
$\frac{30}{t}=K \times 25$
$\mathrm{t}=9 \mathrm{~min}$
8. (a) 16 g per min

## Explanation:

$Q=m s \Delta T=3000 \times 4.2 \times 50$
m = mass of water
$s=$ specific heat of water
heat of combustion is $4.0 \times 10^{4} \mathrm{~J} / \mathrm{g}$
rate of consumption of the fuel = Q / Heat of combustion
$=\frac{3000 \times 4.2 \times 50}{4 \times 10^{4}}=16 \mathrm{gm} / \mathrm{min}$
9. (d) $3.8 \times 10^{-4}$

## Explanation:

Radius $=1.5 A^{\circ}$
molecular volume of oxygen
$V=N_{A} \times \frac{4}{3} \pi r^{3}=6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times\left(1.5 \times 10^{-10}\right)^{3}=8.51 \mathrm{~cm}^{3}$
Actual volume of oxygen at STP $=22400 \mathrm{~cm}^{3}$
ratio of molercular volume to the actual volume $=\frac{8.51}{22400}=3.8 \times 10^{-4}$
10. (b) travel with different speeds

Explanation: As speed of transverse \& longitudinal waves depend on different modulus of elasticity (Young's modulus, Bulk modulus, Modulus of Rigidity) of the medium.

So in the same medium transverse and longitudinal waves
travel with different speeds
11. two

## OR

non-inertial
12. Optics
13. maximum
14. Stress
15. Fusion or Melting
16. Yes, it can be. For example, in projectile motion, the acceleration of the particle acts vertically downwards, while the projectile follows a parabolic path ( which is a 2D motion)
17. Work done by a force is zero even if displacement is not zero, if displacement of the body is in a direction perpendicular to the direction of force applied.
18. Buoyant force acting on a body is given by $\mathrm{F}=\mathrm{mg}$ which is equal to weight of liquid displaced. The buoyant force does not change as the boulder sinks because the boulder displaces the same volume of water at any depth and because water is practically incompressible, its density is practically the same at all depth and hence the weight of water displaced or the buoyant force is same at all depths.
19. The energy of an electromagnetic ware is given by: $\mathbf{E}=\mathbf{h f}$
$h=$ Planck's constant; $f$ = frequency of wave. Since the frequency of thermal radiation is less than that of visible light, the energy associated with thermal radiation is less than associated with visible light.
20. Explosion at the bottom of lake or sea create enormous increase in pressure of medium (water). A shock Wave is thus a longitudinal wave travelling at a speed which is greater than that of ordinary Wave.

## OR

Number of beats/sec $=\left(\mathrm{n}_{1}-\mathrm{n}_{2}\right)$
Hence, time interval between two successive beats = time interval between two successive maxima $=\frac{1}{n_{1}-n_{2}}$, since time is the inverse of the number of beats/sec.
21. From the below graph, $v=$ final velocity of the particle, $u=$ initial velocity of the particle, $a=$ acceleration of the particle.
Slope of v - t graph
$\tan \theta=\frac{v-\nu}{t}$

Again, acceleration $\mathrm{a}=$ slope of v - t graph.
Hence, $\tan \theta=$ acceleration (a)
$\therefore a=\frac{v-u}{t}$
$\Rightarrow v-u=a t$
$\therefore v=u+a t$

22. Given that Horizontal range, $\mathrm{R}=3 \mathrm{~km}$ and angle of projection of the projectile as $30^{\circ}$
$R=\frac{u^{2} \sin 2 \theta}{g}$ or $3=\frac{u^{2} \sin 60^{\circ}}{g}=\frac{u^{2}}{g} \sqrt{3} / 2$
or $\frac{u^{2}}{g}=2 \sqrt{3}$
Given that the muzzle speed is fixed
Therefore, maximum horizontal range, is
$R_{\max }=\frac{u^{2}}{g}=2 \sqrt{3}=3.464 \mathrm{~km}$
Hence, the bullet cannot hit the target at 5 km .
23. $F_{2}=\sqrt{t^{2}+N^{2}}$
$=\sqrt{34.6^{2}+196^{2}}=199.0 \mathrm{~N}$
If $F_{2}$ makes an angle $\alpha$ with the horizontal then
$\tan \alpha=\frac{N}{f}=5.6568$
$\alpha=80^{\circ}$
24. Given that :
$\mathrm{R}=1.5 \times 10^{8} \mathrm{Km}=1.5 \times 10^{11} \mathrm{~m}$
Time period of earth to complete one revolution around the sun is(T) = 365 days $=365$ $\times 24 \times 3600 \mathrm{~s}$
Centripetal force $=$ gravitational force
$M_{s}=\frac{4 \pi^{2} R^{3}}{G T^{2}}=\frac{4 \times 9.87 \times\left(1.5 \times 10^{11}\right)^{3}}{6.64 \times 10^{-11} \times(365 \times 24 \times 3600)^{2}}$
$=\frac{9.87 \times\left(1.5 \times 10^{11}\right)^{3}}{6.64 \times 10^{-11} \times(31536000)^{2}}$
$\mathrm{M}_{\mathrm{S}}=2.01 \times 10^{30} \mathrm{~kg}$
25. initial velocity $\mathrm{u}=126 \mathrm{~km} / \mathrm{hr}=\frac{126 \times 1000}{3600}=35 \mathrm{~m} / \mathrm{s}$
final speed $v=0$
stopping distances $=200 \mathrm{~m}$
using equation $v^{2}-u^{2}=2 a s$
$(0)^{2}-(35)^{2}=2 \times 200 \times(a)$
$a=-3.06 m / s^{2}$ (Retardation)
Now using equation $\mathrm{v}=\mathrm{u}+$ at
$0=35+(-3.06) t$
$\mathrm{t}=11.4 \mathrm{~s}$
26. From the relation of Farenheit and Kelvin scales of temperature, $\frac{F-32}{180}=\frac{T-273}{100}$ $\frac{149-32}{180}=\frac{T-273}{100} \Rightarrow \frac{117}{9}=\frac{T-273}{5}$
$\therefore \mathrm{T}=338 \mathrm{~K}$

## OR

The basic requirements of a cooking utensil are :-
i. A small value of specific heat(C)
ii. A large value of thermal conductivity (K)
iii. Low coefficient of expansion ( $\alpha$ ).
27. The assumptions of kinetic theory of gases are:-
i. A gas consists of a very large number of molecules which should be elastic spheres
and identical for a given gas.
ii. The molecules of a gas are in a state of continuous rapid and random motion.
iii. The size of gas molecules is very small as compared to the distance between them.
iv. The molecules do not exert any force of attraction or repulsion on each other.
v. The collisions of molecules with one another and with walls of the vessel are perfectly elastic.

## OR

Both oxygen and nitrogen gases are diatomic gases and have two degrees of freedom per molecule on account of rotational motion.
$\therefore$ Average rotational kinetic energy per molecule of either gas,
$\bar{E}_{R}=2 \times\left(\frac{1}{2} k_{B} T\right)=\mathrm{k}_{\mathrm{B}} \mathrm{T}$
$\therefore \frac{\left(\bar{E}_{R}\right)_{O_{2}}}{\left(\bar{E}_{R}\right)_{N_{2}}}=1: 1$
28. Given, $\mathrm{l}_{1}=(10.0 \pm 0.1) \mathrm{cm}$ and $\mathrm{l}_{2}=(9.0 \pm 0.1) \mathrm{cm}$
sum $=l_{1}+l_{2}$
Sum in terms of errors $=l_{1}+l_{2}=(10.0 \pm 0.1)+(9.0 \pm 0.1)$
$=(19.0 \pm 0.2) \mathrm{cm}$
Difference $=l_{1}-l_{2}$
Difference $=l_{1}-l_{2}=(10.0-9.0) \pm(0.1+0.1)$
$=(1.0 \pm 0.2) \mathrm{cm}$ ( errors are always added up)
Percentage Error,
For Sum $=\frac{0.2}{19.0} \times 100=1.05 \%$
For Difference $=\frac{0.2}{1.0} \times 100=20 \%$
29. $x=t^{3}-6 t^{2}-15 t+40$
$v=\frac{d x}{d t}=\left(3 t^{2}-12 t-15\right) m / s$
$a=\frac{d v}{d t}=(6 t-12) m / s^{2}$
i) Since , $v=0$
$3 t^{2}-12 t-15=0$
$3 t^{2}-15 t+3 t-15=0$
$3 t(t-5)+3(t-5)=0$
$(3 t+3)(t-5)=0$
Either $\mathrm{t}=-1$ or $\mathrm{t}=5$
Time cannot be negative
$\therefore \mathrm{t}=5$ seconds.
ii) Position at $t=5 \mathrm{~s}$
$\mathrm{x}=(5)^{3}-6(5)^{2}-15(5)+40$
$\mathrm{x}=-60 \mathrm{~m}$

At $t=0 \mathrm{~s}$,
$x=(0)^{3}-6 x(0)^{2}-15 x 0+40=40 m$
Displacement between $t=0 \sec$ to $t=5 \sec$
$S=$ Final Position $\left(x_{5}\right)$ - Initial Position ( $x_{0}$ )
$x_{5}=-60 m$
$\mathrm{x}_{0}=40 \mathrm{~m}$
$s=-60-40$
$\mathrm{s}=-100 \mathrm{~m}$
iii) Acceleration at $t=5 s$
$a=6 t-12$
$a=6(5)-12$
$\mathrm{a}=(30-12)$
$\mathrm{a}=18 \mathrm{~m} / \mathrm{s}^{2}$
30. As per principle of conservation of mechanical energy if there is no dissipation of mechanical energy, then total mechanical energy of a system must remain constant. Of course, potential energy may be converted into kinetic energy or vice versa but sum of the kinetic energy and potential energy of a system must remain unchanged.


Now consider a small-sized object of mass $m$ falling from a point A situated at a height h.

At point A: Since the object is at rest i.e., $u=0$
$\therefore$ K.E. of object at point $\mathrm{A}, \mathrm{K}_{\mathrm{A}}=0$
and P.E. of object $\mathrm{U}_{\mathrm{A}}=\mathrm{mgh}$
$\mathrm{E}_{\mathrm{A}}=\mathrm{K}_{\mathrm{A}}+\mathrm{U}_{\mathrm{A}}=0+\mathrm{mgh}=\mathrm{mgh}$
At point B: Let the objects fall freely through a distance y so as to reach $B$, where its velocity v is given by
$v=\sqrt{2 g y}$
$\therefore$ KE at point B
$K_{B}=\frac{1}{2} m v^{2}=\frac{1}{2} m \cdot 2 g y=\mathrm{mgy}$
and PE at point $\mathrm{B}, \mathrm{U}_{\mathrm{B}}=\mathrm{mg}(\mathrm{h}-\mathrm{y})$
$\therefore \mathrm{E}_{\mathrm{B}}=\mathrm{K}_{\mathrm{B}}+\mathrm{U}_{\mathrm{B}}=\mathrm{mgy}+\mathrm{mg}(\mathrm{h}-\mathrm{y})=\mathrm{mgh}$
At point C: Let the object just reaches the ground at point C , where its final velocity V is given by :
$V=\sqrt{2 g h}$
$\therefore$ K.E at point C, $K_{C}=\frac{1}{2} m V^{2}=\frac{1}{2} m \cdot 2 g h=m g h$
and P.E at point $\mathrm{C}, \mathrm{U}_{\mathrm{C}}=0$
$\therefore \mathrm{E}_{\mathrm{C}}=\mathrm{K}_{\mathrm{C}}+\mathrm{U}_{\mathrm{C}}=\mathrm{mgh}=0=\mathrm{mgh}$
A simple comparison shows that
$\mathrm{E}_{\mathrm{A}}=\mathrm{E}_{\mathrm{B}}=\mathrm{E}_{\mathrm{C}}=\mathrm{mgh}=\mathrm{a}$ constant
A graph showing variation of K.E., P.E. and total energy of a freely falling body is shown below:

31. $m_{1}=m_{2}=M r=10 R$

Let mass $m$ is placed at the mid point $P$ of line joining the centres of $A$ and $B$ sphere

$\left|F_{2}\right|=\left|F_{1}\right|=\frac{G M m}{(5 R)^{2}}$
$\left|F_{1}\right|=\left|F_{2}\right|=\frac{G M m}{25 R^{2}}$
The net force $F_{1}+F_{2}=0,\left(F_{1}=-F_{2}\right)$. m will be in equilibrium.
If now m is displaced by x slightly from P to A then $\mathrm{PA}=(5 R-x)$ and PB
$=(5 R+x)$
$F_{1}=\frac{G M m}{(5 R-x)^{2}}$ and $F_{2}=\frac{G M m}{(5 R+x)^{2}}$
$\therefore F_{2}<F_{1}$
Hence the resultant force acting on P is towards A , resulting in an unstable equilibrium.
32. Let the radius of smaller drop $=r$

Let the radius of bigger drop $=\mathrm{R}$
Volume of smaller drop $=\frac{4}{3} \pi r^{3}$
Volume of bigger drop $=\frac{4}{3} \pi R^{3}$
Now, according to the question,

Volume of bigger drop = Volume of 8 smaller drops.
$\frac{4}{3} \pi R^{3}=8 \times \frac{4}{3} \pi r^{3}$
$\mathrm{R}^{3}=8 \mathrm{r}^{3}$

Taking cube - root
$\mathrm{R}=2 \mathrm{r}$
$=2 \times 1 \mathrm{~mm}$
( $\mathrm{r}=1 \mathrm{~mm}$ (Given))
$=2 \mathrm{~mm}$
$=0.2 \mathrm{~cm}(1 \mathrm{~cm}=10 \mathrm{~mm})$
Now, Terminal velocity of each small drop $\left.\mathrm{N}_{T}=\frac{2}{9} \times \frac{\mathrm{r}^{2}}{\eta}(\mathrm{P}-\sigma) \mathrm{g} \rightarrow 1\right)$
Terminal velocity of bigger drop $\left.\mathrm{V}_{\mathrm{T}}=\frac{2}{9} \times \frac{R^{2}}{\eta}(P-\sigma) g \rightarrow 2\right)$
$\eta=$ Co-efficient of viscosity
P = Density of body
$\sigma=$ Density of fluid
$g$ = acceleration due to gravity
Dividing eq ${ }^{4} 2$ ) by 1 )
$\frac{V_{T}}{N_{T}}=\frac{R^{2}}{r^{2}}$
$V_{T}=N_{T} \times \frac{R^{2}}{r^{2}}$
Given Terminal velocity of small drop $=5 \mathrm{~cm} / \mathrm{s}$
$V_{T}=5 \times \frac{(0.2)^{2}}{(0.1)^{2}}$
$=5 \times \frac{0.04}{0.01}$
$\mathrm{V}_{\mathrm{T}}=20 \mathrm{~cm} / \mathrm{s}$

## OR

Given, $h=0.08 m$,
$d=1000 \mathrm{~kg} / \mathrm{m}^{3}$,
$g=9.80 \mathrm{~m} / \mathrm{s}^{2}$
As we know, $p_{0}=p_{a}+h d g$
$\mathrm{p}_{0}$ is outside pressure $p_{0}=1.01 \times 10^{5} \mathrm{~Pa}+0.08 \mathrm{~m} \times 1000 \mathrm{~kg} / \mathrm{m}^{3} \times 9.80 \mathrm{~m} / \mathrm{s}^{2}$ $p_{0}=1.01784 \times 10^{5} \mathrm{~Pa}$
Calculate inside pressure required in tube in order to blow a hemispherical bubble $\mathrm{p}_{1}=\mathrm{p}_{0}+\frac{2 S}{r}$, where $\mathrm{s}=7.30 \times 10^{-2} \mathrm{pa}-\mathrm{m}$

$$
\begin{aligned}
& \mathrm{p}_{1}=1.01784 \times 10^{5}+\frac{2 \times 7.3 \times 10^{-2}}{10^{-3}} \\
& =(1.01784+0.00146) 10^{5} \mathrm{~Pa} \Rightarrow p_{1}=1.02 \times 10^{5} \mathrm{~Pa}
\end{aligned}
$$

where the radius of the bubble is taken to be equal to the radius of the capillary tube. Since the bubble is hemispherical. The excess pressure in the bubble is 146 Pa .
33. According to First Law of Thermodynamics, if an amount of heat $\Delta Q$ is supplied to a thermodynamic system, a part of it may increase the internal energy of the system by $\Delta \mathrm{U}$ and the remaining part is used up as the external work done $\Delta \mathrm{W}$ by the system. Thus, we have $\Delta Q=\Delta U+\Delta W$

First Law of Thermodynamics follows the conservation law of energy and establishes an exact relation between heat transferred and mechanical work done. It provides a valuable concept of internal energy. It is applicable to every process in nature and to all the three states of matter i.e., solid, liquid and gases. Moreover, change in internal energy of a system may be due to any cause like change in translational or rotational or vibrational kinetic energy or molecular potential energy etc.
34. The frequency ( f ) of a sonometer wire of length $=1$, mass $=\mathrm{m}$ and Tension $=\mathrm{T}$ is given by
$\Rightarrow f=\frac{1}{2 l} \sqrt{\frac{T}{m}}$

Let $k=\frac{1}{2} \sqrt{\frac{T}{m}}$
So, $f=\frac{k}{l}$
In first case;
$\Rightarrow f_{1}=\frac{k}{l_{1}} \rightarrow(1)$
In second case;
$\Rightarrow f_{2}=\frac{k}{l_{2}} \rightarrow(2)$
Subtract equation (1) \& (2)
$\Rightarrow \mathrm{f}_{1}-\mathrm{f}_{2}=\frac{k}{l_{1}}-\frac{k}{l_{2}}=k\left(\frac{1}{l_{1}}-\frac{1}{l_{2}}\right)$
Now, given $l_{1}=50 \mathrm{~cm} ; \mathrm{l}_{2}=50.1 \mathrm{~cm}$
$\Rightarrow \mathrm{f}_{1}-\mathrm{f}_{2}=2$
So, $2=k\left[\frac{1}{50}-\frac{1}{50.1}\right]$
$\Rightarrow 2=k\left[\frac{50.1-50}{50 \times 50.1}\right]$
$2=K\left[\frac{0.1}{2505.0}\right]$
$\Rightarrow \frac{2 \times 2505}{0.1}=k$
$\Rightarrow \frac{5010 \times 10}{01.1}=k$
$\Rightarrow 50100=\mathrm{k}$
So, $f_{1}=\frac{k}{l_{1}}=\frac{50100}{50}=1002 \mathrm{~Hz}$
$\Rightarrow f_{2}=\frac{k}{l_{2}}=\frac{50100 \times 10}{50}=10020 \mathrm{~Hz}$
35. Horizontal force, $\mathrm{F}=60 \mathrm{kgf}=60 \times \mathrm{g} \mathrm{N}=60 \times 10 \mathrm{~N}=600 \mathrm{~N}$

Mass of body A, $\mathrm{m}_{1}=10 \mathrm{~kg}$
Mass of body B, $\mathrm{m}_{2}=20 \mathrm{~kg}$

Total mass of the system, $m=m_{1}+\mathrm{m}_{2}=30 \mathrm{~kg}$
Using Newton's second law of motion, the acceleration (a) produced in the system can be calculated as:
$\mathrm{F}=\mathrm{ma}$
$a=\frac{F}{m}=\frac{600}{30}=20 \mathrm{~m} / \mathrm{s}^{2}$
i. Both the bodies A and B will move with this acceleration as shown in the image. Now when force F is applied on body A, say the tension acting in the string between A and B is T .


The equation of motion can be written as:
$\mathrm{F}-\mathrm{T}=\mathrm{m}_{1} \mathrm{a}$
$\therefore \mathrm{T}=\mathrm{F}-\mathrm{m}_{1} \mathrm{a}$
$=600-10 \times 20=400 \mathrm{~N}$
ii. In this case two bodies A and B will also move with the same acceleration, a = 20 $\mathrm{m} / \mathrm{s}^{2}$ but in the opposite direction as shown in the figure. Now when the force F is applied on body B, say the tension in the string between A and B is T' in this case.


The equation of motion can be written as:
$\mathrm{F}-\mathrm{T}=\mathrm{m}_{2} \mathrm{a}$
$\mathrm{T}^{\prime}=\mathrm{F}-\mathrm{m}_{2} \mathrm{a}$
$\therefore T^{\prime}=600-20 \times 20=200 \mathrm{~N}$

## OR

i. The principle of conservation of linear momentum states that, "If no external forces act on the system of two colliding objects, then the vector sum of the linear momentum of each body remains constant and is not affected by their mutual interaction." i.e. if $\bar{F}$ ext $=0$ then $\bar{P}=$ constant . To prove this principle, we consider a collision between two spheres $\mathbf{A}$ and $\mathbf{B}$ having masses of $\mathbf{m}_{\mathbf{1}}$ and $\mathbf{m}_{\mathbf{2}}$ respectively.Let $\mathbf{u}_{\mathbf{1}}$ and $\mathbf{u}_{\mathbf{2}}$ be the velocities of the spheres before
collision such that $\mathbf{u}_{\mathbf{1}}>\mathbf{u}_{\mathbf{2}}$ and moving on the same straight line. After collision, let their velocities be $\mathbf{v}_{\mathbf{1}}$ and $\mathbf{v}_{\mathbf{2}}$ on the same line. If they collide with each other for a short time interval $\mathbf{t}$, each sphere exerts a force on the other sphere and so, the force experienced by $\mathbf{A}$ is given as $F_{2}=\frac{\text { change in momentum }}{\text { time }}=\frac{m_{1} v_{1}-m_{1} u_{1}}{t}$ Similarly, force experienced by $\mathbf{B}$ is $F_{1}=\frac{\text { change in momentum }}{\text { time }}=\frac{m_{2} v_{2}-m_{2} u_{2}}{t}$ According to Newton's third law of motion, the force experienced by A and B are equal and opposite, $\overrightarrow{F_{2}}=-\overrightarrow{F_{1}}$
$\Rightarrow m_{2} v_{2}-m_{2} u_{2}=-\left(m_{1} v_{1}-m_{1} u_{1}\right)$
$\Rightarrow m_{2} v_{2}+m_{1} v_{1}=m_{1} u_{1}+m_{2} u_{2}$
$\Rightarrow P_{f}=P_{i}$
That is, total momentum before collision is equal to total momentum after collision if no external forces act on them which proves the principle of conservation of linear momentum.
ii. A ball rebounding between two walls located between at $x=0 \mathrm{~cm}$ and $x=2 \mathrm{~cm}$; after every 2 s , the ball receives an impulse of magnitude $0.08 \times 10^{-2} \mathrm{kgm} / \mathrm{s}$ from the walls.
If we take any one of the triangular portion of the graph, we can see that the position of the ball is increasing uniformly in first 2 s and then decreasing at the same rate in the next 2 s. i.e. The ball is coming back to the same position after every 4 s . The given graph shows that a body changes its direction of motion after every 2 s. Physically, this situation can be visualized as a ball rebounding to and fro between two stationary walls situated between positions $x=0 \mathrm{~cm}$ and $\mathrm{x}=2 \mathrm{~cm}$. Since the slope of the x-t graph reverses after every 2 s , the ball collides with a wall after every 2 s . Therefore, ball receives an impulse after every 2 s .
Mass of the ball, $\mathrm{m}=0.04 \mathrm{~kg}$
The slope of the graph gives the velocity of the ball. Using the graph (in first 2s), we can calculate initial velocity ( $u$ ) as:
$u=\frac{(2-0) \times 10^{-2}}{(2-0)}=10^{-2} \mathrm{~m} / \mathrm{s}$
Velocity of the ball before collision(taking any one of the triangle of the graph and time for the first 2 s ), $\mathrm{u}=10^{-2} \mathrm{~m} / \mathrm{s}$
Velocity of the ball after collision(taking the same triangle and time for next 2 s ), v
$=-10^{-2} \mathrm{~m} / \mathrm{s}$
(Here, the negative sign arises as the ball reverses its direction of motion i.e. the decrease of position of the ball for next 2s.)
Now from the mathematical explanation of Newton's 2nd law of motion,
Magnitude of impulse = Change in momentum
$=|m v-m u|$
$=|0.04(\mathrm{v}-\mathrm{u})|$
$=\left|0.04\left(-10^{-2}-10^{-2}\right)\right|$
$=0.08 \times 10^{-2} \mathrm{kgm} / \mathrm{s}$
36. Radii of the ring and the disc, $r=10 \mathrm{~cm}=0.1 \mathrm{~m}$ Initial angular speed, $\omega_{z}=10 \pi \mathrm{rad} \mathrm{s}^{-1}$
Coefficient of kinetic friction, $\mu_{k}=0.2$
Initial velocity of both the objects, $\mathrm{u}=0$
Motion of the two objects is caused by frictional force. As per Newton's second law of motion, we have frictional force, $f=m a$
$\mu_{k} \mathrm{mg}=\mathrm{ma}$
Where,
$a=$ Acceleration produced in the objects
$\mathrm{m}=$ Mass
$\therefore a=\mu_{k} \mathrm{~g} \ldots$ (i)
As per the first equation of motion, the final velocity of the objects can be obtained as:
$v=u+a t$
$=0+\mu_{\mathrm{k}} \mathrm{gt}$
$=\mu_{\mathrm{k}} \mathrm{gt} . .$. (ii)
The torque applied by the frictional force will act in a perpendicularly outward direction and cause a reduction in the initial angular speed.
Torque, $\mathrm{T}=-\mathrm{Ia}$
$\alpha=$ Angular acceleration
$\mathrm{u}_{\mathrm{z}} \mathrm{mgr}=-\mathrm{I} \alpha$
$\therefore a=\frac{-\mu_{k} m g r}{I}$.......(iii)
Using the first equation of rotational motion to obtain the final angular speed:
$\omega=\omega_{e}+a t$
$=\omega_{x}+\frac{-\mu_{k} m g r}{I} t$
Rolling starts when linear velocity, $v=r u$
$\therefore v=r\left(\omega_{0}-\frac{\mu_{k} g m r t}{I}\right)$
Equating equations (ii) and (v), we get:
$\mu_{k} g t=r\left(\omega_{0}-\frac{\mu_{k} g m r t}{I}\right)$
$=r \omega_{0}-\frac{\mu_{i} g m r^{2} t}{I}$
For the ring $I=m r^{2}$
$\therefore \mu_{k} g t=r \omega_{0}-\frac{\mu_{k} g m r^{2} t}{m r^{2}}$
$=r \omega_{0}=u_{k}-\frac{u_{k} g m r^{2} t}{m r^{2}}$
$2 \mu_{k} g t=r \omega_{0}$
$\therefore t_{r}=\frac{r \omega_{0}}{2 \mu_{k} g}$
$=\frac{0.1 \times 10 \times 3.14}{2 \times 0.2 \times 9.8}=0.80 s$
For the ring $I=\frac{1}{2} m r^{2}$
$\therefore \mu_{k} g t_{d}=r \omega_{0}-\frac{\mu_{k} g m r^{2} t}{\frac{1}{2} m r^{2}}$
$=r \omega_{0}-2 \mu_{k} g t$
$3 \mu_{k} g t_{d}=r \omega_{0}$
$\therefore t_{d}=\frac{r \omega_{0}}{3 \mu_{k} g}$
$=\frac{0.1 \times 10 \times 3.14}{3 \times 0.2 \times 9.8}=0.53 \mathrm{~s}$
Since $t_{d}>t_{r}$, the disc will start rolling before the ring.

## OR

a. The moment of inertia (M.I.) of a sphere about its diameter $=\frac{2}{5} M R^{2}$

MI. $-\frac{2}{5} M R^{2}$

Given,
Moment of inertia of the sphere about its diametre $=\left(\frac{2}{5}\right) \mathrm{mR}^{2}$
Use, parallel axis theorem ,
Moment of inertia of the sphere about tangent $=\mathrm{I}+\mathrm{mR}^{2}$

$$
\begin{aligned}
& =\left(\frac{2}{5}\right) \mathrm{mR}^{2}+\mathrm{mR}^{2} \\
& =(7 / 5) \mathrm{mR}^{2}
\end{aligned}
$$

b. Moment of inertia of disc of mass m and radius R about any of its diametre $=$ $\mathrm{mR}^{2} / 4$
Moment of inertia about diametre $=\mathrm{Ix}=\mathrm{Iy}=\left(\frac{1}{4}\right) \mathrm{mR}^{2}$
Using , perpendicular axis theorem ,
Iz = Ix + Iy
Where Iz is moment of inertia about perpendicular axis of plane of disc .
$\mathrm{Iz}=\left(\frac{1}{4}\right) \mathrm{mR}^{2}+\left(\frac{1}{4}\right) \mathrm{mR}^{2}$
$=\left(\frac{1}{2}\right) \mathrm{mR}^{2}$


Moment of inertia of disc about passing through a point of its edge
Use , parallel axis theorem,
$\mathrm{I}=\mathrm{Iz}+\mathrm{mR}^{2}$
$=\left(\frac{1}{2}\right) \mathrm{mR}^{2}+\mathrm{mR}^{2}$
$=\left(\frac{1}{2}\right) \mathrm{mR}^{2}$
37. Given, displacement equation $\mathrm{x}(\mathrm{t})=\operatorname{Acos}(\omega t+\phi)$...(i)

At $\mathrm{t}=0 ; \mathrm{x}(0)=1 \mathrm{~cm}$, velocity of the particle $v=\omega \mathrm{cm} / \mathrm{s}$
Angular frequency $\omega=\pi \mathrm{s}^{-1}$
$\Rightarrow 1=A \cos (\omega t+\phi)$
For $\mathrm{t}=0,1=\mathrm{A} \cos \phi$
Now, $v(t)=\frac{d x(t)}{d t}=\frac{d}{d t} A \cos (\omega t+\phi)$
$=-A \omega \sin (\omega t+\phi)$
Again at $\mathrm{t}=0, v=\omega \mathrm{cm} / \mathrm{s}$
$\Rightarrow \omega=-A \omega \sin \phi$
$\Rightarrow-1=A \sin \phi$
Squaring and adding eqs.(i) and (ii),
$A^{2} \cos ^{2} \phi+A^{2} \sin ^{2} \phi=(1)^{2}+(-1)^{2}$
$A^{2}=2 \Rightarrow A= \pm \sqrt{2} \mathrm{~cm}$

Hence, the amplitude of the $\mathrm{SHM}=\sqrt{2} \mathrm{~cm}$
Dividing Eq. (ii) by (i), we get
$\frac{A \sin \phi}{A \cos \phi}=\frac{-1}{1}$ or $\tan \phi=-1$
$\Rightarrow \phi=-\frac{\pi}{4}$ or $\frac{7 \pi}{4}$
Now, if instead of cosine, we choose the sine function in the displacement equation, then
$\mathrm{x}(\mathrm{t})=\mathrm{B} \sin (\omega t+\alpha)$
At $\mathrm{t}=0, \mathrm{x}=1 \mathrm{~cm}, \Rightarrow 1=B \sin (0+\alpha)$
or $B \sin \alpha=1 \ldots \ldots \ldots .$. (iii)
Velocity $\mathrm{v}(\mathrm{t})=\frac{d x(t)}{d t}=\frac{d}{d t}[B \sin (\omega t+\alpha)]$
$=+B \omega \cos (\omega t+\alpha)$
Again at $\mathrm{t}=0, \mathrm{v}(\mathrm{t})=\omega \mathrm{cm} / \mathrm{s}$
B $\cos \alpha=+1$ (iv)

Squaring and adding Eqs.(iii) and (iv),
$B^{2} \sin ^{2} \alpha+B^{2} \cos ^{2} \alpha=(1)^{2}+(+1)^{2}$
$\Rightarrow B^{2} \sin ^{2} \alpha+B^{2} \cos ^{2} \alpha=2$
$B^{2}\left(\sin ^{2} \alpha+\cos ^{2} \alpha\right)=2$
$B^{2} 1=2 \Rightarrow B= \pm \sqrt{2} \mathrm{~cm}$
Hence, amplitude of the simple harmonic motion in both types of trigonometric wave equation expression $=\sqrt{2} \mathrm{~cm}$
Dividing Eq. (iii) by (iv), we get
$\frac{B \sin \alpha}{B \cos \alpha}=\frac{1}{1}$ or $\tan \alpha=1$
$\therefore \alpha=\frac{\pi}{4}$, only the phase angle differs for sine and cosine wave equation.

## OR

L et the liquid column in both arms of the V-tube were at $h_{0}$ heights initially. Now due to pressure difference the liquid columns in A arm is pressed by $x$ and in arm B is liftted by $x$ (so difference in vertical height between two levels $=2 x$ ) Consider an element of liquid of height dx inside the tube.


Then its mass $d m=$ volume $\times d e n s i t y=A \cdot d x \rho$ (where, $\mathrm{A}=$ area of cross-section of tube, $\rho=$ density of the liquid inside the tube)
Potential energy of the right arm with $d m$ elementary mass column $=(d m) g h$
Potential energy of $d m$ elementary mass in left arm column $=A \rho g x d x$ (putting the value of $\mathrm{dm}=A . d x . \rho$ and $\mathrm{h}=\mathrm{x}$ )
$\therefore$ Total potential energy in left column $=\int_{0}^{h_{1}} A \rho g x d x=A \rho g\left[\frac{x^{2}}{2}\right]_{0}^{h_{1}}$
$=A \rho g \frac{h_{1}^{2}}{2}$
From above given figure $\sin 45^{\circ}=\frac{h_{1}}{l} \quad \therefore h_{1}=h_{2}=l \sin 45^{\circ}=\frac{l}{\sqrt{2}}$
$\therefore h_{1}^{2}=h_{2}^{2}=\frac{l^{2}}{2}$
$\therefore$ Potential energy in the left column $=A \rho g \frac{l^{2}}{4}$
Similarly potential energy in right column $=A \rho g \frac{l^{2}}{4}$
$\therefore$ Total potential energy $=A \rho g \frac{l^{2}}{4}+A \rho g \frac{l^{2}}{4}=\frac{A \rho g l^{2}}{2}$
Due to pressure difference, left element moves towards right side by 'y' units and the same element rises in the right arm by 'y' units.
Then the liquid column length in the left arm becomes by decreasing $=(l-y)$
And the liquid column length in the right arm becomes by increasing $=(l+y)$
Now decreased potential energy of liquid column in the left arm
$=A \rho g(l-y)^{2} \sin ^{2} 45^{\circ}$
Similarly increased potential energy of liquid column in the right arm
$=A \rho g(l+y)^{2} \sin ^{2} 45^{\circ}$
$\therefore$ Total potential energy due to two liquid columns in the left and right arm
respectively $=A \rho g\left(\frac{1}{\sqrt{2}}\right)^{2}\left[(l-y)^{2}+(l+y)^{2}\right]$
Final potential energy due to difference in liquid columns in the two arms,
$=\frac{A \rho g}{2}\left[l^{2}+y^{2}-2 l y+l^{2}+y^{2}+2 l y\right]$
$\therefore$ Final potential energy $=\frac{A \rho g}{2}\left(2 l^{2}+2 y^{2}\right)$
Now change in potential energy = Final potential energy due to liquid columns in the two arms- Initial potential energy due to liquid columns in the two arms
$=\frac{A \rho g}{2}\left(2 l^{2}+2 y^{2}\right)-\frac{A \rho g l^{2}}{2}$
$=\frac{A \rho g}{2}\left[2 l^{2}+2 y^{2}-l^{2}\right]$
$\therefore$ Change in potential energy $=\frac{A \rho g}{2}\left(l^{2}+2 y^{2}\right)$
If change in velocity ( v ) of total liquid column be v then change in kinetic energy,
$\Delta K E=\frac{1}{2} m v^{2}$
Again $m=$ volume $\times$ density $=(A .2 l) \rho$
$\therefore \Delta K E=\frac{1}{2}(A 2 l \rho) v^{2}=A \rho l v^{2}$
$\therefore$ Change in Total energy $=$ change in potential energy + change in kinetic energy
$=\frac{A \rho g}{2}\left(l^{2}+2 y^{2}\right)+A \rho l v^{2}$
Again, from the law of conservation of energy, total change in energy
$\Delta P E+\Delta K E=0$
$\therefore \frac{A \rho g}{2}\left[l^{2}+2 y^{2}\right]+A \rho l v^{2}=0$
$\therefore \frac{A \rho}{2}\left[g\left(l^{2}+2 y^{2}\right)+2 l v^{2}\right]=0$
$\because \frac{A \rho}{2} \neq 0$
$\therefore g\left(l^{2}+2 y^{2}\right)+2 l v^{2}=0$
Differentiating on both sides of the above equation with respect to time,t we get
$g\left[0+2 \times 2 y \frac{d y}{d t}\right]+2 l .2 v \cdot \frac{d v}{d t}=0$
$\therefore 4 g y \frac{d y}{d t}+4 v l \frac{d^{2} y}{d t^{2}}=0\left[\because a=\frac{d v}{d t}=\frac{d^{2} y}{d t^{2}}\right]$
$\Rightarrow 4 g y \cdot v+4 v l \frac{d^{2} y}{d t^{2}}=0 \Rightarrow 4 v\left[g y+l \cdot \frac{d^{2} y}{d t^{2}}\right]=0$
$\Rightarrow \frac{d^{2} y}{d t^{2}}+\frac{g}{l} y=0 \quad \because 4 v \neq 0 \ldots$ (i)
It is the equation of a simple harmonic motion and can be compared with the standard equation of a simple harmonic motion i.e. $\frac{d^{2} y}{d t^{2}}+\omega^{2} y=0 \ldots .$. (ii) [ $\omega$ is the angular acceleration or angular frequency of the particle executing simple harmonic motion]
Comparing the above two equations (i) and (ii) we get, $\therefore \omega^{2}=\frac{g}{l}$
$\therefore \frac{2 \pi}{T}=\sqrt{\frac{g}{l}} \Rightarrow T=2 \pi \sqrt{\frac{l}{g}}\left[\because \omega=\frac{2 \pi}{T}\right.$, T being time period of the simple harmonic motion]

