CBSE Class 11 Mathematics
Sample Papers 09 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The relation R = {1, 1), (2, 2), (3, 3)} on the set {1, 2, 3} is
   a. an equivalence relation
   b. reflexive only
   c. symmetric only
   d. transitive only
2. The number of ways in which 6 " + " and 4 " – " signs can be arranged in a line such that no two " – " signs occur together is
   a. 5040

b. 35

c. 120

d. 210

3. The largest term in the expansion of $(1+x)^{19}$ when $x=\frac{1}{2}$ is

a. 6^{th}

b. 8^{th}

c. 5^{th}

d. 7^{th}

4. The number of ways in which 4 persons can be seated at a round table so that all shall not have same neighbours in any two arrangements is:

a. 3

b. none of these

c. 24

d. 6

5. The number of bijective functions from the set A to itself when A constrains 106 elements is

a. $(106)^2$

b. 106

c. 2^{106}

d. $\underline{|106}$

6. The greatest positive integer, which divides $(n+1)(n+2)(n+3)\ldots\ldots\ldots\ldots\ldots\ldots(n+r)\,\forall n \in W$, is

a. n+r

b. r

c. $(r+1)!$

d. $r!$

7. The chance that an event E ‘occurs’ or does not occur’s

a. 1

b. 0

c. none of these

d. 2

8. The centre of the sphere , which passes through (a , 0 , 0) , (0 , b , 0) (0 , 0 , c) and (0 , 0 ,0) is ? where abc $\neq$ 0

a. (a/2 , 0 ,0)

b. (a/2 , b/2 , c/2)

c. (0 , 0 , c/2)

d. (0 , b/2 , 0)

9. Both A and B throw a dice. The chance that B throws a number not less than that thrown by A is

a. 1/ 2

b. 21/ 36

c. 19/36

d. 15/ 36

10. $(\sqrt{3}+1)^{2n+1}+(\sqrt{3}-1)^{2n+1}$ is

a. an even positive integer

b. an irrational number

c. an odd positive integer

d. a rational number

11. Fill in the blanks:

If A and B are non-empty sets and either A and B is an infinite set, then A $\times$ B will also be an ________ set.

12. Fill in the blanks:

The coefficient of x^n in the binomial expansion of $(1 + x)^{2n-1}$ = ________.

13. Fill in the blanks:

The value of 4! - 3! is ________.

14. Fill in the blanks:

The coordinates of a point are the perpendicular distance from the ________ on the respective axes.

OR

Fill in the blanks:

The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimension are 10, 13 and 8 units are ________.

15. Fill in the blanks:

The derivative of secx is ________.

OR

Fill in the blanks:

The value of limit $\lim\limits_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2}$ is ________.

16. If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12,16, 20}, C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10,

15, 20}, find: C - B

17. Find the number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same lines.

18. Solve $9x^2 + 16 = 0$.

OR

Express (5 + 4 i) + (5 - 4 i) in the form of a + ib.

19. Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

20. If ${}^{n}C_{10} = {}^{n}C_{12}$, then find the value of ${}^{23}C_{n}$.

21. Assume that P(A) = P(B) then show that A = B.

OR

State whether A = {x : x is a letter in the word LOYAL} and B = {x: x is a letter of the word ALLOY} are equal? Justify your answer.

22. An experiment consists of recording boy-girt composition of families with 2 children.
(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?
(ii) What is the sample space if we are interested, in the number of girls in the family?

23. If the term free from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then find the value of k.

24. A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k - y_1 = m(h - x_1)$.

OR

Find the equation of line passing through the points (-1, 1) and (2, - 4).

25. Consider the statement, S: 80 is a multiple of 5 and 4. Check its validity.

26. Prove that $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$ = tan 6x

27. Let A and B be two sets. If $A \cap X = B \cap X = \phi$and $A \cup X = B \cup X$ for some set X, prove that A = B.

28. Let A = {-2, -1 , 0 ,1 , 2} and f : A $\rightarrow$Z be a function defined by f(x) = x^2 - 2x - 3. Find range of f i.e. f (A).

OR

If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}, then verify that

i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
ii. $A \times C$ is a subset of $B \times D$.

29. Evaluate $\lim\limits_{x \rightarrow 3} \frac{e^x - e^3}{x-3}$

30. Find the modulus and argument of complex number $\frac{1}{1+i}$.

31. Solve the inequality 2x + y > 3 graphically.

OR

Solve the inequation $\frac{x+3}{x-2} \geqslant 4$.

32. Prove by the principle of mathematical induction that for all n $\in$ N:
$\frac{1}{1.3}$ + $\frac{1}{3.5}$ + $\frac{1}{5.7}$ + ...+ $\frac{1}{(2n-1)(2n+1)}$ = $\frac{n}{2n+1}$

33. Prove that: cos 20° cos 40° cos 60° cos 80° = $\frac{1}{16}$.

OR

Find the general solution of the equation, sin x - 3 sin 2x + sin 3x = cos x - 3 cos2x + cos 3x.

34. The Sum of two no. is 6 times their geometric mean, show that no. are in the ratio (3 + 3 $\sqrt{2}$) : (3 - 2 $\sqrt{2}$)

35. Find the equation of a circle passing through the points (2, - 6), (6, 4) and (- 3, 1).

OR

Find the equation of a circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point (5, 4).

36. Calculate the coefficient of variation from the following data:

Income (in Rs.):	1000-1700	1700-2400	2400-3100	3100-3800	3800-4500	4500-5200
No. of families:	12	18	20	25	35	10

Solution
Section A

1. (a) an equivalence relation
2. (b) 35

   Explanation: Since all the plus signs are identical, we have number of ways in which 6 plus signs can be arranged=1.

   Now we will have 7 empty slots between these 6 identical + signs

   Hence number of possible places of - sign =7

   Therefore number of ways in which the 4 minus sign can take any of the possible 7 places=$^7C_4$ $=\frac{7!}{4!(7-4)!}=\frac{5.6.7}{1.2.3}=35$

3. (d) 7^{th} **Explanation:** We have the general term in the expansion of $(1+x)^{19}$ is given by $T_{r+1}={}^{19}C_r \quad (x)^r$

   Now $\frac{T_{r+1}}{T_r}=\frac{19C_r \quad (x)^r}{19C_{r-1} \quad (x)^{r-1}}$

   $=\frac{19!(x)^r}{(19-r)!\cdot r!}\times\frac{(20-r)!(r-1)!}{19!(x)^{r-1}}$

   $=\frac{(20-r)x}{r}$

   $=\frac{(20-r)}{2r}$ since given x = $\frac{1}{2}$

   Now $\frac{T_{r+1}}{T_r}\geq 1$

   $\Rightarrow\frac{(20-r)}{2r}\geq 1$

   $\Rightarrow 20-r\geq 2r$

   $\Rightarrow r\leq\frac{20}{3}$

   $\Rightarrow r\leq 6\frac{2}{3}$

   Hence the maximum value of r is 6 and hence the greatest term is $T_{r+1}=T_7$

4. (a) 3 **Explanation:**

The number of ways of seating 4 persons at a round table=$(4-1)!=3!$

But in clockwise and anticlockwise arrangements the same persons are neighbours and hence these two arrangements will be same.

Therefore the required number of arrangements=$\frac{3!}{2}=\frac{6}{2}=3$

5. (d) $\underline{|106}$ **Explanation:**

   If x and y are any two infinite sets having the same number if elements sayn, then the number of injective function from x to y is 106!.

6. (d) r ! **Explanation:**

   If n = 0 the given expression becomes 1.2.3.4........r = r! Also when n = 1 one more extra term will be there in the product 2.3.4........$(r+1)$ which is also divisible by r!.

7. (a) 1 **Explanation:**

   Let E denotes the given event. Then, $\overline{E}$ denotes E does not occur

   We have, $E \cup \overline{E}=S$

   $\Rightarrow P(E \cup \overline{E})=P(S)=1[\because P(S)=1]$

   $\therefore$ Required probability is, $P(E \cup \overline{E})=1$

8. (b) (a/2 , b/2 , c/2)
   Explanation:

   General equation of the sphere is $x^2+y^2+z^2+2gx+2fy+2hz+d=0$ ---------------------1)

   Since 1) passes through the point (0,0,0) using this in 1) we get d=0

   Similarly 1) passes through (a , 0 , 0) , (0 , b , 0) (0 , 0 , c) using these values in 1)

   $$a^2+2ag=0 => a(a+2g)=0$$
   $$b^2+2bf=0 => b(b+2f)=0$$
   $$c^2+2ch=0 => c(c+2h)=0$$

But as abc$\neq$0 So , a $\neq$0 ,b $\neq$0 ,c $\neq$0

So from above equations , we have a = - 2g , b= - 2f , c = -2h

centre is (-f ,-g , -h) = (a/2 , b/2 , c/2)

9. (b) 21/ 36

Explanation:

Given B getting number not less than A means B can get number on dice greater than or equal to A.

For throwing A and B one dice, the number of elements in the sample space is 6x6=36 i.e. n(S)= 36

Let E be the event of "B getting number not less than A", So it can happen in 21 ways out of 36 ways.

The ways are E={(1,1), (1,2),(1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,4),(4,5),(4,6),(5,5),(5,6),(6,6)} i.e. n(E)= 21

Therefore, P(E)= n(E)/n(S) = 21/36

10. (b) an irrational number

Explanation: We have $(a+b)^n+(a-b)^n$

= [$^nC_0\ a^n + {}^nC_1\ a^{n-1}b + {}^nC_2\ a^{n-2}b^2 + {}^nC_3\ a^{n-3}b_3$ + ... + $^nC_n\ b^n$] + [$^nC_0\ a^n - {}^nC_1\ a^{n-1}b + {}^nC_2\ a^{n-2}b_2 - {}^nC_3\ a^{n-3}b_3$ + ... +$(-1)^n . {}^nC_n\ b^n$]

$= 2\left[{}^nC_0 \quad a^n + {}^n C_2 \quad a^{n-2}b^2 + \ldots\ldots\ldots\ldots\right]$

Let $a=\sqrt{3}$ and b = 1 and n = 2n + 1

Now we get $(\sqrt{3}+1)^{2n+1}+(\sqrt{3}-1)^{2n+1}=2$ [$^{2n+1}C_0\ (\sqrt{3})^{2n+1}$ + $2\ ^{n+1}C_2\ (\sqrt{3})2^{n-1}1_2 + {}^{2n+1}C_4\ (\sqrt{3})^{2n-3}1_4$+....]

Since there are odd powers of $\sqrt{3}$ we have $(\sqrt{3}+1)^{2n+1}+(\sqrt{3}-1)^{2n+1}$ is alwasys an irrational number

11. infinite

12. ${}^{2n-1}C_n$

13. 18

14. given point

OR

$\sqrt{333}$

15. secx tanx

OR

$-\frac{1}{4}$

16. Here A = {3, 6,9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20}, C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}

C - B = {2,4,6,8,10,12,14,16} - {4,8,12,16,20}
= {2,6,10,14}

17. We know that a triangle will be formed by taking three points at a time.

$\therefore$ Required number of triangles = ${}^{12}C_3$ - ${}^{7}C_3$
$= \frac{12\times11\times10}{3\times2\times1} - \frac{7\times6\times5}{3\times2}$
= 220 - 35 = 185
Hence, the total number of such triangle is 185.

18. We have, $9x^2 + 16 = 0$

$\Rightarrow 9x^2 - 16i^2 = 0$ [$\because i^2 = -1$]

$\Rightarrow (3x)^2 - (4i)^2 = 0$

$\Rightarrow (3x - 4i)(3x + 4i) = 0$

$\Rightarrow 3x - 4i = 0$ or $3x + 4i = 0$

$\therefore x = \frac{4}{3}i$ or $x = -\frac{4}{3}i$
Hence, the roots of the given equation are $\frac{4}{3}i$ and $-\frac{4}{3}i$.

OR

Given, $(5+4i)+(5-4i)$

= (5 + 5) + i(4 - 4) = 10 + 0i

19. Here A ={x, y, z} and B = {1, 2}

Number of elements in set A = 3

Number of elements in set B = 2

Number of subsets of $A \times B = 3 \times 2 = 6$

Number of relations from A to B = 2^6

20. We know that, ${}^nC_x = {}^nC_y \Leftrightarrow x + y = n$ or $x = y$

Here $x \neq y$, so x + y = n.

$\Rightarrow$ n = 10 + 12 = 22

Now, ${}^{23}C_n = {}^{23}C_{22} = 23$

21. Let $X \in A \Rightarrow \{x\} \in P(A)$

$\Rightarrow \{x\} \in P(B)\ [\because P(A) = P(B)]$

$\Rightarrow x \in B$

$\therefore A \subset B\ldots$ (i)

Let $x \in B \Rightarrow \{x\} \in P(B)$

$\Rightarrow \{x\} \in P(A)[\because P(A) = P(B)]$

$\Rightarrow x \in A\ldots$ (ii)

$\therefore B \subset A$

From (i) and (ii) we have A = B

OR

We have, A = {L ,0 ,Y ,A ,L} = {L ,0 ,Y , A}

and, B = {A ,L ,L ,0 ,Y } = {L ,0 ,Y ,A }

Clearly, A = B.

22. There are two children in a family which are to be boys or girls.

(i) Hence S = {BB, BG, GB, GG}

(ii) When the outcome is BB then number of girl is 0

When the outcome is BG then number of girl is 1

When the outcome is GB then number of girl is 1

When the outcome is GG then number of girl is 2

Hence, when we are interested to know the number of girls in the family then the required sample space S = {0, 1, 2}.

23. General term of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is $T_{r+1} = {}^{10}C_r\,(\sqrt{x})^{10-r}\left(-\frac{k}{x^2}\right)^r$

$={}^{10}C_r x^{\frac{10-r}{2}-2r}(-k)^r = {}^{10}C_r(-k)^r x^{\frac{10-5r}{2}}$

For the term free from x,

Put $\frac{10-5r}{2}$ = 0 ⇒ 10 - 5r = 0 ⇒ r = 2

On putting r = 2, we get

$T_{2+1} = {}^{10}C_2\,(-k)^2$

But term free from x = 405 [given]

$\therefore {}^{10}C_2 k^2 = 405$

$\Rightarrow \frac{10\times9}{2}\times k^2 = 405 \Rightarrow k^2 = \frac{405\times2}{10\times9} = 9$

$\therefore k = \pm 3$

24. Let $A(x_1, y_1)$ and B(h, k) be two points.

$\therefore$ Slope of AB $= \frac{k-y_1}{h-x_1}$

Slope of AB = m (given)

$\therefore m = \frac{k-y_1}{h-x_1} \Rightarrow k-y_1 = m(h-x_1)$

OR

Given points are $A(x_1, y_1) = (-1, 1)$ and $B(x_2, y_2) = (2, -4)$, then equation of line AB is

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$\Rightarrow \quad y-1 = \frac{-4-1}{2+1}(x+1)[\because x_1=-1, y_1=1, x_2=2, y_2=-4]$

$\Rightarrow \quad y-1 = \frac{-5}{3}(x+1) \Rightarrow 3y-3 = -5x-5$

⇒ 5x + 3y + 2 = 0

25. The given compound statement is

S : 80 is a multiple of 5 and 4.

The component statements of the given compound statement are p : 80 is a multiple of 5.

q : 80 is a multiple of 4.

Then, $S \equiv p \wedge q$

Now, since we know that 80 is multiple of 5 as well as 4.

Therefore, the statements p and q are true.

Hence, the compound statement $S \equiv p \wedge q$ is also true i.e., $S \equiv p \wedge q$ is a valid statement.

26. LHS = $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

= $\frac{2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)+\ 2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)+\ 2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)}$

[$\because sin\ x + sin\ y = 2sin\left(\frac{x+y}{2}\right) \times cos\left(\frac{x-y}{2}\right)$ and $cos\ x + cos\ y = 2cos\left(\frac{x+y}{2}\right) \times cos\left(\frac{x-y}{2}\right)$]

= $\frac{2\sin 6x\cdot\cos x+2\sin 6x\cdot\cos 3x}{2\cos 6x\cdot\cos x+2\cos 6x\cdot\cos 3x}$

= $\frac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)} = \frac{\sin 6x}{\cos 6x} = \tan 6x$

= RHS

Hence proved.

27. Here $A \cup X = B \cup X$ for some set X

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow A = (A \cap B) \cup (A \cap X)\ [\because A \cap (A \cup X) = A]$

$\Rightarrow A = (A \cap B) \cup \phi\ [\because A \cap X = \phi]$

$\Rightarrow A = A \cap B$

$\Rightarrow A \subset B \ldots$(i)

Also $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = B\ [\because B \cap (B \cup X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B\ [\because B \cap X) = \phi]$

$\Rightarrow B \cap A = B$

$\Rightarrow B \subset A \ldots$ (ii)

From (i) and (ii), we have

A = B.

28. We have,

f(x) = x^2 -2x - 3

Now,

f(-2) = $(-2)^2$ - 2(-2) - 3

= 4 + 4 - 3

= 5

f (-1) = $(-1)^2$ -2(-1)-3

= 1 + 2 - 3

= 0

f (0) = $(0)^2$ - 2 $\times$ 0 - 3

= -3

f(1) = $(1)^2 - 2\times1 - 3$

= 1 - 2 - 3

= - 4

f(2) = $(2)^2$ - 2 $\times$ 2 - 3

= 4 - 4 - 3

= -3

Range(f) = {-4, -3, 0, 5}

OR

i. We have, $A=\{1,2\}$, $B=\{1,2,3,4\}$, $C=\{5,6\}$ and $D=\{5,6,7,8\}$

$\therefore (B\cap C)=\{1,2,3,4\}\cap\{5,6\}=\phi$

and $A\times(B\cap C)=\{1,2\}\times\phi=\phi$ .... (i)

Now, $A\times B$ = {1, 2} $\times$ {1, 2, 3, 4}

= $\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

and $A\times C=\{1,2\}\times\{5,6\}$

= $\{(1,5),(1,6),(2,5),(2,6)\}$

$\therefore \quad (A\times B)\cap(A\times C)=\phi$........ (ii)

From Eqs. (i) and (ii),

$A\times(B\cap C)=(A\times B)\cap(A\times C)$

ii. Now, $A\times C=\{1,2\}\times\{5,6\}$

= $\{(1,5),(1,6),(2,5),(2,6)\}$ ......(iii)

and $B \times D$ ={1, 2, 3, 4} $\times$ {5, 6, 7, 8}

= {

$(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4$

..... (iv)

From Eqs. (iii) and (iv), we can say,

$A \times C$ is a subset of $B \times D$.

29. $\lim\limits_{x \to 3} \frac{e^x - e^3}{x-3} = \lim\limits_{x \to 3} e^3 \left[\frac{e^{x-3} - 1}{x-3}\right]$

$= e^3 \lim\limits_{x \to 3} \left[\frac{e^{x-3} - 1}{x-3}\right]$

$= e^3 \lim\limits_{y \to 0} \left[\frac{e^{y} - 1}{y}\right]$ [put y = x - 3 so that $y \to 0$ as $x \to 3$]

= $e^3 \times 1$

= e^3

30. We have, $\frac{1}{1+i} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1^2 - i^2} = \frac{1-i}{1+1}$

= $\frac{1-i}{2} = \frac{1}{2} - \frac{i}{2}$

Let r cosθ = $\frac{1}{2}$ ...(i)

and r sinθ = - $\frac{1}{2}$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$r^2 \cos^2\theta + r^2 \sin^2\theta = \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2$

$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \Rightarrow r = \frac{1}{\sqrt{2}}$ [$\because$ r is positive]

On putting the value of r in Eqs. (i) and (ii), we get

cosθ = $\frac{1}{\sqrt{2}}$ and sinθ = - $\frac{1}{\sqrt{2}}$

Since, cosθ is positive and sinθ is negative.

So, θ lies in IV quadrant.

$\therefore$ θ = - $\frac{\pi}{4}$

Hence, modulus of $\frac{1}{1+i}$ is $\frac{1}{\sqrt{2}}$ and argument is $\frac{-\pi}{4}$.

31. We have, 2x + y > 3

Step I In equation form, given inequality can be written as

2x + y = 3 ... (i)

Step II On putting x = 0 in Eq. (i), we get

2 (0) + y = 3 $\Rightarrow$ y = 3

$\therefore$ The line meets Y - axis at y = 3 i.e., at point A (0,3).

On putting y = 0 in Eq. (i), we get

2x + 0 = 3 $\Rightarrow$ x = $\frac{3}{2}$

$\therefore$ The line meets X - axis at x = $\frac{3}{2}$ i.e., at point B $\left(\frac{3}{2}, 0\right)$

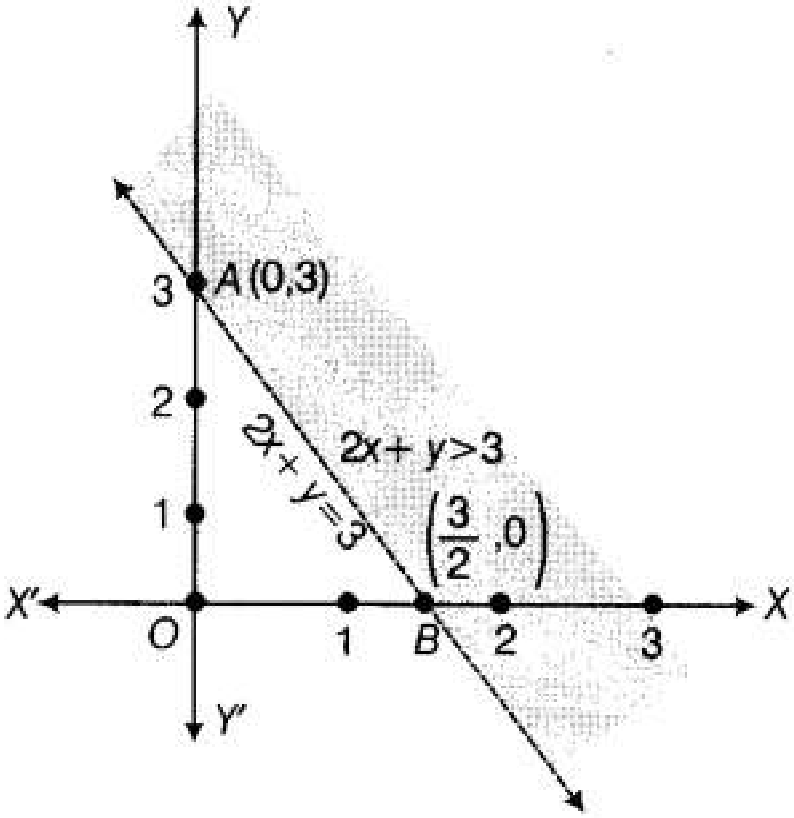
Step III Now, join the points obtained in **step II**. Since given inequality has the sign >, so we join the points by a dotted line.

Step IV Now, take a point not lying on the line say (0, 0) to check whether it satisfies the given inequality or not.

On putting x = 0 and y = 0 in given inequality, we get,

2 (0) + 0 > 3 $\Rightarrow$ 0 > 3 which is not correct.

Since, (0, 0) does not satisfy the given inequality, so we shade the portion which does not contain (0, 0) i.e., the region above the dotted line AB. The shaded region gives the solution set.



OR

Here $\frac{x+3}{x-2} \geqslant 4, x \neq 2$
$\Rightarrow \frac{x+3}{x-2} - 4 \geqslant 0$
$\Rightarrow \frac{x+3-4x+8}{x-2} \geqslant 0$
$\Rightarrow \frac{-3x+11}{x-2} \geqslant 0$
$\Rightarrow -3x + 11 \geqslant 0$
$\Rightarrow -3x \geqslant -11$
Dividing both sides by -3
$\therefore \frac{-3x}{-3} \leqslant \frac{-11}{-3}$
$\Rightarrow x \leqslant \frac{11}{3}$
Thus the solution set of given inequation is $\left(-\infty, \frac{11}{3}\right] - \{2\}$.

32. Suppose $P(n)$ be the statement given by
$P(n)$: $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$
We have, $P(1)$: LHS = $\frac{1}{1.3} = \frac{1}{3}$
RHS = $\frac{1}{(2\times1+1)} = \frac{1}{3}$
So, $P(1)$ is true.
Suppose $P(m)$ be true.
$\Rightarrow \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2m-1)(2m+1)} = \frac{m}{2m+1}$ .....(i)
We shall now show that $P(m+1)$ is true. For this we shall that
$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2m-1)(2m+1)} + \frac{1}{(2m+1)(2m+3)} = \frac{m+1}{2m+3}$
Considering LHS
$= \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2m-1)(2m+1)} + \frac{1}{(2m+1)(2m+3)}$
$= \frac{m}{2m+1} + \frac{1}{(2m+1)(2m+3)}$ [Using (i)]
$= \frac{2m^2+3m+1}{(2m+1)(2m+3)} = \frac{(2m+1)(m+1)}{(2m+1)(2m+3)} = \frac{m+1}{2m+3}$ = RHS
$\therefore P(m+1)$ is true
$\Rightarrow P(m)$ is true $\therefore P(m+1)$ is true
$\therefore$ by the principle of mathematical induction, the given result is true for all $n \in N$.

33. We have,
LHS = cos 20° cos 40° cos 60° cos 80°
$\Rightarrow$ LHS = cos 60° (cos 20° cos 40°) cos 80°
$\Rightarrow$ LHS = $\frac{1}{2} \times \frac{1}{2}$ (2 cos 20° cos 40°) cos 80° $\left[\because \cos\frac{\pi}{3} = \frac{1}{2}\right]$

$\Rightarrow$ LHS = $\frac{1}{4}$ [{cos (40° + 20°) + cos (40° - 20°)} cos 80°] [$\because$ 2 cos A cos B = cos (A + B) + cos (A - B)]
$\Rightarrow$ LHS = $\frac{1}{4}$ {(cos 60° + cos 20°) cos 80°}
$\Rightarrow$ LHS = $\frac{1}{4}$ {($\frac{1}{2}$ + cos 20°) cos 80°}
$\Rightarrow$ LHS = $\frac{1}{4}$ {$\frac{1}{2}$ cos 80° + cos 80° cos 20°}
$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° + 2 cos 80° cos 20°}
$\Rightarrow$ LHS = $\frac{1}{8}$ [cos 80° + {cos (80° + 20°) + cos (80° - 20°)}]
$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° + cos 100° + cos 60°}
$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° + cos (180° - 80°) + cos 60°}
$\Rightarrow$ LHS = $\frac{1}{8}$ {cos 80° - cos 80° + cos 60°} [$\because$ cos (180° - x) = -cos x]
$\Rightarrow$ LHS $= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ = RHS

OR

Given, $sin\ x - 3sin\ 2x + sin\ 3x = cos\ x - 3cos2x + cos\ 3x$
$\Rightarrow (sin\ x - cos\ x) - 3(sin\ 2x - cos\ 2x) + (sin\ 3x - cos\ 3x) = 0$
$\Rightarrow$ [cos $\left(\frac{\pi}{2} - x\right) - cos\ x] - 3[cos\ \left(\frac{\pi}{2} - 2x\right) - cos 2x +$ [cos $\left(\frac{\pi}{2} - 3x\right)$ $-cos 3x] = 0$
[$\because$ sin θ = cos $\left(\frac{\pi}{2} - \theta\right)$]
$\Rightarrow$ - 2 sin $\left(\frac{\frac{\pi}{2} - x + x}{2}\right)$ sin $\left(\frac{\frac{\pi}{2} - x - x}{2}\right)$ - 3 [- 2 sin $\left(\frac{\frac{\pi}{2} - 2x + 2x}{2}\right)$ sin $\left(\frac{\frac{\pi}{2} - 2x - 2x}{2}\right)$] + [- 2 sin $\left(\frac{\frac{\pi}{2} - 3x + 3x}{2}\right)$ sin $\left(\frac{\frac{\pi}{2} - 3x - 3x}{2}\right)$] = 0
[$\because cosx - cosy = -2sin\ \left(\frac{x+y}{2}\right)$ sin $\left(\frac{x-y}{2}\right)$]
$\Rightarrow$ - 2 sin $\left(\frac{\pi}{4}\right)$ sin $\left(\frac{\frac{\pi}{2} - 2x}{2}\right)$ + 3 [2 sin $\left(\frac{\pi}{4}\right)$ sin $\left(\frac{\frac{\pi}{2} - 4x}{2}\right)$] - 2 sin $\left(\frac{\pi}{4}\right)$ sin $\left(\frac{\frac{\pi}{2} - 6x}{2}\right)$ = 0
$\Rightarrow$ 2 sin $\left(\frac{\pi}{4}\right)$ [- sin $\left(\frac{\pi}{4} - x\right)$ + 3 sin $\left(\frac{\pi}{4} - 2x\right)$ - sin $\left(\frac{\pi}{4} - 3x\right)$] = 0
$\Rightarrow$ [- sin $\left\{-\left(x - \frac{\pi}{4}\right)\right\}$ + 3 sin $\left\{-\left(2x - \frac{\pi}{4}\right)\right\}$ - sin $\left\{-\left(3x - \frac{\pi}{4}\right)\right\}$ = 0
$\Rightarrow$ [sin $\left(x - \frac{\pi}{4}\right)$ - 3 sin $\left(2x - \frac{\pi}{4}\right)$ + sin $\left(3x - \frac{\pi}{4}\right)$] = 0
[$\because$ sin (- θ) = - sin θ]
$\Rightarrow$ [sin $\left(x - \frac{\pi}{4}\right)$ + sin $\left(3x - \frac{\pi}{4}\right)$] - 3 sin $\left(2x - \frac{\pi}{4}\right)$ = 0
$\Rightarrow$ 2 sin $\left(\frac{x - \frac{\pi}{4} + 3x - \frac{\pi}{4}}{2}\right)$ cos $\left(\frac{x - \frac{\pi}{4} - 3x + \frac{\pi}{4}}{2}\right)$ - 3 sin $\left(2x - \frac{\pi}{4}\right)$ = 0

[$\because sin(x+y) = 2sin\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$\Rightarrow$ 2 sin $\left(\frac{4x-\frac{\pi}{2}}{2}\right)$ cos $\left(\frac{-2x}{2}\right)$ - 3 $\left(2x-\frac{\pi}{4}\right)$ = 0

$\Rightarrow$ 2 sin $\left(2x-\frac{\pi}{4}\right)$ cos (- x) - 3 sin $\left(2x-\frac{\pi}{4}\right)$ = 0

$\Rightarrow$ 2 sin $\left(2x-\frac{\pi}{4}\right)$ cos x - 3 sin $\left(2x-\frac{\pi}{4}\right)$ = 0

[$\because cos(-x) = cosx$]

$\Rightarrow$ sin $\left(2x-\frac{\pi}{4}\right)\left[2cos\ x-3\right] = 0$

$\Rightarrow$ sin $\left(2x-\frac{\pi}{4}\right)$ = 0 as cos x $\neq \frac{3}{2}$

[$\because$ cos x cannot be greater than 1]

$\therefore$ sin $\left(2x-\frac{\pi}{4}\right)$ = sin 0

$\Rightarrow$ 2x - $\frac{\pi}{4} = n\pi$

$\Rightarrow$ 2x = $n\pi + \frac{\pi}{4}$

$\Rightarrow$ x = $\frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$

34. a + b = 6 $\sqrt{a}b$

$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

by C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{2}{1}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$

again by C and D

$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\frac{a}{b} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2}$ (on squaring both sides)

$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$

$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

a : b = (3 + 2$\sqrt{2}$) : (3 - 2$\sqrt{2}$)

35. Let the equation of the circle passing through the given points be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, circle passes through the point (2, - 6).

So, put x = 2, y = - 6 in Eq. (i), we get

$4 + 36 + 4g - 12f + c = 0$

$\Rightarrow 4g - 12f + c = - 40$ ...(ii)

Also, circle passes through the point (6, 4).

So, put x = 6, y = 4 in Eq. (i), we get

$36 + 16 + 12g + 8f + c = 0$

$\Rightarrow 12g + 8f + c = - 52$ ...(iii)

Also, cirlce passes through the point (- 3, 1).

So, put x = - 3 and y = 1 in Eq. (i), we get

$9 + 1 - 6g + 2f + c = 0$

$\Rightarrow - 6g + 2f + c = - 10$ ...(iv)

On subtracting Eq. (iii) from Eq. (ii), we get

$- 8g - 20f = 12$ ...(v)

On subtracting Eq. (iv) from Eq. (iii), we get

$18g + 6f = - 42$ ...(vi)

On solving Eqs. (v) and (vi) for g and f, we get

$g = -\frac{32}{13}, f = \frac{5}{13}$

On putting the values of g and f in Eq. (ii), we get

$c = -\frac{332}{13}$

Now, on putting $g = -\frac{32}{13}$, $f = \frac{5}{13}$ and $c = -\frac{332}{13}$ in Eq. (i), we get

$x^2 + y^2 - \frac{64}{13}x + \frac{10}{13}y - \frac{332}{13} = 0$

$\Rightarrow 13x^2 + 13y^2 - 64x + 10y - 332 = 0$

which is the required equation of circle.

OR

Here, the equation of circle is $x^2 + y^2 + 4x + 6y + 11 = 0$

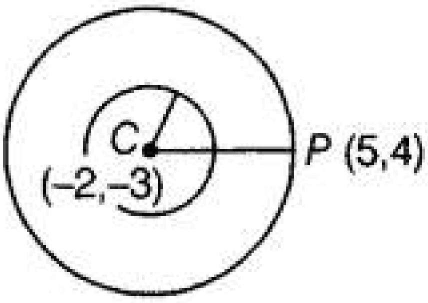
$\Rightarrow (x^2 + 4x) + (y^2 + 6y) = - 11$

On adding 4 and 9 both sides to make perfect squares, we get

$(x^2 + 4x + 4) + (y^2 + 6y + 9) = - 11 + 4 + 9$

$\Rightarrow (x + 2)^2 + (y + 3)^2 = (\sqrt{2})^2$ ...(i)

Its centre is (- 2, - 3)



The required circle is concentric with circle 1, therefore its centre is (- 2 , - 3) . Since, it passes through (5, 4), therefore radius is

r = CP = $\sqrt{(5+2)^2+(4+3)^2}$ [$\because$ distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

= $\sqrt{49+49}=7\sqrt{2}$

Hence, the equation of required circle having centre (- 2, - 3) and radius $7\sqrt{2}$ is,

$(x + 2)^2 + (y + 3)^2 = (7\sqrt{2})^2$

$\Rightarrow x^2 + 4x + 4 + y^2 + 6y + 9 = 98$

$\Rightarrow x^2 + 4x + y^2 + 6y - 85 = 0$

36. To calculate the coefficient of variation we need to make the following table,

Income	f_i	x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 2750}{700}$	f_iu_i	u_i^2	$f_iu_i^2$
1000-1700	12	1350	-2	-24	4	48
1700-2400	18	2050	-1	-18	1	18
2400-3100	20	2750	0	0	0	0
3100-3800	25	3450	1	25	1	25
3800-4500	35	4150	2	70	4	140
4500-5200	10	4850	3	30	9	90
	120			**83**		**321**

Here, N = 120, A = 2750, $\Sigma f_iu_i = 83$, $\Sigma f_iu_i^2 = 321$ and h = 700

First we need to find the mean of the given data,

$\therefore$ Mean = $\overline{X}$ = A + h $\left(\frac{1}{N}\Sigma f_iu_i\right)$

$\Rightarrow \overline{X}$ = 2750 + 700 $\left(\frac{83}{120}\right)$ = 3234.17

Next, we need to find standard deviation from the data.

var (x) = $h^2 \left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right]$ = 490000 $\left[\frac{321}{120} - \left(\frac{83}{120}\right)^2\right]$ = 1076332.64

$\therefore$ S.D. = $\sqrt{\operatorname{var}(x)} = \sqrt{1076332 \cdot 64}$ = 1037.46

Now,

Coefficient of variation (in percentage) = $\frac{S.D}{\overline{X}} \times$ 100 = $\frac{1037.46}{3234.17} \times$ 100 = 32.08%