CBSE Class 11 Mathematics
Sample Papers 06 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The domain of the function f (x) = $\sqrt{\sin x - 1}$ is
   a. $-\phi$
   b. $\left\{x \in R : x = 2n\pi \pm \frac{\pi}{2}, n \in I\right\}$
   c. $\left\{\frac{\pi}{2}\right\}$
   d. $\left\{x \in R : x = 2n\pi + \frac{\pi}{2}, n \in I\right\}$.
2. The number of all odd divisors of 3600 is
   a. 9
   b. 18

c. none of these

d. 45

3. $(\sqrt{5}+1)^{2n+1}-(\sqrt{5}-1)^{2n+1}$is

a. 0

b. an even positive integer

c. an odd positive integer

d. not an integer

4. The number of all selections which a student can make for answering one or more questions out of 8 given questions in a paper, when each question has an alternative, is:

a. 255

b. 6561

c. 6560

d. 256

5. If f : $R \to R$ satisfies f (x + y) = f (x) + f (y) for all x, y $\in$ R and f (1) = 7, then $\sum_{r=1}^{n} f(r)$ is

a. 7 n(n +1)

b. $\frac{7(n+1)}{2}$

c. $\frac{7n(n+1)}{2}$

d. $\frac{7n}{2}$

6. For all positive integers n, the number $4^n+15n-1$ is divisible by :

a. 16

b. 24

c. 9

d. 36

7. A box contains n pairs of shoes and 2r shoes are selected.(r<n).The probability that there is exactly one pair is

a. $\frac{(n.^{n-1}C_{r-1})2^{r-1}}{^{2n}C_{2r}}$

b. $\frac{^{n-1}C_{r-1}}{^{2n}C_{2r}}$

c. $\frac{n.^{n-1}C_{r-1}}{^{2n}C_{2r}}$

d. none of these

8. The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

a. at right angles to plane YOZ

b. none of these

c. at right angles to x –axis

d. is parallel to Y- axis

9. 8 coins are tossed at a time. The probability of getting 6 heads up is

a. 229/ 256

b. 37/ 256

c. 57/ 64

d. 7/ 64

10. The 1st three terms in the expansion of $(2+\frac{x}{3})^4$ are

a. $16+\frac{32x}{3}+\frac{24x^2}{9}$

b. $16+\frac{34x}{3}+\frac{24x^2}{9}$

c. $16+12x+\frac{3}{16}x^2$

d. $16+3x-\frac{3}{16}x^2$

11. Fill in the blanks:

    Two ordered pairs are equal if their corresponding elements are ________.

12. Fill in the blanks:

    The total number of terms in the expansion of $(x + a)^{51} - (x + a)^{51}$ after simplification is ________.

13. Fill in the blanks:

    The value of 2 $\times$ 6! - 3 $\times$ 5! is ________.

14. Fill in the blanks:

    The three coordinate planes divide the space into ________ parts.

    OR

    Fill in the blanks:

    If a point P lies in yz-plane, then the coordinates of a point on yz-plane is of the form ________.

15. State true or false:

    The derivative of 5 sin x - 6 cos x + 7 is ________.

    OR

    Fill in the blanks:

    The value of the given limit $\lim_{x\to 0} x \sec x$ is ________.

16. Taking the set of natural numbers as the universal set, write down the complement of the set: {x : x is an odd natural number}

17. Evaluate $2\times 6!-3\times 5!$

18. Solve the inequalities: $2 \leqslant 3x - 4 \leqslant 5$

OR

Express the complex numbers 3(7 + i7) + i(7 + i7) in standard form

19. Find x and y, if (x + 3, 5) = (6, 2x + y).

20. How many 3 letter words can be made, using the letters of the word 'ORIENTAL?

21. Find AΔB, if A = {1, 3, 6, 11, 12} and B = {1, 6}.

OR

If A is any set, prove that: $A \subseteq \phi \Leftrightarrow A = \phi$

22. A and B are two events such that P(A) = 0.54, P(B) = 0.69 and $P(A \cap B) = 0.35$. Find

    i. $P(A \cup B)$
    ii. $P(A' \cap B')$
    iii. $P(A \cap B')$
    iv. $P(B \cap A')$

23. If the middle term in the binomial expansion of $\left(\frac{1}{x} + x \sin x\right)^{10}$ is equal to $\frac{63}{8}$, find the value of x.

24. Find the angle between the lines joining the points (0, 0), (2, 3) and the points (2, - 2), (3, 5).

OR

Find the angle between the lines $y = (2 - \sqrt{3})(x + 5)$ and $y = (2 + \sqrt{3})(x - 7)$.

25. Rewrite each of the following statements in the form "p if and, only if q"
(i) p : If you watch television, then your mind is free and if your mind is free, then you watch television.
(ii) q : For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.
(iii) r : If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a

rectangle, then it is equiangular.

26. Prove that cot A + cot (60^o + A) - cot (60^o - A) = 3 cot 3 A

27. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers.
Find: the number of people who read exactly one newspaper.

28. If f (x) = x^2 find $\frac{f(1.1)-f(1)}{(1.1-1)}$

OR

If A = {1, 2, 3}, B = {3, 4} and C = {1, 3, 5}, find

i. $A \times (B \cup C)$
ii. $A \times (B \cap C)$
iii. $(A \times B) \cap (A \times C)$

29. Evaluate $\lim\limits_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$.

30. If $(x + iy)^{1/3}$ = a + ib, where x, y, a, b $\in$ R, then show that $\frac{x}{a} - \frac{y}{b}$ = - 2 (a^2 + b^2).

31. Solve the linear inequality $\frac{-3x+10}{x+1} > 0$.

OR

Solve $x + \frac{x}{2} + \frac{x}{3} < 11$

32. Prove by Mathematical Induction that $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$, for all n$\in$N.

33. If α and β be the two distinct real numbers satisfying the equation a cosθ + b sin θ = c, then prove that sin $(\alpha + \beta) = \frac{2ab}{a^2+b^2}$ and $\sin \alpha \cdot \sin \beta = \frac{c^s - a^2}{a^2+b^2}$

OR

Prove that $\cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15} = \frac{1}{16}$

34. A farmer buys a used tractor for Rs. 12000. He pays Rs. 6000 cash and agrees to pay the balance in annual installments of Rs. 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

35. Find out the equation of parabola, if the focus is at (- 6, - 6) and the vertex is at (- 2, 2).

OR

A visitor with sign board 'DO NOT LITTER' is moving on a circular path in an exhibition. During the movement he stops at points represented by (3, - 2) and (-2, 0). Also, centre of the circular path is on the line 2x - y = 3. What is the equation of the path? What message he wants to give to the public?

36. A student obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later found that one observation was wrongly copied as 50, the correct figure being 40. Find the correct mean and S.D.

Solution

Section A

1. (d) $\left\{x \in R : x = 2n\pi + \frac{\pi}{2}, n \in I\right\}$. **Explanation:**

   this exists only if

   $$\begin{aligned}&\sin x - 1 \geq 0\\&\sin x \geq 1\\&\sin x > 1, \text{notpossible}\\&\therefore \sin x = 1\end{aligned}$$

   $$\Rightarrow \sin x = \sin(2n\pi + \tfrac{\pi}{2})$$
   $$\Rightarrow x = 2n\pi + \tfrac{\pi}{2}(n \in I)$$

2. (a) 9 **Explanation:**

   $we \quad have \quad 3600 = 2^4 . 3^2 . 5^2$

   To get the odd factors we will get rid of 2's

   We will make the selection from only 3's and 5's

   Number of ways 3 can be selected from a lot of two 3's= 3 ways (one 3,two 3's or three 3's)

   Number of ways 5 can be selected from a lot of two 5's= 3 ways (one 5,two 5's or three 5's)

   Therefore the number of odd factors is 3600= 3 X 3 =9

3. (b) an even positive integer
   Explanation: We have $(a+b)^n - (a-b)^n$
   = [$^nC_0$ a^n + $^nC_1$ $a^{n-1}b$+$^nC_2$ $a^{n-2}b_2$+$^nC_3$ $a^{n-3}b^3$ + ... ^nC_n b^n] - [$^nC_0$ a^n-$^nC_1$ $a^{n-1}b$+$^nC_2$ $a^{n-2}b_2$-$^nC_3$ $a^{n-3}b_3$ + ... + $(-1)^n$.^nC_n b^n]

$= 2\left[{}^{n}C_1 \quad a^{n-1}b + {}^{n}C_3 \quad a^{n-3}b^3 + \ldots\ldots\ldots\ldots\right]$
Let $a = \sqrt{5}$ and b =1 and n = 2n + 1
Now we get $(\sqrt{5}+1)^{2n+1} - (\sqrt{5}-1)^{2n+1} = 2^{[2n+1]}C_1$
$(\sqrt{3})^{2n} + {}^{2n+1}C_3 \quad (\sqrt{3})^{2n-2}1^3 + {}^{2n+1}C_5\,(\sqrt{3})^{2n-4}1^5 + \ldots\ldots\ldots\ldots\ldots]$
$= 2\left[{}^{2n+1}C_1 \quad (3)^n + {}^{2n+1}C_3 \quad (3)^{n-1} + {}^{2n+1}C_5 \quad (3)^{n-2} + \ldots\ldots\ldots\ldots\right]$

= 2 (a positive integer)
Hence we have $(\sqrt{5}+1)^{2n} - (\sqrt{5}-1)^{2n+1}$ is an even positive integer

4. (c) 6560 **Explanation:**

   Since a student can solve every question in three ways- either he can attempt the first alternative , or the second alternative or he does not attemp that question

   Hence the total ways in which a sudent can attempt one or more of 8 questions =3^8

   Therefore to find the number of all selections which a student can make for answering one or more questions ou tof 8 given questions =$3^8 - 1 = 6560$ [we will have to exclude only the case of not answering all the 8 questions]

5. (c) $\frac{7n(n+1)}{2}$ **Explanation:** Given $f(x+y) = f(x) + f(y) \ldots (i)$
   and f(1) = 7
   Put x = 1, y = 1 in equation (i), we obtain f(1+1) = f(1) + f(1) = 14 $\Rightarrow$ f(2) = 14
   Similarly f(1+1+1) = f(2) + f(1) = 14 + 7 $\Rightarrow$ f(3) = 21
   Since we have f(1) = 1 $\times$ 7 = 7, f(2) = 2 $\times$ 7 = 14, f(3) = 3 $\times$ 7 = 21,.....
   We can get f(n) = n x 7 = 7n
   Now $\sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + \ldots\ldots\ldots\ldots\ldots + f(n)$
   = 7 + 14 + 21 + ... + 7n
   = 7 [1+2+3+....+n]$= 7\frac{n(n+1)}{2}$

6. (c) 9

   Explanation:

   Replace n = 1 we get 18 n = 2 we get 45.... By the principle of mathematical induction it is divisible by 9.

7. (a) $\frac{(n.^{n-1}C_{r-1})2^{r-1}}{^{2n}C_{2r}}$ **Explanation:**

The box contains 2n shoe. we can choose 2r shoes out of 2n shoes in $^{2n}C_{2r}$ ways. We can choose one complete pair out of n pairs in $^{n}C_{1}$ ways. Now, we have to avoid a complete pair. While choosing (2r-2)shoes out of remaining (n-1)pairs of shoes,we first choose (r-1) pairs out of (n-1) pairs. This can be done in $^{n-1}C_{r-1}$ways. From each of these (r-1) pairs, choose (r-1) single (unmatching) shoes from each pair.This can be done in 2^{r-1} ways. Thus, the number of favourable ways is $(^{n}C_{1})(^{n-1}C_{r-1}).2^{r-1}$.

Hence, the probability of the required event

= $\frac{n(^{n-1}C_{r-1}).2^{r-1}}{^{2n}C_{2r}}$

8. (c) at right angles to x –axis
Explanation:

Since the drs i.e. a=0.hence the line is perpendicular to x-axis.

9. (d) 7/ 64
Explanation:

Total ways of getting 6 heads out of 8 toss of coins is 28.

Total number of outcome is 2^8 = 256

Therefore probability is $\frac{28}{256}=\frac{7}{64}$

10. (a) $16+\frac{32x}{3}+\frac{24x^2}{9}$ Explanation:
We have
$(x+a)^n={}^nC_0 \quad x^n+{}^nC_1 \quad x^{n-1}a+{}^nC_2 \quad x^{n-2}a^2+{}^nC_3 \quad x^{n-3}a^3+$ .... + ^nC_n a^n

Now consider $\left(2+\frac{x}{3}\right)^4$
Here x = 2, a = $\frac{x}{3}$, n= 4
$\left(2+\frac{x}{3}\right)^4={}^4C_0 \quad (2)^4+{}^4C_1 \quad (2)^3\left(\frac{x}{3}\right)+{}^4C_2 \quad (2)^2\left(\frac{x}{3}\right)^2+{}^4C_3 \quad (2)^1\left(\frac{x}{3}\right)^3+$

$= 16 + \frac{32x}{3} + \frac{24x^2}{9} + \ldots\ldots\ldots$

11. equal

12. 26

13. 1080

14. Eight

OR

(0, y, z)

15. 5 cos x + 6 sin x

OR

0

16. Here $U = \{x : x \in N\}$
Let A = {x : x is an odd natural number}
$A' = U - A = \{x : x \in N\}-${x : x is an odd natural number}
= {x : x is an even natural number}

17. We have,
$2 \times 6! - 3 \times 5! = 2 \times 6 \times 5! - 3 \times 5!$
= 5! (12 - 3)
= 5! $\times$ 9
$= 5 \times 4 \times 3 \times 2 \times 1 \times 9$
= 1080

18. We have $2 \leqslant 3x - 4 \leqslant 5$
$\Rightarrow 2 + 4 \leqslant 3x \leqslant 5 + 4 \Rightarrow 6 \leqslant 3x \leqslant 9$
$\Rightarrow 2 \leqslant x \leqslant 3$

OR

3(7 + i7) + i (7 + i7)

$= 21 + 21i + 7i + 7i^2 = 21 + 28i - 7$

$= 14 + 28\,i$

19. Given,

    (x + 3, 5) = (6, 2x + y)

    Since the ordered pairs are equal, corresponding elements are also equal.

    Thus, x + 3 = 6 and 5 = 2x + y

    On solving we get, x = 3 and y = -1

20. We have word 'ORIENTAL'

    Total number of letters, n = 8

    Number of letters to be used in forming a word, r = 3

    Thus, total number of words thus formed = $^8P_3$

    $= \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\times7\times6\times5!}{5!} = 336$

21. We know that $A\Delta B$ represents the Symmetric Difference between sets A and B.

    That is, $A\Delta B = (A - B) \cup (B - A)$

    Now, according to the question,

    A = {1, 3, 6, 11, 12} and B = {1, 6}

    Then, (A - B) = {1, 3, 6, 11, 12} - {1, 6} = {3, 11, 12}

    and (B - A) = {1, 6} - {1, 3, 6, 11, 12} = ϕ

    where ϕ represents null set or empty set.

    $\therefore$ $A\Delta B = (A - B) \cup (B - A)$

    = {3, 11, 12} $\cup$ ϕ

    = {3, 11,12}

OR

The symbol '$\Leftrightarrow$' stands for if and only if (in short if).

In order to show that two sets A and B are equal, we show that $A \subseteq B$ and $B \subseteq A$.

We have $A \subseteq \phi, \because \phi$is a subset of every set,

$\therefore \quad \phi \subseteq A$

Hence A = ϕ

To show the backward implication, suppose that $A = \phi$.

$\because$ every set is a subset of itself

$\therefore \quad \phi = A \subseteq \phi$

Hence, proved.

22. Here P(A) = 0.54, P(B) = 0.69 and $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$ = 0.35

    i. We know that
    $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
    = 0.54+ 0.69 - 0.35
    = 1.23 - 0.35 = 0.88

    ii. $\mathrm{P}(\mathrm{A}^{'} \cap \mathrm{B}^{'}) = \mathrm{P}(\mathrm{A} \cup \mathrm{B})^{'} = 1 - \mathrm{P}(\mathrm{A} \cup \mathrm{B})$
    = 1 - 0.88 = 0.12

    iii. $\mathrm{P}(\mathrm{A} \cap \mathrm{B}^{'}) = \mathrm{P}(\mathrm{A}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$
    = 0.54 - 0.35 = 0.19

    iv. $P(B \cap A') = P(B) - P(A \cap B)$
    = 0.69 - 0.35 = 0.34

23. In the binomial expansion of $\left(\frac{1}{x} + x \sin x\right)^{10}$, $\left(\frac{10}{2} + 1\right)^{th}$ i.e. 6th term is the middle term.

    It is given that

    $T_6 = \frac{63}{8}$

    $\Rightarrow \quad {}^{10}\mathrm{C}_5 \left(\frac{1}{x}\right)^{10-5}$ (x sinx)$^5 = \frac{63}{8}$

    $\Rightarrow \quad \frac{10!}{5!5!} (\sin x)^5 = \frac{63}{8}$

    $\Rightarrow \quad (\sin x)^5 = \left(\frac{1}{2}\right)^5$

    $\Rightarrow \quad \sin x = \frac{1}{2} = \sin \frac{\pi}{6}$

    $\Rightarrow \quad x = n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

24. Let θ be the angle between the given lines.

    We have,

    m_1 = Slope of the line joining (0, 0) and $(2, 3) = \frac{3-0}{2-0} = \frac{3}{2}$

    m_2 = Slope of the line joining (2, -2) and (3, 5) $= \frac{5+2}{3-2} = 7$

$\therefore \tan\theta = \pm\left|\frac{m_1-m_2}{1+m_1m_2}\right| = \pm\left|\frac{7-3/2}{1+7(3/2)}\right|$

$= \pm\left|\frac{11/2}{23/2}\right| = \pm\left|\frac{11}{23}\right|$

$\Rightarrow \theta = \tan^{-1}\left|\frac{11}{23}\right|$

OR

Given equation of lines are

$y = (2-\sqrt{3})(x+5)$ and $y = (2+\sqrt{3})(x-7)$

$\Rightarrow \quad y = (2-\sqrt{3})x + 5(2-\sqrt{3})$

and $y = (2+\sqrt{3})x - 7(2+\sqrt{3})$

Slope of above lines are,

$m_1 = 2-\sqrt{3}, m_2 = 2+\sqrt{3}$

Angle between two lines,

$\tan\theta = \left|\frac{m_2-m_1}{m_1+m_2}\right| = \left|\frac{2+\sqrt{3}-(2-\sqrt{3})}{1+(2+\sqrt{3})(2-\sqrt{3})}\right|$

$\Rightarrow \quad \tan\theta = |\sqrt{3}|$

i.e., $\tan\theta = \sqrt{3}$ and $\tan\theta = -\sqrt{3}$

$\therefore \quad \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

25. (i) You watch television if and only if your mind is free.

(ii) You get an A grade if and only if you do all the homework regularly.

(iii) A quadrilateral is equiangular if and only if it is a rectangle.

26. **LHS:** cot A + cot (60^0 + A) - cot (60^0 - A)

= $\frac{1}{\tan A} + \frac{1}{\tan(60^\circ+A)} - \frac{1}{\tan(60^\circ-A)}$

= $\frac{1}{\tan A} + \frac{1-\sqrt{3}\tan A}{\sqrt{3}+\tan A} - \frac{1+\sqrt{3}\tan A}{\sqrt{3}-\tan A}$ [$\because$tan (A + B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$ and tan 60^0 = $\sqrt{3}$]

= $\frac{1}{\tan A} - \frac{8\tan A}{3-\tan^2 A}$

= $\frac{3-9\tan^2 A}{3\tan A-\tan^3 A} = 3\left(\frac{1-3\tan^2 A}{3\tan A-\tan^3 A}\right)$

= $\frac{3}{\tan 3A}$

$= 3cot\ 3A =$ **RHS**

Hence proved.

27. Here

n(U) = a + b + c + d + e + f + g + h = 60 ...... (i) n (H) = a + b + c +d = 25 ...... (ii)

n(T) = b + c + f + g = 26 ....(iii)

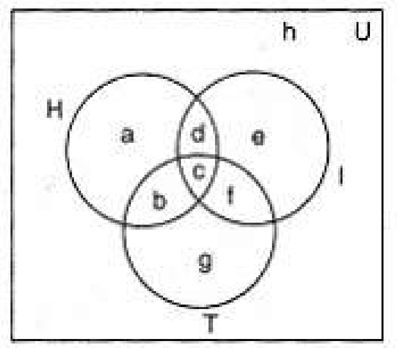
n(I) = c + d + e + f = 26 ...... (iv)

$n(H \cap I) = c + d = 9$...... (v)

$n(H \cap T) = b + c = 11$....... (vi)

$n(T \cap I) = c + f = 8$....... (vii)

$n(H \cap T \cap I) = c = 3$...... (viii)



Putting value of c in (vii),

3+ f = 8 $\Rightarrow$f = 5

Putting value of c in (vi),

3 +b = 11$\Rightarrow$b = 8

Putting values of c in (v),

3 + d = 9$\Rightarrow$d = 6

Putting value of c, d, f in (iv),

3 + 6 + e + 5 = 26$\Rightarrow$e = 26 - 14= 12

Putting value of b, c, f in (iii),

8 + 3 + 5 + g = 26 $\Rightarrow$g = 26 - 16= 10

Putting value of b, c, d in (ii)

a + 8 + 3 + 6 = 25 $\Rightarrow$a = 25 - 17 = 8

Number of people who read exactly one newspapers

= a + e + g

= 8 + 12 +10 = 30

28. Here f (x) = x^2

At x = 1.1

f(1.1) = $(1.1)^2$ = 1.21

f(1)=$(1)^2$ = 1

$\therefore \frac{f(1.1)-f(1)}{(1.1-1)} = \frac{1.21-1}{0.1} = \frac{0.21}{0.1} = 2.1$

OR

Given: A = {1, 2, 3}, B = {3, 4} and C = {1, 3, 5}

i. Clearly, $B \cup C$ = {1, 3 , 4, 5}
$\therefore A \times (B \cup C)$,= {1, 2, 3} $\times$ {1, 3, 4, 5}
= {(1, 1), (1, 3), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), (3, 1), (3, 3), (3, 4), (3, 5)}

ii. Clearly, $B \cap C$ = {3}.
$\therefore A \times (B \cap C)$ = {1, 2, 3} $\times$ {3} = {(1, 3), (2, 3), (3, 3)}

iii. A $\times$ B = {1, 2, 3} $\times$ {3, 4} = {(1, 3), (1, 4), (2, 3),(2, 4), (3, 3), (3, 4)},
and, A $\times$ C = {1, 2, 3} $\times$ {1, 3, 5} = {(1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)}
$\therefore (A \times B) \cap (A \times C)$ = {(1, 3), (2, 3), (3, 3)}

29. Given, $\lim\limits_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$

On multiplying and dividing by $\sqrt{x} + \sqrt{a}$, we get

= $\lim\limits_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$

$= \lim\limits_{x \to a} \frac{(\sin x - \sin a)(\sqrt{x} + \sqrt{a})}{x - a}$

On putting x - a = h, as x $\to$ a, then h $\to$ 0

$\therefore \lim\limits_{h \to 0} \frac{[\sin(a+h) - \sin a][\sqrt{a+h} + \sqrt{a}]}{h}$

$= \lim\limits_{h \to 0} \frac{2\cos\frac{a+h+a}{2}\sin\left(\frac{a+h-a}{2}\right)(\sqrt{a+h} + \sqrt{a})}{h}$

$\left[\because \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$= \lim\limits_{h \to 0} \frac{2\cos\frac{2a+h}{2}\left(\sin\frac{h}{2}\right)(\sqrt{a+h} + \sqrt{a})}{2\left(\frac{h}{2}\right)}$

$= \frac{2\cos\left(\frac{2a+0}{2}\right)(\sqrt{a+0} + \sqrt{a})}{2} \left[\because \lim\limits_{h \to 0} \frac{\sin\ h/2}{h/2} = 1\right]$

$= 2\sqrt{a}\cos a$

30. We have, $(x + iy)^{1/3} = a + ib$

$\Rightarrow$ $x + iy = (a + ib)^3$ [cubing on both sides]

$\Rightarrow$ $x + iy = a^3 + i^3 b^3 + 3 iab (a + ib)$

$\Rightarrow$ $x + iy = a^3 - ib^3 + i 3a^2 b - 3ab^2$

$\Rightarrow$ $x + iy = a^3 - 3ab^2 + i (3a^2 b - b^3)$

On equating real and imaginary parts from both sides, we get

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

$\Rightarrow$ $\frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

Now, $\frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$

$= - 2a^2 - 2b^2 = - 2 (a^2 + b^2)$

Hence proved.

31. We have $\frac{-3x+10}{x+1} > 0$

$\Rightarrow$ $\frac{-3x+10}{x+1} \times (x+1)^2 > 0 \cdot (x+1)^2$ [multiplying both sides by $(x + 1)^2$]

$\Rightarrow$ $(-3x + 10)(x + 1)>0$

Therefore, Product of $(-3x + 10)$ and $(x + 1)$ will be positive.

Case I: if both are positive.

i.e., $(-3x +10) > 0$ and $(x + 1)>0$

$\Rightarrow$ $3x < 10$ and $x > -1$

$\Rightarrow \quad x < \frac{10}{3}$ and $x > -1$

$\Rightarrow \quad -1 < x < \frac{10}{3}$

$\Rightarrow \quad x \in \left(-1, \frac{10}{3}\right)$

Case II: If both are negative.

i.e., $(-3x + 10) < 0$ and $(x + 1) < 0$

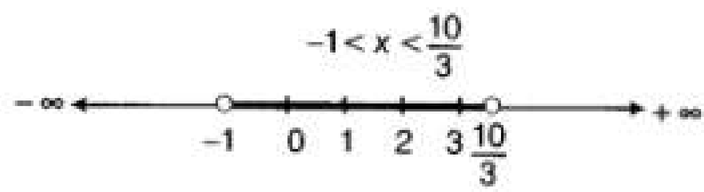
$\Rightarrow$ $-3x < -10$ and $x < -1$

$\Rightarrow$ $3x > 10$ and $x < -1$

$\Rightarrow \quad x > \frac{10}{3}$ and $x < -1$

So, this is impossible. [since, system of inequalities have no common solution]

Thus, the solution is $\left(-1, \frac{10}{3}\right)$.



OR

Here $x+\frac{x}{2}+\frac{x}{3}<11$
$\Rightarrow \frac{6x+3x+2x}{6}<11$
$\Rightarrow \frac{11x}{6}<11$
Multiplying both sides by 6, we have

11x < 66

Dividing both sides by 11, we have

x < 6

Thus the solution set is $(-\infty, 6)$

32. **StepI:** Let $P(n)$ be the given statement. Then,
$P(n): \frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$
Step II: For $n=1$, we have
LHS = $\frac{1}{(2\cdot1-1)(2\cdot1+1)}=\frac{1}{(1)(3)}=\frac{1}{3}$
RHS = $\frac{1}{2\cdot1+1}=\frac{1}{3}$
$\therefore P(1)$ is true.
Step III: For $n=k$, assume that $P(k)$ is true
i.e, $P(k): \frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\ldots+\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$ .....(i)
Step IV: For $n=k+1$, we have to show that
$P(k+1): \frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\ldots+\frac{1}{[(2k-1)(2k+1)\}}+\frac{1}{[2(k+1)-1][2(k+1)+1]}=\frac{k+1}{2(k+1)+1}$
Now, consider **LHS**
$=\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\ldots+\frac{1}{(2k-1)(2k+1)}+\frac{1}{[2(k+1)-1][2(k+1)+1]}$
$=\frac{k}{2k+1}+\frac{1}{[2k+2-1][2k+2+1]}$[From Eq.(i)]
$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$
$=\frac{1}{(2k+1)}\left(k+\frac{1}{2k+3}\right)$
$=\frac{1}{(2k+1)}\left[\frac{k(2k+3)+1}{(2k+3)}\right]=\frac{1}{(2k+1)}\frac{[2k^2+3k+1]}{(2k+3)}$
$=\frac{2k^2+2k+k+1}{(2k+1)(2k+3)}=\frac{2k(k+1)+1(k+1)}{(2k+1)(2k+3)}$

$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3} = \frac{k+1}{2(k+1)+1}$

= **RHS**

So, $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, $P(n)$ is true for all n $\in$ N.

33. Given, $a\ cos\theta\ + b\ sin\theta\ = c$ ...(i)

$\Rightarrow b\ sin\theta = c - a\ cos\theta$

On squaring both sides, we get

$b^2 sin^2\theta\ = c^2 + a^2cos^2\theta\ - 2ac\ cos\theta$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow b^2(1 - cos^2\theta) = c^2 + a^2cos^2\theta - 2ac\ cos\theta$

$[\because cos^2x + sin^2x = 1 \Rightarrow sin^2x = 1 - cos^2x]$

$\Rightarrow (a^2 + b^2)cos^2\theta\ - 2ac\ cos\theta\ + c^2 - b^2 = 0$ ...(ii)

It is a quadratic equation in cosθ

Since, α and β are the roots of Eq. (i).

So, cosα and cosβ are the roots of Eq. (ii).

$\therefore$ cosα $\times$ cosβ = $\frac{c^2-b^2}{a^2+b^2}$ ... (iii)

[$\because$ product of roots in quadratic equation = $\frac{\text{constant term}}{\text{coefficient of square term}}$]

On squaring Eq. (i), we get

$a^2cos^2\theta\ + b^2sin^2\theta\ + 2ab\ sin\theta\ cos\theta\ = c^2$

$\Rightarrow$ $a^2 + b^2 \tan^2\theta + 2ab \tan\theta = c^2 \sec^2\theta$

[dividing both sides by $\cos^2\theta$]

$\Rightarrow$ $a^2 + b^2 \tan^2\theta + 2ab \tan\theta = c^2 (1 + \tan^2\theta)$

[$\because$ $\sec^2 x = 1 + \tan^2 x$]

$\Rightarrow$ $(b^2 - c^2) \tan^2\theta + 2ab \tan\theta + a^2 - c^2 = 0$ ... (iv)

Since, tanα and tanβ are roots of Eq. (iv).

$\therefore \tan\alpha + \tan\beta = \frac{-2ab}{b^2-c^2}$

and $\tan\alpha \times \tan\beta = \frac{a^2-c^2}{b^2-c^2}$

$\Rightarrow \frac{\sin\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta} = \frac{a^2-c^2}{b^2-c^2}$

$\Rightarrow$ sinα $\times$ sinβ = $\frac{a^2-c^2}{b^2-c^2}$ $\times$ cosα $\times$ cosβ

= $\frac{a^2-c^2}{b^2-c^2} \times \frac{c^2-b^2}{a^2+b^2}$ [using Eq. (iii)]

$= \frac{c^2-a^2}{a^2+b^2}$

We have, $\tan\alpha + \tan\beta = \frac{-2ab}{b^2-c^2}$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \frac{-2ab}{b^2-c^2}$

$\Rightarrow \frac{\sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha}{\cos\alpha \cdot \cos\beta} = \frac{-2ab}{b^2-c^2}$

$\Rightarrow \sin(\alpha+\beta) = \frac{-2ab}{b^2-c^2}$ $(\cos\alpha \times \cos\beta)$

$= \frac{-2ab}{b^2-c^2} \times \frac{c^2-b^2}{a^2+b^2}$ [using Eq. (iii)]

$\therefore \sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

OR

LHS = $\cos\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{16\pi}{15}$

$= cos\,\frac{2\pi}{15}\ cos2\left(\frac{2\pi}{15}\right) cos4\left(\frac{2\pi}{15}\right) cos8\left(\frac{2\pi}{15}\right)$

Put $\frac{2\pi}{15} = \alpha$

$\Rightarrow$LHS = $\cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha$

$= \frac{2\sin\alpha[\cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha]}{2\sin\alpha}$ [multiplying numerator and denominator by $2sin\ \alpha$]

$= \frac{(2\sin\alpha \cdot \cos\alpha) \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha}{2\sin\alpha}$

$= \frac{2(\sin 2\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha)}{2(2\sin\alpha)}$ [$\because 2sin\alpha\ cos\alpha = sin2\alpha$ and multiplying numerator and denominator by 2]

$= \frac{(2\sin 2\alpha \cdot \cos 2\alpha) \cdot \cos 4\alpha \cdot \cos 8\alpha}{4\sin\alpha}$

$= \frac{2(\sin 4\alpha \cdot \cos 4\alpha)\cos 8\alpha}{2(4\sin\alpha)}$ [$\because 2sin\alpha\ cos\alpha = sin2\alpha$ and multiplying numerator and denominator by 2]

$= \frac{2(\sin 8\alpha \cdot \cos 8\alpha)}{2(8\sin\alpha)}$

$= \frac{\sin 16\alpha}{16\sin\alpha} = \frac{\sin(15\alpha+\alpha)}{16\sin\alpha}$

Now, $15\alpha = 2\pi$,

$= \frac{\sin(2\pi+\alpha)}{16\sin\alpha} = \frac{\sin\alpha}{16\sin\alpha} = \frac{1}{16}$ = RHS

$\therefore$ **LHS = RHS**

Hence proved.

34. Total cost of the tractor = Rs. 12000, Cash paid = Rs. 6000

Balance to be paid = 12000 – 6000 = Rs. 6000

Annual installment = Rs. 500

$\therefore$ Number of installment $= \frac{6000}{500} = 12$

Interest of 1st installment = $\frac{6000 \times 12 \times 1}{100}$ = Rs. 720

Amount of 1st installment = 500 + 720 = Rs. 1220

Interest of 2nd installment = $\frac{5500 \times 12 \times 1}{100}$ = Rs. 660

Amount of 2nd installment = 500 + 660 = Rs. 1160

Interest of 3rd installment = $\frac{5000 \times 12 \times 1}{100}$ = Rs. 600

Amount of 3rd installment = 500 + 600 = Rs. 1100

$\therefore$ Sequence of installments is 1220, 1160, 1100, ... which is in A.P

Here, a = 1220, d = 1160 - 1220 = -60 and n = 12

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

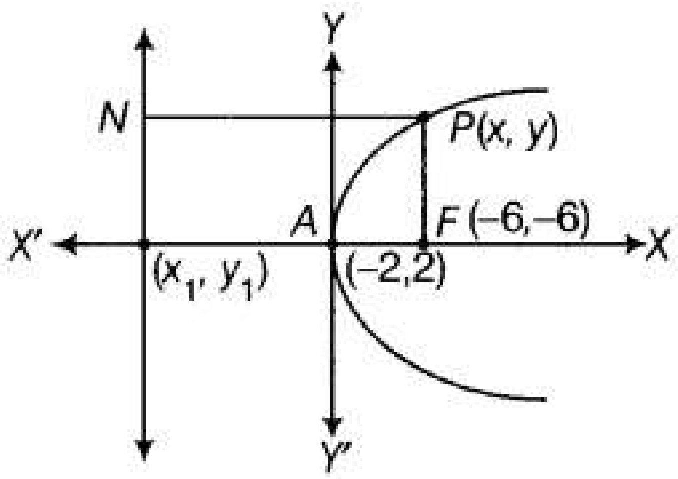
= $\frac{12}{2}[2 \times 1220 + (12-1) \times (-60)]$

= 6 [2440 - 660] = Rs. 10680

Therefore, the total cost of tractor is (10680 + 6000) = Rs. 16680.

35. Given, focus is at (- 6 , - 6) and vertex is at (- 2, 2).
Now we have to find equation of the parabola with focus (-6 ,-6) and vertex (-2 , 2)
Let (x_1, y_1) be the coordinate of the point of intersection of axis and directrix.

Then, (-2, 2) is the mid-point of the line segment joining (- 6, - 6) and (x_1, y_1).



$\therefore -2 = \frac{x_1 - 6}{2} \Rightarrow x_1 = 2$

and $2 = \frac{y_1 - 6}{2} \Rightarrow y_1 = 10$

Thus, the point (2, 10) is the point of intersection of axis and directrix.

Now, the slope of line segment joining vertex and focus is given by

$m_1 = \frac{-6-2}{-6+2} = 2$

=> $m_1 = 2$

also Slope of directrix,

that is , $m_2 = \frac{-1}{2}$

Now using the values in the general we get the equation of directrix which is given by

$y - 10 = \frac{-1}{2}(x - 2)$

$\Rightarrow 2y - 20 = -x + 2$

=> $x + 2y = 2 + 20$

$\Rightarrow x + 2y = 22$

Let P (x, y) be any point on parabola and PN be the length of perpendicular from P on directrix and FP be the distance between focus F and point P.

So, FP = PN $\Rightarrow (FP)^2 = (PN)^2$

$\Rightarrow (x + 6)^2 + (y + 6)^2 = \left(\frac{x+2y-22}{\sqrt{1+4}}\right)^2$

$\Rightarrow x^2 + 36 + 12x + y^2 + 36 + 12y = \frac{x^2+4y^2+484+4xy-44x-88y}{5}$

$\Rightarrow 5x^2 + 180 + 60x + 5y^2 + 60y + 180 = x^2 + 4y^2 + 484 + 4xy - 44x - 88y$

$\Rightarrow 4x^2 + y^2 - 4xy + 104x + 148y - 124 = 0$

which is the required equation of the parabola.

OR

Let the equation of circle whose centre (- g, - f) be

$x^2 + y^2 + 2\ gx + 2\ fy + c = 0$ ...(i)

Since, is passes through points (3, - 2) and (- 2, 0)

$\therefore (3)^2 + (-2)^2 + 2g\ (3) + 2f\ (-2) + c = 0$

and $(-2)^2 + (0)^2 + 2g\ (-2) + 2f\ (0) + c = 0$

$\Rightarrow 9 + 4 + 6g - 4f + c = 0$

and $4 + 0 - 4g + 0 + c = 0$

$\Rightarrow 6g - 4f + c = -13$

and $c = 4g - 4$ ...(ii)

$\therefore 6g - 4f + (4g - 4) = -13$

$\Rightarrow 10g - 4f = -9$ ...(iii)

Also, centre (- g, - f) lies on the line $2x - y = 3$

$\therefore -2g + f = 3$ ...(iv)

On solving Eqs. (iii) and (iv), we get

$g = \frac{3}{2}$ and $f = 6$

On putting the values of g and f in Eq. (ii), we get

$c = 4\left(\frac{3}{2}\right) - 4 = 6 - 4 = 2$

On putting the values of g, f and c in Eq. (i), we get

$x^2 + y^2 + 2\left(\frac{3}{2}\right) x + 2(6)\ x + 2 = 0$

$\Rightarrow x^2 + y^2 + 3x + 12x + 2 = 0$

which is the required equation of the path

The message which he wants to give to the public is 'Keep your place clean'.

36. We have, $n = 100$, $\overline{x} = 40$ and $\sigma = 5.1$

$\therefore \overline{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \Sigma x_i = n\overline{x} = 100 \times 40 = 4000$

$\therefore$ Incorrect $\Sigma x_i = 4000$

and,

$\sigma = 5.1$

$\Rightarrow \sigma^2 = 26.01$

$\Rightarrow \frac{1}{n}\Sigma {x_i}^2 - (\text{mean})^2 = 26.01$

$\Rightarrow \frac{1}{100}\Sigma x_i^2$ - 1600 = 26.01

$\Rightarrow \Sigma x_i^2$ = 1626.01 $\times$ 100

$\therefore$ Incorrect Σx_i^2 = 162601

To correct the $\sum x_i$, we need to subtract the incorrect observation 50 and add correct observation is 40.

We have, incorrect Σx_i = 4000

$\therefore$ Correct Σx_i = 4000 - 50 + 40 = 3990

and,

Similarly, to obtain correct $\sum x_i^2$ we need to subtract 50^2 and add 40^2 to incorrect one.

Incorrect Σx_i^2 = 162601

$\therefore$ Correct Σx_i^2 = 162601 - 50^2 + 40^2 = 161701

Now, Correct mean = $\frac{3990}{100}$ = 39.90

Correct variance = $\frac{1}{100}$ (Correct Σx_i^2) - (Correct mean)^2

$\Rightarrow$ Correct variance = $\frac{161701}{100} - \left(\frac{3990}{100}\right)^2$

$\Rightarrow$ Correct variance = $\frac{161701 \times 100 - (3990)^2}{(100)^2}$

$\Rightarrow$ Correct variance = $\frac{16170100 - 15920100}{10000} = 25$

$\therefore$ Correct standard deviation = $\sqrt{25}$ = 5