CBSE Class 11 Mathematics

Sample Papers 05 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. For all x $\in$ (0, 1)

   a. $\log_e x > x$

   b. sin x $>$ x

   c. $\log_e(1+x) < x$

   d. $e^x < 1+x$

2. 5 persons board a lift on the ground floor of an 8 storey building. In how many ways can they leave the lift?

   a. ${}^7P_5$

b. none of these

c. 5^7

d. 7^5

3. The number 111111.............1 (91 times) is

a. not an odd number

b. an even number

c. not a prime

d. Has a factor as 6

4. A convex polygon of n sides has twice as many diagonals as the number of sides. The value of n is

a. 8
b. 7
c. 6
d. 5

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5. Let x be any real, then [x + y] = [x] + [y] holds for

a. y $\in$ R

b. y $\in$ R, y $\in$ Q.

c. y $\in$ Q

d. y $\in$ I

6. A student was asked to prove a statement P (n) by method of induction . He proved P (k + 1) is true whenever P (k) Is true for all k ≥ 5 , $k \in N$ and P (5) is true . On the basis of this he could conclude that P (n) is true

a. none of these

b. for all n $>$ 5

c. for all n < 5

d. for all n ≥ 5

7. A box contains 10 eggs out of which 4 are rotten. Two eggs are taken out together. If one of them is found to be good, the probability that other is also is

a. $\frac{2}{3}$
b. $\frac{5}{13}$
c. $\frac{8}{15}$
d. $\frac{1}{3}$

8. Distance between two parallel planes 2x+y+2z=8 and 4x+2y+4z+5=0 is

a. $\frac{5}{2}$

b. $\frac{3}{2}$

c. none of these

d. $\frac{7}{2}$

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9. One ticket is selected at random from 100 tickets numbered 00, 01, 02..., 99. Suppose S and T are the sum and product of the digits of the number on the ticket, then the probability of getting S = 7 and T = 0 is

a. $\frac{1}{50}$

b. $\frac{1}{4}$

c. $\frac{2}{19}$

d. $\frac{19}{100}$

10. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

a. $(-1)^{n-1}(1+n)$

b. $(-1)^n(1-n)$

c. $(-1)^n(1+n)$

d. $(-1)^{n-1}(n)$

11. Fill in the blanks:

The function f defined by f(x) = mx + c, x $\in$ R is called ________function, where m and c are constants.

12. Fill in the blanks:

The general term in the expansion of $(x^2 - y^2)^6$ is equal to ________.

13. Fill in the blanks:

The number of words which can be formed out of letters of the word ARTICLE, so that the vowels occupy the even place is ________.

14. Fill in the blanks:

The equation of yz-plane is ________.

OR

Fill in the blanks:

The distance of the point P(2, 3, 5) from the xy-plane is ________.

15. Fill in the blanks:

The value of the limit $\lim\limits_{x\to 0} \frac{ax+b}{cx+1}$ is ________.

OR

Fill in the blanks:

The derivative of $2x - \frac{3}{4}$ is ________.

16. Write the set in roster form: F = The set of all letters in the word BETTER.

17. Compute $\frac{8!}{6!\times 2!}$

18. Express the complex number sin 50^0 + i cos 50^0 in the polar form.

OR

Find the modulus of the conjugate of the following complex number - 3 i.

19. A and B are two sets having 3 elements in common. If n(A) = 5, n(B) = 4, find n(AxB) and $n[(A \times B) \cap (B \times A)]$

20. How many word with or without meaning can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

21. Let B be a subset of a set A and let P(A : B) = $\{X \in P(A) : X \supset B\}$

   i. Show that: P (A : ϕ) = P (A)
   ii. If A = {a, b, c, d} and B = {a, b} , list all the members of the set P(A : B)

OR

In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all the people speak at least one of the two languages. How many people speak only English and not Hindi? How many people speak English?

22. In a simultaneous throw of a pair of dice, find the probability of getting a doublet of prime numbers.

23. If p is a real number and the middle term in the expansion of $\left(\frac{p}{2}+2\right)^8$ is 1120, then find the value of p.

24. Find the equation of the line upon which the length of perpendicular p from origin and the angle a made by this perpendicular with the positive direction of X-axis are p = 4, α = 120^0.

OR

The perpendicular distance of a line from the origin is 7 cm and its slope is -1. Find the equation of line.

25. Write the negation of the following statements.

i. If I become a doctor, then I will open a hospital.
ii. If 2 + 3 = 5, then 5 is an odd number.

26. Prove $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

27. In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali? How many can speak both Hindi and Bengali?

28. Let f = {(1, 1), (2, 3), (0, - 1), - 1, - 3)} be a function from Z to Z defined by f (x) = ax + b for some integers a, b. Determine a,b.

OR

Find the domain and range of the following function:

i. $\left\{\left(x, \frac{x^2-1}{x-1}\right) : x \in R, x \neq 1\right\}$
ii. $\{(x, -|x|) : x \in R\}$

29. Evaluate $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 5\theta}$.

30. Solve $x^2 + 3x + 9 = 0$

31. Solve the inequalities graphically in two dimensional plane: $3y - 5x < 30$

OR

Solve $\frac{3(x-2)}{5} \leqslant \frac{5(2-x)}{3}$

32. Prove the following by using the principle of mathematical induction for all $n \in N$:
$a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n-1)}{r-1}$.

33. Prove that $\cos 2x \cdot \cos \frac{x}{2} - \cos 3x \cdot \cos \frac{9x}{2} = \sin 5x \cdot \sin \frac{5x}{2}$

OR

Find all the angles between 0^o and 360^o, which satisfy the equation $\sin^2 \theta = \frac{3}{4}$.

Suppose a student substituted θ = 60^{o} and checked that it is true. He concluded statement is always true. What type of quality he should acquire to reach at correct result?

34. If a, b, c, d are in G.P., prove that $(a^n + b^n)$, $(b^n + c^n)$, $(c^n + d^n)$ are in G.P.

35. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

OR

Find the equation of the hyperbola whose vertices are (- 8, -1) and (16, - 1) and focus is (17, - 1).

36. The runs of two players for 10 innings each are as follows:

A	58	59	60	54	65	66	52	75	69	52
B	94	26	92	65	96	78	14	34	98	13

Who may be regarded as the more consistent player?

Solution

Section A

1. (c) $\log_e(1+x) < x$ **Explanation:**

   Let f(x) = x - log(1+x) in [0,x]; x $\in$ $(0,1)$ clearly, f is contineous on [0,x] & differentiable on (0,x).

   Therefore by langrange's mean value theorem, there exits c $\in$ $(0, \mathrm{x})$such that,

   $\mathrm{f'(c)} = \frac{\mathrm{f(x)} - \mathrm{f(0)}}{\mathrm{x-0}}$

   $\Rightarrow 1 - \frac{1}{\mathrm{c}} = \frac{\mathrm{x-log(1+x)} - 0}{\mathrm{x}} \{\because \mathrm{f'(x)} = 1 - \frac{1}{1+\mathrm{x}}\}$

   $\Rightarrow \frac{\mathrm{x-log(1+x)}}{\mathrm{x}} = 1 - \frac{1}{1+\mathrm{c}} \Rightarrow \frac{\mathrm{x-log(1+x)}}{\mathrm{x}} > 0 \left[\because \mathrm{c} \in (0,1) \Rightarrow 1 - \frac{1}{1+\mathrm{c}} > 0\right]$

   $\Rightarrow \mathrm{x} - \log(1+\mathrm{x}) > 0 \; [\because \mathrm{x} \in (0,1)]$

   $\Rightarrow \log(1+\mathrm{x}) < \mathrm{x}$

2. (d) 7^5 **Explanation:**

   Since they are boarding from ground floor and we are considering the number of ways they leave the lift ,we can consider there are 7 floor sas we exclude the ground floor)

   As each of the 5 persons can leave the lift in 7 ways, required number of ways= 7^5

3. (c) not a prime
   Explanation:

   111...111(91times) can be expressed as:-
   $\frac{1}{9}\left(10^{91}-1\right) \Rightarrow \frac{1}{9}\left(10^7-1\right) \times x$
   $\Rightarrow 111111 \times x$
   $\Rightarrow$ where $x = \left(10^7\right)^{12} + \left(10^7\right)^{11} + \cdots + 1$

4. (b) 7 **Explanation:** Consider a convex polygon which has n sides

   We have an nsided polygon has n vertices.If you join ever ydistinct pair of vertices you will get lines.

   These lines account for the n sides of the polygon as well as for the diagonals

   Then we have the number of diagonals = ${}^nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

   But given number of diagonals=2.number of sides
   $\Rightarrow 2n = \frac{n(n-3)}{2}$
   $\Rightarrow 4n = n(n-3)$
   $\Rightarrow n - 3 = 4$
   $\Rightarrow$ n = 7

5. (d) y $\in$ I
   Explanation:

   $[x] = x \ \forall \ x \in I$

   so,

   $[x+y] = [x] + [y]$ only if x and y $\in I$

6. (d) for all n ≥ 5
   Explanation:

   Yes because if it is true for n = k then it is true for all n ,which is the basic concept of mathematical induction.

7. (b) $\frac{5}{13}$ **Explanation:**

   Let $E_1 =$one of the two eggs is good and

   $E_2 =$both the selected eggs are good.

   Here,E_2 is subset of

   $\therefore p(\frac{E_2}{E_1}) = \frac{p(E_1 \cap E_2)}{p(E_1)}$

$= \frac{p(E_2)}{p(E_1)} = \frac{{}^6C_2}{{}^6C_2 + {}^6C_1.4C_3} = \frac{15}{15+24} = \frac{5}{13}$

8. (d) $\frac{7}{2}$ **Explanation:**

   Given planes are

   2x+2y+2z-8=0

   and 2x+y+2z+$\frac{5}{2}$ =0

   Distance between planes =$\frac{|c_1-c_2|}{\sqrt{a^2+b^2+c^2}}$.

   $= \left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right|$

   $= \frac{\frac{21}{2}}{3} = \frac{7}{2}$

9. (a) $\frac{1}{50}$ **Explanation:**

   Given 100 tickets numbered 00, 01, 02..., 99. , S and T are the sum and product of the digits of the number on the ticket. here the number of elements in the sample space = 100

   Now,S=7 i.e. sum of the digits is 7,S={07,70,16,61,25,52,34,43}, and T=0 i.e. product of the digits = 0 ,T={ 00,01,02,03,04,05,06,07,08,09,10,20,30,40,50,60,70,80,90}

   T=0 and S=7 $\Rightarrow$ T $\cap$ S ={07,70} $\Rightarrow$ n(T $\cap$ S) = 2

   Hence P(T=0 and S=7)= P(T $\cap$ S) = 2/100 = 1/50

10. (b) $(-1)^n(1-n)$ **Explanation:** (1+x) $(1-x)^n$ = (1+x) $(1-{}^nC_1 x+{}^nC_2 x^2-{}^nC_3 x^3+{}^nC_4 x^4 +$ ...... $+ (-1)^{n-1}.{}^nC_{n-1} x^{n-1} + (-1)^n.{}^nC_n x^n)$

    Coefficients of

    $x^n = (-1)^n \cdot {}^n C_n \quad + (-1)^{n-1} \cdot {}^n C_{n-1} \quad = (-1)^n \cdot 1 + (-1)^{n-1} \cdot n =$ $(-1)^n$ (1-n)

11. linear

12. ${}^6C_r\, x^{12-2r}(-y)^r$

13. 144

14. x = 0

OR

5

15. b

OR

2

16. F = The set of all letters in the word BETTER
$\therefore F$ = {B, E, T, R}

17. $\frac{8!}{6!\times 2!} = \frac{8\times 7\times 6!}{6!(2\times 1)} = \frac{8\times 7}{2} = 28$

18. Suppose, $z = sin50^o + i\ cos50^o$
$= sin(90^o\ - 40^o) + i\ cos(90^o\ - 40^o)$
= cos 40^o + i sin 40^o

OR

Suppose, $z = -3i = 0 - 3i$
Then, $\overline{z} = 0 + 3i$
$\therefore |\overline{z}| = \sqrt{(0)^2 + (3)^2} = \sqrt{9}$ = 3

19. As given,
n(A) = 5 and n(B) = 4

We know that if A and B are two finite sets, then n (A $\times$ B) = n(A) $\times$ n(B)
$\therefore n(A \times B) = 5 \times 4 = 20$

From set theory we know that,

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

so here, $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

As given, sets A and B have 3 elements in common

So,

$n[(A \times B) \cap (B \times A)] = 3 \times 3 = 9$

20. There are 8 letters in the word EQUATION including 5 vowels and 3 consonants. Now 5 vowels can be arranged in 5! Ways and 3 consonants can be arranged in 3! Ways. Also the two groups of vowels and consonants can be arranged in 2! Ways.

$\therefore$ Total number of permutation $= 5! \times 3! \times 2!$

$= 120 \times 6 \times 2 = 1440$

21. i. We have, P (A : B) = $\{X \in P(A) : X \supset B\}$

 $= \{X \in P(A) : B \subset X\}$

 = Set of all those subsets of A which contain B

 $\therefore$ P(A : ϕ) = Set of all subsets of set A = P (A).

 ii. If A = {a, b, c, d} and B = {a, b}.

 Then, P (A : B)

 = Set of all those subsets of set A which contain B

 = {a, b}, {a, b, c}, {a, b, d}, {a, b, c, d}

OR

Let H denote the set of people speaking Hindi and E the set of people speaking English.

Then, it is given that:

n(H $\cup$ E) = 50, n (H) = 35 , n (H $\cap$ E) = 25.

Now, n(E - H) = n (H $\cup$ E) - n(H)

= 50 - 35 = 15

Thus, the number of people speaking English but not Hindi is 15.

Now, $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\Rightarrow 50 = 35 + n(E) - 25 \Rightarrow n(E) = 40$

Hence, the number of people who speak English is 40.

22. Suppose E be the event that a doublet of prime number appear.

$\therefore$ E = {(2,2), (3,3), (5,5)}

$n(E) = 3$

$\therefore P(E) = \frac{3}{36} = \frac{1}{12}$

23. Given expression is $\left(\frac{p}{2} + 2\right)^8$

Here, n = 8 (even)

So, middle term = $\left(\frac{8}{2} + 1\right)$ th term = 5th term

$\therefore$ T_5 = ${}^8C_4$ $(p/2)^{8-4}$ $(2)^4$

= $\frac{8\times7\times6\times5}{4\times3\times2\times1} \times \frac{p^4}{2^4} \times 2^4 = 70p^4$

But given, middle term = 1120

$\therefore$ 70 p^4 = 1120 $\Rightarrow$ p^4 = 16 $\Rightarrow$ p = $\pm$ 2

24. Equation of the line upon which the length of perpendicular from origin is p and this perpendicular makes an angle α with the positive direction of X-axis is,

x cos α + y sin α = p

Here, p = 4 and α =120^o, therefore, required equation of the line is

x cos 120^o + y sin 120^o = 4

$\Rightarrow$ x cos (90^o + 30^o) + y sin (90^o + 30^o) =4

$\Rightarrow$ -x sin 30^o + y cos 30^o = 4

$\Rightarrow -x \times \frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 4$

$\Rightarrow -x + 3\sqrt{y} = 8$

OR

Here, slope m = tan α = -1 $\Rightarrow$ α = $\tan^{-1}$ (-1)

$\therefore$ α = 135^o

and p = 7

Thus, the required line is x cos135^o + y sin 135^o = 7

$\Rightarrow$ x cos (180^o - 45^o) + y sin (180^o - 45^o) = 7

$\Rightarrow$ -x$\cdot$ cos 45^o + y sin 45^o = 7

$\Rightarrow \quad -x \cdot \frac{1}{\sqrt{2}} + y \cdot \frac{1}{\sqrt{2}} = 7 \Rightarrow \quad x - y + 7\sqrt{2} = 0$

25. i. Let p and q be the component statements.

p : I become a doctor.

q : I will open a hospital.

The given proposition is $p \Rightarrow q$

Now, $\sim(p \Rightarrow q) \equiv p \wedge (\sim q)$

Therefore, the negation of the given proposition is 'I will become a doctor and I will not open a hospital.'

ii. Similarly, we get, p:2 + 3 = 5

and q:5 is an odd number.

$\therefore \sim(p \Rightarrow q) \equiv p \wedge \sim q$

So, 2 + 3 = 5 and 5 is an even number.

26. We have L.H.S. $= \sin^2 6x - \sin^2 4x$

$= \sin(6x + 4x) \cdot \sin(6x - 4x)$

$[\because \sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)]$

$= \sin 10x \cdot \sin 2x = R.H.S.$

27. Let,

n(P) denote the total number of people,

n(H) denote the number of people who speak Hindi

and n(B) denote the number of people who speak Bengali

Then,

n(P) = 1000, n(H) = 750, n(B) = 400

We have,

$P = (H \cup B)$

$\therefore \quad n(P) = n(H \cup B)$

= n(H) + n(B) - n(H⋂B)

$\Rightarrow$ 1000 = 750 + 400 - n(H⋂B)

$\Rightarrow$1000 = 1150 - n(H⋂B)

$\Rightarrow n(H \cap B)$ = 150

Hence, 150 people can speak both Hindi and Bengali now H = (H - B)∪$(H \cap B)$, the union being disjoint

$\therefore \quad n(H) = n(H - B) + n(H \cap B)$

$\Rightarrow \quad 750 = n(H - B) + 150$

$\Rightarrow \quad n(H-B)=750-150$

= 600

Hence, 600 people can speak Hindi Only.

On similar lines, we have

$B=(B-H)\cup(H\cap B)$

$\Rightarrow \quad n(B)=n(B-H)+n(H\cap B)$

$\Rightarrow \quad 400=n(B-H)+150$

$\Rightarrow \quad n(B-H)=400-150$

= 250

Hence, 250 people can speak Bengali only.

28. Here f(x) = ax + b

f = {(1, 1), (2, 3), (0, -1), (-1, -3)}

$\Rightarrow$ f (1) = 1, f (2) = 3, f(0) = -1, f(-1) = -3

Now f(1) = 1 $\Rightarrow a\times 1+b=1 \Rightarrow$ a+ b = 1 . . . (i)

$f(2)=3 \Rightarrow a\times 2+b=3 \Rightarrow 2a+b=3 \ldots$ (ii)

Subtracting (i) from (ii) we get

2a+b-(a+b)=3-1 $\Rightarrow$ a = 2

Putting a = 2 in (i)

2 + b = 1 $\Rightarrow$ b = -1

OR

i. Suppose $f=\left\{\left(x,\frac{x^2-1}{x-1}\right): x\in R, x\neq 1\right\}$

   Clearly, f is not defined, when $x=1$

   So, f is defined for all real values of x, except x = 1.

   $\therefore$ Domain $(f)=R-\{1\}$

   Let $y=\frac{x^2-1}{x-1}$

   $=\frac{(x-1)(x+1)}{(x-1)}=x+1$

   $\Rightarrow x=y-1$

   Clearly, x is not defined, when $y=2$ as $x\neq 1$.

   Range $(f)=R-\{2\}$

ii. Let $f=\{(x,-|x|): x\in R\}$

Clearly, $f(x)=-|x|$, which is ≤ 0 for all $x \in R$

$\therefore$ Domain $(f)=R$

and range $(f)=$ {x: x $\in$ R and x $\leq$ 0}

29. We have, $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 5\theta} = \lim\limits_{\theta \to 0} \frac{2\sin^2(2\theta)}{2\sin^2\left(\frac{5\theta}{2}\right)}$ $\left[\because 1-\cos 2\theta = 2\sin^2\theta\right]$

$$= \lim_{\theta \to 0} \frac{\frac{\sin^2(2\theta)}{4\theta^2} \times 4\theta^2}{\frac{\sin^2 \frac{5\theta}{2}}{\frac{25\theta^2}{4}} \times \frac{25\theta^2}{4}} = \frac{16}{25} \times \frac{\lim\limits_{2\theta \to 0}\left(\frac{\sin 2\theta}{2\theta}\right)^2}{\lim\limits_{\frac{5\theta}{2} \to 0}\left(\frac{\sin 5\theta/2}{5\theta/2}\right)^2}$$

[as $\theta \to 0$, then $2\theta \to 0$ and $\frac{5\theta}{2} \to 0$]

$$= \frac{16}{25} \times \frac{(1)^2}{(1)^2} \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{16}{25}$$

30. Here x^2 + 3x + 9 = 0

Comparing the given quadratic equation with ax^2 + bx + c = 0 we have

a = 1 , b= 3 and c = 9

$$\therefore x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 9}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2}$$

Thus $x = \frac{-3+3\sqrt{3}i}{2}$ and $x = \frac{-3-3\sqrt{3}i}{2}$

31. The given inequality is 3y - 5x < 30.

Draw the graph of the line 3y - 5x = 30

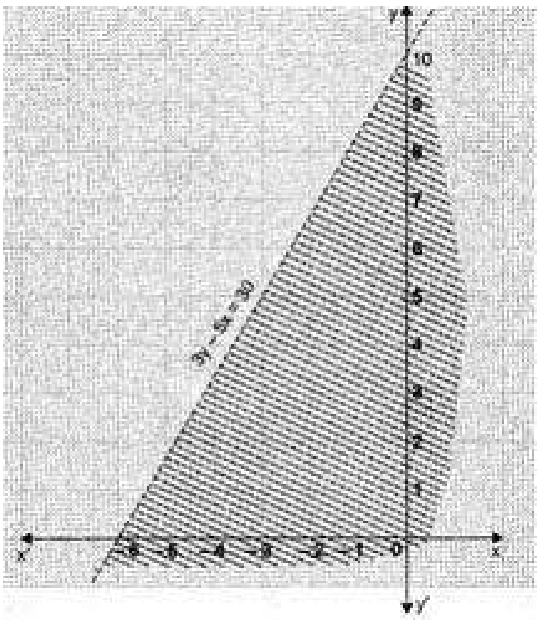


Table of values satisfying the equation 3y - 5x = 30

X	-6	0
Y	0	10

Putting (0, 0) in the given in equation, we have $-3 \times 0 - 5 \times 0 < 30 \Rightarrow 0 < 30$, which is true.

$\therefore$ Half plane of 3y - 5x < 0 is towards origin.

OR

Here $\frac{3(x-2)}{5} \leqslant \frac{5(2-x)}{3}$

$\Rightarrow \frac{3x-6}{5} \leqslant \frac{10-5x}{3}$

$\Rightarrow \frac{3x}{5} - \frac{6}{5} \leqslant \frac{10}{3} - \frac{5x}{3}$

$\Rightarrow \frac{3x}{5} + \frac{5x}{3} \leqslant \frac{10}{3} + \frac{6}{5}$

$\Rightarrow \frac{9x+25x}{15} \leqslant \frac{50+18}{15}$

$\Rightarrow \frac{34x}{15} \leqslant \frac{68}{15}$

Multiplying both sides by 15, we have

$34x \leqslant 68$

Dividing both sides by 34, we have

$x \leqslant 2$

Thus the solution set is $(-\infty, 2]$

32. Let P(n) = a + ar + ar^2 + ... + $ar^{n-1}$$= \frac{a(r^n-1)}{r-1}$.

For n = 1

$P(1) = ar^{1-1} = \frac{a(r^1-1)}{r-1} \Rightarrow a = a$

$\therefore$ P(1) is true

Let P(n) be true for n = k

$\therefore P(k) = a + ar + ar^2 + \ldots + ar^{k-1} = \frac{a(r^k-1)}{r-1}$ .... (i)

For n = k + 1

R.H.S. $= \frac{a(r^{k+1}-1)}{r-1}$

L.H.S. $= \frac{a(r^k-1)}{r-1} + ar^k$ [Using (i)]

$= \frac{ar^k}{r-1} - \frac{a}{r-1} + ar^k$

$= ar^k \cdot \left(\frac{1}{r-1} + 1\right) - \frac{a}{r-1} = \frac{ar^{k+1}}{r-1} - \frac{a}{r-1} = \frac{ar^{k+1}-a}{r-1}$
$= \frac{a(r^{k+1}-1)}{r-1}$
$\therefore$ P(k + 1) is true

Thus P(k) is true $\Rightarrow$ P(k + 1) is true

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.

33. **LHS** = $cos2x \times \ cos\ \frac{x}{2}\ - cos3x\ \times\ cos\ \frac{9x}{2}$
= $\frac{1}{2}$ [2 cos 2x $\times$ cos $\frac{x}{2}$ - 2 cos $\frac{9x}{2}$ $\times$ cos 3x] [multiplying numerator and denominator by 2]
= $\frac{1}{2}$ [cos $\left(2x + \frac{x}{2}\right)$ + cos $\left(2x - \frac{x}{2}\right)$ - cos $\left(\frac{9x}{2} + 3x\right)$ - cos $\left(\frac{9x}{2} - 3x\right)$] [$\because$
$2cos\ x\ \times\ cos\ y = cos(x+y) + cos(x-y)$]
= $\frac{1}{2}$ [cos $\frac{5x}{2}$ + cos $\frac{3x}{2}$ - cos $\frac{15x}{2}$ - cos $\frac{3x}{2}$]
= $\frac{1}{2}$ [cos $\frac{5x}{2}$ - cos $\frac{15x}{2}$]
= $\frac{1}{2}$ [- 2 sin $\left(\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right)$ sin$\left(\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right)$] [$\because cos\ x - cos\ y = -2sin$
$\left(\frac{x+y}{2}\right)$ $\cdot$ sin$\left(\frac{x-y}{2}\right)$]
= - sin 5x sin $\left(\frac{-5x}{2}\right) = sin5x\ \times\ \ sin\ \frac{5x}{2}$ [$\because$ $sin(-\theta) = -sin\,\theta$]

= RHS

$\therefore$ LHS = RHS

Hence proved.

OR

Given,
$sin^2\ \theta = \frac{3}{4}$
$\Rightarrow sin\ \theta = \pm\frac{\sqrt{3}}{2}$
$\therefore$ Either $sin\ \theta = \frac{\sqrt{3}}{2}$ or sin $\theta = \frac{-\sqrt{3}}{2}$
Case 1: $sin\ \theta = \frac{\sqrt{3}}{2}$ implies that θ lies in Ist or IInd quadrant.

We know,
$sin60^o = \frac{\sqrt{3}}{2}$ and $sin(180^o - 60^o)$ = $\frac{\sqrt{3}}{2}$
$\therefore \theta = 60^o, 120^o$
Case 2: sin $\theta = \frac{-\sqrt{3}}{2}$ implies that θ lies in III or IV quadrant.

We have, $sin(180^o + 60^o) = -sin60^o = \frac{-\sqrt{3}}{2}$

$\Rightarrow \theta = 180^o + 60^o = 240^o$

$sin(360^o - 60^o) = -sin60^o = \frac{-\sqrt{3}}{2}$

$\therefore \theta = 360^o - 60^o = 300^o$

Hence, the required angles are $60^o, 120^o, 240^o\ and\ 300^o$.

He should acquire the quality of full satisfaction to reach at correct result.

34. Given: a, b, c, d are in G.P.

To prove: $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

$\Rightarrow \frac{b^n+c^n}{a^n+b^n} = \frac{c^n+d^n}{b^n+c^n}$

Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$\therefore \frac{b}{a} = k$

$\Rightarrow$ b = ak

And $\frac{c}{b} = k$

$\Rightarrow$ c = bk = (ak)k = ak^2

Also $\frac{d}{c} = k$

$\Rightarrow$ d = ck = $(ak^2)k = ak^3$

Now, $\frac{b^n+c^n}{a^n+b^n} = \frac{c^n+d^n}{b^n+c^n}$

$\Rightarrow \frac{(ak)^n+(ak^2)^n}{a^n+(ak)^n} = \frac{(ak^2)^n+(ak^3)^n}{(ak)^n+(ak^2)^n}$

$\Rightarrow \frac{a^nk^n+a^nk^{2n}}{a^n+a^nk^n} = \frac{a^nk^{2n}+a^nk^{3n}}{a^nk^n+a^nk^{2n}}$

$\Rightarrow \frac{a^nk^n(1+k^n)}{a^n(1+k^n)} = \frac{a^nk^{2n}(1+k^n)}{a^nk^n(1+k^n)}$

$\Rightarrow k^n = k^n$

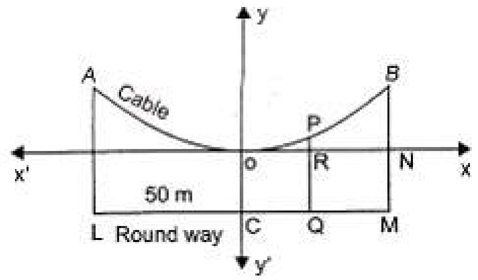
Therefore, $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

35. Let AOB be the cable of uniformly loaded suspension bridge. Let AL and BM be the longest wires of length 30 m each. Let OC be the shortest wire of length 6 m and LM be the roadway.

Now AL = BM = 30 m, OC = 6 m and LM = 100 m.

$\therefore$ LC = CM = $\frac{1}{2}$ LM= 50 m

Let O be the vertex and axis of the parabola be y-axis. So the equation of parabola in standard form is $x^2 = 4ay$



Coordinates of point B are (50, 24)

Since point B lies on the parabola $x^2 = 4ay$

$\therefore (50)^2 = 4a \times 24 \Rightarrow a = \frac{2500}{4\times24} = \frac{625}{24}$

So equation of parabola is $x^2 = \frac{4\times625}{24}y \Rightarrow x^2 = \frac{625}{6}y$

Let length of the supporting wire PW at a distance of 18 m be h.

$\therefore$ OR = 18 m and PR = PQ – QP = PQ - OC = h - 6

Coordinates of point P are (18, h - 6)

Since the point P lies on parabola $x^2 = \frac{625}{6}y$

$\therefore (18)^2 = \frac{625}{6}(h - 6) \Rightarrow 324 \times 6 = 625h - 3750$

$\Rightarrow 625h = 1944 + 3750 \Rightarrow h = \frac{5694}{625}$ = 9.11 m approx.

OR

The centre of the hyperbola is the mid-point of the line joining the two vertices.

So, the coordinates of the centre are $\left(\frac{16-8}{2}, \frac{-1-1}{2}\right)$ i.e., (4, -1).

Let 2a and 2b be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$\frac{(x-4)^2}{a^2} - \frac{(y+1)^2}{b^2} = 1$ ....(i)

Now, The distance between two vertices = 2a

$\therefore \sqrt{(16+8)^2 + (-1+1)^2} = 2a$ [$\because$ vertices = (-8, -1) and (16, -1)]

$\Rightarrow$ 24 = 2a

$\Rightarrow$ a = 12

$\Rightarrow$ a^2 = 144

and, the distance between the focus and vertex is = ae - a

$\therefore \sqrt{(17-16)^2+(-1+1)^2}$ = ae - a

$\Rightarrow \sqrt{1^2}$ = ae - a

$\Rightarrow$ ae - a = 1

$\Rightarrow$ 12 $\times$ e - 12 = 1

$\Rightarrow$ 12e = 1 + 12

$\Rightarrow$ e = $\frac{13}{12}$

$\Rightarrow$ e^2 = $\frac{169}{144}$

Now,

$b^2 = a^2 (e^2 - 1)$

= $(12)^2 \left(\frac{169}{144}-1\right)$

= 144 $\times \left(\frac{169-144}{144}\right)$

= 144 $\times \frac{25}{144}$

= 25

Putting a^2 = 144 and b^2 = 25 in equation (1), we get

$\frac{(x-4)^2}{144}-\frac{(y+1)^2}{25}$ = 1

$\Rightarrow \frac{25(x-4)^2-144(y+1)^2}{3600}$ = 1

$\Rightarrow 25\left[x^2+16-8x\right]-144\left[y^2+1+2y\right]=3600$

$\Rightarrow$ $25x^2 + 400 - 200x - 144y^2 - 144 - 288y = 3600$

$\Rightarrow$ $25x^2 - 144y^2 - 200x - 288y + 256 = 3600$

$\Rightarrow$ $25x^2 - 144y^2 - 200x - 288y - 3344 = 0$

This is the equation of the required hyperbola.

36.

Player A			Player B		
x_i	x_i - 61	$(x_i - 61)^2$	y_i	y_i - 61	$(y_i - 61)^2$
58	-3	9	94	33	1089

59	-2	4	26	-35	1225
60	-1	1	92	31	961
54	-7	49	65	4	16
65	4	16	96	35	1225
66	5	25	78	17	289
52	-9	81	14	-47	2209
75	14	196	34	-27	729
69	8	64	98	37	1369
52	-9	81	13	-48	2304
610		526	610		11416

For player A,

$\sum x_i = 610, \sum (x_i - 61)^2 = 526$ and $n = 10$

Mean = $\frac{\sum x_i}{n}$ = $\frac{610}{10}$ $= 61$

SD = $\sqrt{\frac{\sum (x_i - 61)^2}{N}}$ = $\sqrt{\frac{526}{10}}$ = $\sqrt{52.6} = 7.25$

For player B,

$\sum y_i = 610, \sum (y_i - 61)^2 = 11416$ and $n = 10$

Mean = $\frac{\sum y_i}{n}$ = $\frac{610}{10}$ $= 61$

SD = $\sqrt{\frac{\sum (y_i - 61)^2}{N}}$ = $\sqrt{\frac{11416}{10}}$

=$\sqrt{1141.6} = 33.79$

Since, SD for player A is 7.25 < SD for player B is 33.79.

$\therefore$ Player A is more consistent player.