CBSE Class 11 Mathematics
Sample Papers 04 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. A function f : [0, ∞] $\rightarrow$ [0, ∞) defined as $f(x) = \frac{x}{1+x}$ is

   a. onto but not one-one

   b. one-one and onto

   c. neither one-one nor onto

   d. one-one but not onto

2. The number of three-digit numbers having at least one digit as 5 is

   a. 648

b. 225

c. 252

d. 246

3. If $x = 99^{50} + 100^{50}$ and $y = (101)^{50}$ then

a. x $<$ y

b. x $>$ y

c. x = y

d. $x \geq y$

4. In how many ways can a mixed doubles tennis game be arranged from a group of 10 players consisting of 6 men and 4 women

a. 48

b. 180

c. 90

d. 120

5. Let function f : R $\rightarrow$ R be defined by f (x)= 2x + sin x for x $\in$ R. Then f is

a. one-to-one and onto

b. one-to-one but NOT onto

c. onto but NOT one-to-one

d. neither one-to-one nor onto

6. The inequality $n! > 2^{n-1}$ is true

a. for all n> 2 ,$n \in N$

b. for all n $>$ 1

c. for all n ∈ N

d. for no n ∈ N

7. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

a. 0.375

b. 0.39

c. 0.89

d. 0.86

8. The graph of the equation $x^2 + y^2 = 0$ in the three-dimensional space is

a. Z-axis

b. X-axis

c. Y-axis

d. XY – plane

9. If E is an event, then P $\left(\bar{E}\right)$ is equal to

a. - P (E)

b. P (E)

c. 1 – P (E)

d. 1 + P (E)

10. If three successive terms in the expansion of $(1 + x)^n a$ have their coefficients in the ratio 6 : 33 : 110, then n is equal to

a. 10

b. 13

c. 12

d. 11

11. Fill in the blanks:

    If the set A has 3 elements and set B has 4 elements, then the number of elements in A $\times$ B is ________.

12. Fill in the blanks:

    In the objective function Z = ax + by, x and y are called ________ variables.

13. Fill in the blanks:

    If there are two events such that they can be performed independently in m and n ways respectively, then either of the two events can be performed in ________ ways.

14. Fill in the blanks:

    If the mid-points of the sides of a triangle AB;BC;CA are D(1, 2, -3), E(3, 0, 1) and F(-1, 1, -4), then the centroid of the triangle ABC is ________.

OR

Fill in the blanks:

The equation of z-axis, are ________.

15. Fill in the blanks:

    The value of limit $\lim_{r \to 1} \pi r^2$ is ________.

OR

Fill in the blanks:

The derivative of cosecx is ________.

16. Write the interval in set-builder form: [-23, 5)

17. In how many ways, 5 flags in which 3 are red, 1 is white and 1 is blue, be arranged on staff, one below the other, if flags of one colour are not distinguishable?

18. Find the real and imaginary parts of the complex number 3 i.

OR

If | z | = 1, then find the value of $\frac{1+z}{1+\bar{z}}$

19. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$,then find the values of x and y.

20. Given, 5 Different green dyes, 4 different blue dyes and 3 different red dyes. Find the number of combinations of dyes which can be chosen taking at least one green and one blue dye.

21. Decide among the following sets which sets are subsets of each another:
$A=\{X : X \in R$ and x satisfies $x^2-8x+12=0\}$, B = {2, 4, 6} , C = {2, 4, 6, 8, . . . }, D = {6}

OR

Is B = {x : x^2 + 2x + 1 = 0, x ∈ N} a singleton set?

22. In a simultaneous throw of a pair of dice, find the probability of getting 8 as the sum.

23. Using binomial theorem. evaluate $(103)^3$.

24. Find the equation of straight line which passes through the point (5, 6) and has intercepts on the axes equal in magnitude but opposite in sign.

OR

Find the equation of the line intersecting the X-axis at a distance of 3 units to the left of origin with slope -2.

25. Write the component statement of the compound statement and check whether the compound statement is true or false:
To enter into a public library children need an identity card from the school or a letter from the school authorities.

26. Find the maximum and minimum values of 6sin x cos x + 4 cos 2x.

27. Let A, B and C be three sets such that $A \cup B = C$ and $A \cap B = \phi$ then prove that A = C - B.

28. If h denotes the number of honest people and p denotes the number of punctual people and a relation between honest people and punctual people is given as h = p +16. If P denotes the number of people who progress in life and a relation between number of people who progress and honest people is given as
P = $\left(\frac{h}{8}\right)$ + 5
Find the relation between number of people who progress in life and punctual people. How does the punctuality is important in the progress of life?

OR

Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in $A \times B$, find A and B, where x, y and z are distinct elements.

29. Find $\frac{dy}{dx}$ where y = 3 tan x + 5 log $x + \frac{1}{x^2} + 5e^x$

30. Find the real numbers x and y if (x - iy) (3+ 5i) is the conjugate of -6 - 24i.

31. Solve the inequation $\frac{2x+4}{x-3} \leqslant 4.$

OR

A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

32. Prove by the principal of mathematical induction that for all n $\in$ N.

$1 + 4 + 7 + ... + (3n - 2) = \frac{1}{2} n(3n - 1)$

33. Solve: 4 sinx cosx + 2 sinx + 2 cosx + 1 = 0.

OR

If tan $(\pi \cos\theta)$ = cot $(\pi \sin\theta)$ and then prove that cos $\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

34. The sum of p, q, r terms of an AP are a, b, c respectively. Show that
$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

35. Find the vertex, axis, focus, directrix and length of latus rectum of parabola $y^2 - 8y - x + 19 = 0$.

OR

A rod of length 12 m moves with its ends always touching the coordinates axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the X-axis.

36. From the following data, state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Solution
Section A

1. (d) one-one but not onto

   Explanation:

   f : $[0, \omega] \rightarrow (0, \omega)$ is defined by,

   $f(x) = \frac{x}{1+x}$

   one - one : $f(x_1) = f(x_2)$

   $\Rightarrow \frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1(1 + x_2) = x_2(1 + x_1) \Rightarrow x_1 = x_2$

   $\Rightarrow$ f is one - one

   onto: Let $y \in (0, \omega)$ (co - domain) be arbitrary, such that

   f(x) = y

   $\Rightarrow \frac{x}{1+x = y} \Rightarrow x = \frac{y}{1-y} \notin (0, \omega) \forall y \in (0, \omega)$

   $e.g\ y = 2 \Rightarrow x = \frac{2}{1-2} = \frac{2}{-1} = -2 \notin (0, \omega)$

   $\therefore$ f is not onto

2. (c) 252

   Explanation: First we will find the number of three-digit numbers (i.e, numbers from 100 to 999)which can be formed using the digits 0,1,2,3,4,5,6,7,8and 9 with repetition allowed.

   Now we have the first place can be filled by any of the 9 digits other than 0 and since repetition is allowed the second and third can be filled by any of the ten digits.

   Hence the total number of three-digit numbers will be = $9 \times 10 \times 10 = 900$

Now we will consider the case that the number does not have the digit 5.

Now the first place can be filled by any of the 8 digits other than 0 and 5 and since repetition is allowed the second and third can be filled by any of the 9 digits other than 5.

Hence the total number of ways we can form a three-digit number without 5 will be = $8 \times 9 \times 9 = 648$

Therefore the number of three-digit numbers with at least one 5=$900 - 648 = 252$

3. (a) x $<$ y

   Explanation: Given $x = 99^{50} + 100^{50}$ and $y = (101)^{50}$

   Now

   $y = (101)^{50} = (100+1)^{50} = ^{50}C_0 \quad (100)^{50} + ^{50}C_1 \quad (100)^{49} + ^{50}C_2 \quad (100)^{48} + \ldots\ldots + 50\text{C}_{50} \ldots\ldots\ldots.(i)$

   Now subtract equation (ii) from equation (i), we get

   $(101)^{50} - (99)^{50} = 2\left[^{50}C_1 \quad (100)^{49} + ^{50}C_3 \quad (100)^{47} + \ldots\ldots.\right]$

   $= 2\left[50(100)^{49} + \frac{50\times49\times48}{3\times2\times1}(100)^{47} + \ldots\ldots.\right]$

   $= (100)^{50} + 2\left(\frac{50\times49\times48}{3\times2\times1}(100)^{47}\right)$

   $\Rightarrow (101)^{50} - (99)^{50} > (100)^{50}$

   $\Rightarrow (101)^{50} > (100)^{50} + (99)^{50} \Rightarrow y < x$

4. (b) 180

   Explanation:

   A team of 4 players are to be selected.

   2 out of 6 men can be done in $^6C_2$ ways. 2 out of 4 women can be done in $^4C_2$ways.

   So the number of ways to select 4 players is $^6C_2$x $^4C_2$= 90.

   Now we can arrange these people to form mixed doubles.

If M_1, M_2, W_1, W_2, are the 4 members selected then one team can be chosen as $(M_1, W_1) \quad or \quad (M_1, W_2)$ in 2 different ways

Therefore the required number of arrangements= 90x2 = 180.

5. (a) one-to-one and onto

   Explanation:

   one - one: $f(x_1) = f(x_2)$

   $\Rightarrow 2x_1 + \sin x_1 = 2x_2 + \sin x_2$

   $\Rightarrow 2(x_1 - x_2) + (\sin x_1 - \sin x_2) = 0$

   $\Rightarrow x_1 - x_2 = 0$

6. (a) for all n> 2, n∈N

   Explanation:

   Since when n = 1 , we get the inequality as $1 > 1$, which is not true. also for n = 2 , we get $2 > 2$, which is false. Hence the given statement is true for $n > 2$

7. (b) 0.39

   Explanation:

   We have, P(A) = 0.25, P(B) = 0.50, $P(A \cap B) = 0.14$

   $\therefore$ Required Probability $= P(\overline{A} \cap \overline{B})$

   $= 1 - P(A \cap B)$

   $= 1 - [P(A) + P(B) - P(A \cap B)]$

   $= 1 - [0.25 + 0.50 - 0.15]$

   = 0.39

8. (a) Z - axis

   Explanation:

$x^2 + y^2 = 0 \Rightarrow x \ = 0 \quad and \quad y \ = \ 0$ represents a point (0,0) in space in 2D. In 3D the collection of all these points forms Z axis.

9. (c) 1 – P (E)

Explanation:

we have, $(E \cup \overline{E}) = S$

$\therefore P(E \cup \overline{E}) = P(S)$

$\Rightarrow P(E) + P(\overline{E}) = P(S)$ {U sing additiion thereom and E and $\overline{E}$ are mutually exclusive}

$\Rightarrow P(E) + P(\overline{E}) = 1$ {Probability in sample space is unque}

$\Rightarrow P(\overline{E}) = 1 - P(E)$

10. (c) 12

Explanation: We have

$(1+x)^n =^n C_0 \quad 1^n +^n C_1 \quad (1)^{n-1}(x) +^n C_2 \quad (1)^{n-2}(x)^2 + \ldots +^n C_r \quad (1)^{n-r}(x)^r \ldots\ldots. +^n C_n \quad (x)^n$

Suppose the three consecutive terms be T_{r-1}, T_r and T_{r+1}, whose coefficients are in the ratio 6 : 33 : 110

Now we have $T_{r+1} =^n C_r \quad (x)^r, \quad T_r =^n C_{r-1} \quad (x)^{r-1} \quad$ and $\quad T_{r-1} =^n C_{r-2} \quad (x)^{r-2}$

Now $\frac{T_r}{T_{r-1}} = \frac{^nC_{r-1}}{^nC_{r-2}} = \frac{33}{6}$ and $\frac{T_{r+1}}{T_r} = \frac{^nC_r \quad (x)^r}{^nC_{r-1} \quad (x)^{r-1}} = \frac{110}{33}$

$\Rightarrow \frac{n!}{(n+1-r)!(r-1)!} \times \frac{(n+2-r)!(r-2)!}{n!} = \frac{11}{2}$ and $\frac{n!}{(n-r)! \cdot r!} \times \frac{(n+1-r)!(r-1)!}{n!}$ = $\frac{10}{3}$

$\Rightarrow \frac{(n+2-r)}{(r-1)} = \frac{11}{2}$ and $\frac{(n+1-r)}{r} = \frac{10}{3}$

$\Rightarrow 2n - 2r + 4 = 11r - 11$ and $3n - 3r + 3 = 10r$

$\Rightarrow 13r - 2n = 15 \ldots\ldots.(i) \quad$ and $\quad 13r - 3n = 3 \ldots.(ii)$

Solving (i) and (ii) we get n = 12

11. 12

12. decision

13. (m + n)

14. (1, 1, -2)

OR

x = 0, y = 0

15. π

OR

-cosecx cotx

16. The interval [-23, 5) can be written in set builder form as
$\{x : x \in R, -23 \leqslant x < 12\}$

17. Here, n = 5, p_1 = 3, p_2 = 1, p_3 = 1
Required number of ways $= \frac{n!}{p_1!p_2!p_3!}$
$= \frac{5!}{3!1!1!}$
$= \frac{5 \times 4 \times 3!}{3!}$
= 5 $\times$ 4 = 20
$\therefore$ Required number of ways is 20.

18. Suppose, $z = 3i = 0 + 3i$
Here, Re (z) = 0 and Im (z) = 3

OR

Given, $|z|$ = 1
$\Rightarrow |z|^2$ = 1 $\Rightarrow z\overline{z}$ = 1
Now, $\frac{1+z}{1+\overline{z}} = \frac{z\overline{z}+z}{1+\overline{z}}$ [$\because$ 1 = $z\overline{z}$]
= $\frac{z(\overline{z}+1)}{(\overline{z}+1)}$ = z = 1

19. Given, $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$
Comparing corresponding elements,
$\Rightarrow \frac{x}{3} + 1 = \frac{5}{3}$ and $y - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \quad \frac{x}{3} = \frac{5}{3} - 1$ and $y = \frac{1}{3} + \frac{2}{3}$
$\Rightarrow \quad \frac{x}{3} = \frac{5-3}{3}$ and $y = \frac{3}{3}$
$\Rightarrow \quad \frac{x}{3} = \frac{2}{3}$ and $y = 1$
$\therefore x = 2$ and $y = 1$

20. In each dye of chosen, there are two possibilities either choose or reject it.
$\therefore$ The total number of ways in which at least one green and one blue dyes are chosen
= $(2^5 - 1)(2^4 - 1)2^3$ = (32 - 1) (16 - 1)8
$= 31 \times 15 \times 8$ = 3720

21. Here $A = \{X : X \in R$ and x satisfies $x^2 - 8x + 12 = 10\}$
$= \{x : x \in R$ and $(x-6)(x-2) = 0\}$
={2,6}
B = {2, 4, 6}
C = {2, 4, 6, 8, . . . } and D = {6}
Now $A \subset B, A \subset C, B \subset C, D \subset A, D \subset B and D \subset C.$

OR

We have, $B = \{x : x^2 + 2x + 1 = 0, x \in N]$
Now, $x^2 + 2x + 1 = 0$
$\Rightarrow (x+1)^2 = 0$
$\Rightarrow x = -1$ which is not a natural number.
Thus, B = {} = ϕ
Hence, B is not a singleton set.

22. Since a pair of dice have been thrown,
$\therefore$ Numbers of elementary events in sample space is $6^2 = 36$
Suppose E be the event that the sum 8 appear on the faces of dice,
$\therefore$ E = {(2,6), (3,5), (4,4), (5,3), (6,2)}
$\therefore n(E) = 5$
$\therefore P(E) = \frac{5}{36}$

23. Here, 103 can be written as 100 + 3.
$(103)^3 = (100 + 3)^3$

$= {}^3C_0\,(100)^3 + {}^3C_1\,(100)^2\,3 + {}^3C_2\,(100)^1\,3^2 + {}^3C_3(100)^0\,3^3$

$= 1000000 + 9 \times 10000 + 3 \times 9 \times 100 + 27$

$= 1000000 + 90000 + 2700 + 27$

$= 1092727$

24. Let the equation of line in intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

We have, a = - b ...(i)

Now, equation of line becomes

$\frac{x}{-b} + \frac{y}{b} = 1$ ...(ii)

It passes through (5, 6).

$\therefore \quad \frac{5}{-b} + \frac{6}{b} = 1 \Rightarrow \frac{-5+6}{b} = 1$

$\Rightarrow$ b = 1

$\therefore$ a = -1

On putting a = -1 and b = 1 in Eq. (ii), we get

$\frac{x}{-1} + \frac{y}{1} = 1 \Rightarrow -x + y = 1$

$\Rightarrow$ x - y + 1 = 0

OR

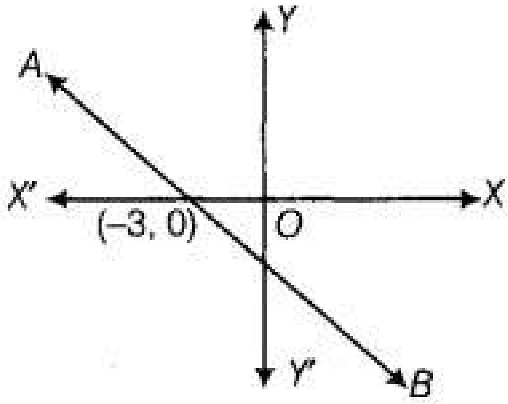
Given, the line intersecting the X-axis to the left of the origin. It means it intersect the negative X-axis. Clearly, line AB passes through the point (-3, 0) and m = -2.

Equation of line in point slope from is

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = -2(x + 3)$

$\Rightarrow$ y = -2x - 6

$\Rightarrow$ 2x + y + 6 = 0



25. The component statements of the compound statement.
p: To get into a public library children need an identity card.
q: To get into a public library children need a letter from the school authorities.
We know that if p and q are true then p or q must also be true.
Hence, the compound statement is true.

26. Given, $6sinx\ cosx + 4cos2x = 3sin2x + 4cos2x$ [∵sin 2A = 2 sin A cos A
which is of the from $asin\theta \ \ + bcos\theta$.
$\therefore -\sqrt{3^2+4^2} \leq 3sin2x + 4cos2x \leq 5$
$\Rightarrow -5 \leq 3sin2x + 4cos2x \leq 5$
Thus, maximum and minimum values of 6sin x cos x + 4 cos 2x are 5 and - 5, respectively.

27. Here $A \cup B = C$
$\therefore (A \cup B) - B = C - B$
$\Rightarrow (A \cup B) \cap B' = C - B (\because A - B = A \cap B')$
$\Rightarrow (A \cap B') \cup (B \cap B') = C - B$
$\Rightarrow (A \cap B') \cup \phi = C - B$
$\Rightarrow (A \cap B') = C - B$
$\Rightarrow A - B = C - B$
$\Rightarrow A = C - B \ (\because A \cap B = \phi)$

28. According to the question, the relation between honest and punctual people is
$h = p + 16$
And the relation between progress and honest people is
$P = \left(\frac{h}{8}\right) + 5$
Required relation between the number of people who progress in life and punctual is given by,
$P = \left(\frac{p+16}{8}\right) + 5 \quad [\because \ h = p + 16]$
P = $\left(\frac{p}{8}\right) + 2 + 5$
P = $\left(\frac{p}{8}\right) + 7$
We can complete our work on time and the quality of work will also good if we are punctual. This helps us to get progress in our life.

OR

Here $(x, 1) \in A \times B \Rightarrow x \in A$ and $1 \in B$

(y, 2) $\in A \times B \Rightarrow y \in A$and $2 \in B$

(z, 1) $\in A \times B \Rightarrow z \in A$ and 1

It is given that n(A) = 3 and n(B)= 2

$\therefore$ A = {x, y, z }

and B = {1, 2}

29. Here y = 3 tan x + 5 log $x + \frac{1}{x^2} + 5e^x$.

= 3 tan x + 5 log x + (x^{-2}) + 5e^x

$\therefore \frac{dy}{dx} = \frac{d}{dx}(3\tan x) + \frac{d}{dx}(5\log x) + \frac{d}{dx}(x^{-2}) + \frac{d}{dx}(5e^x)$

$= 3\sec^2 x + \frac{5}{x} - 2x^{-3} + 5e^x$

$= 3\sec^2 x + \frac{5}{x} - \frac{2}{x^3} + 5e^x$

30. Here $\overline{-6 - 24i} = -6 + 24i$

Now (x - iy) (3 + 5i) = -6 + 24i

$\Rightarrow 3x + 5xi - 3yi - 5yi^2 = 6 + 24i$

$\Rightarrow (3x + 5y) + (5x - 3y)i = -6 + 24i$

Comparing both sides, we have

3x + 5y = -6. . . . (i)

and 5x - 3y = 24 .... (ii)

Multiplying (i) by 3 and (ii) by 5 and then adding

$9x + 15y = -18$

$\underline{25x - 15y = 120} \Rightarrow x = 3$

$34x = 102$

Putting x = 3 in (i)

3(3)+5y=-6

Thus y=-3

31. Here $\frac{2x+4}{x-3} \leqslant 4, x \neq 3$

$\Rightarrow \frac{2x+4}{x-3} - 4 \leqslant 0$

$\Rightarrow \frac{2x+4-4x+12}{x-3} \leqslant 0$

$\Rightarrow \frac{-2x+16}{x-3} \leqslant 0$

$\Rightarrow -2x + 16 \leqslant 0$
$\Rightarrow -2x \leqslant -16$
Dividing both sides by -2

$\Rightarrow x \geqslant 8$
the solution set of given in equation is $[8, \infty)$.

OR

Let the length of the shortest board be x cm
Then length of the second board = (x + 3) cm
length of the third board = 2x cm
Now $x + (x+3) + 2x \leqslant 91$ and $2x \geqslant (x+3) + 5$
$\Rightarrow 4x + 3 \leqslant 91$ and $2x - (x+3) \geqslant 5$
$\Rightarrow 4x \leqslant 91 - 3$ and $2x - x - 3 \geqslant 5$
$\Rightarrow 4x \leqslant 88$ and $x \geqslant 5 + 3$
$\Rightarrow x \leqslant 22$ and $x \geqslant 8$
Thus minimum length of shortest board is 8 cm and maximum length is 22 cm.

32. Let P(n) be the statement given by
$P(n) : 1 + 4 + 7 + \ldots + (3n-2) = \frac{1}{2}n(3n-1)$
We have,
$P(1)$:
LHS = 1
RHS = $\frac{1}{2} \times$ (1) $\times$ (3 $\times$ 1 - 1) = 1
So, $P(1)$ is true
Let P(m) be true. Then,
$1 + 4 + 7 + \ldots + (3m-2) = \frac{1}{2}m(3m-1)$ ...(i)
We wish to show that P(m + 1) is true. For this we have to show that
$1 + 4 + 7 + \ldots + (3m-2) + [3(m+1) - 2]$ = $\frac{1}{2}$ $(m+1)[3(m+1) - 1]$
Now,
$1 + 4 + 7 + \ldots + (3m-2) + [3(m+1) - 2]$
= $\frac{1}{2}$ $m(3m-1) + [3(m+1) - 2]$ [Using (i)]
= $\frac{1}{2}$ $m(3m-1) + (3m+1) = \frac{1}{2}[3m^2 - m + 6m + 2]$
= $\frac{1}{2}[3m^2 + 5m + 2] = \frac{1}{2}$(m + 1)(3m + 2) = $\frac{1}{2}$(m + 1)[3(m + 1)-1]

$\therefore$ P(m + 1) is true

Thus, P(m) is true $\Rightarrow$ P(m + 1) is true.

Hence, by the principle of mathematical induction, the given result is true for all n $\in$ N.

33. $4sin\ x\ cos\ x\ +2sin\ x\ +2cos\ x+1=0$

$\Rightarrow 2sin\ x(2cos\ x+1)+1(2cos\ x+1)=0$

$\Rightarrow (2sin\ x+1)(2cos\ x+1)=0$

$\Rightarrow 2sin\ x+1=0$

or $2cos\ x+1=0$

$\Rightarrow sinx=-\frac{1}{2}$

or $cosx=-\frac{1}{2}$

Now, if sin x = - $\frac{1}{2}$

$\Rightarrow$ sin x = sin $\left(-\frac{\pi}{6}\right)$

$\therefore$ The general solution of this equation is

x = $n\pi+(-1)^n\left(-\frac{\pi}{6}\right)$= $n\pi+(-1)^{n+1}\left(\frac{\pi}{6}\right)$

$\Rightarrow$ x = $\pi\left[n+\frac{(-1)^{n+1}}{6}\right]$ ...(i)

and if cos x = $\frac{-1}{2}$

$\Rightarrow$ cos x = cos $\left(\pi-\frac{\pi}{3}\right)$ = cos $\frac{2\pi}{3}$

The general solution of this equation is

x = $2n\pi\pm\frac{2\pi}{3}$

$\Rightarrow$ x = $2\pi\left(n\pm\frac{1}{3}\right)$ ... (ii)

From Eqs. (i) and (ii), we have x = $\pi\left[n+\frac{(-1)^{n+1}}{6}\right]$ or $2\pi\left(n\pm\frac{1}{3}\right)$ where n $\in$ Z

These are the required solutions.

OR

Given,

$tan\ (\pi\cos\theta)=cot\ (\pi\sin\theta)$

$\Rightarrow \frac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)}=\frac{\cos(\pi\sin\theta)}{\sin(\pi\sin\theta)}$

$\Rightarrow$ sin $(\pi\cos\theta)\times$ sin $(\pi\sin\theta)$ = cos $(\pi\sin\theta)\times$ cos $(\pi\cos\theta)$

$\Rightarrow$ cos $(\pi\cos\theta)$ $\times$cos $(\pi\sin\theta)$ - sin $(\pi\cos\theta)$ $\times$ sin $(\pi\sin\theta)$ = 0

$\Rightarrow$ cos $[\pi\cos\theta+\pi\sin\theta]$ = 0

$[\because cos\ x \times\ \ cos\ y - sin\ x \times\ \ sin\ y = cos(x+y)]$

$\Rightarrow$ cos $(\pi\cos\theta+\pi\sin\theta)$ = cos $\left(\pm\frac{\pi}{2}\right)$ $[\because$ cos $\left(\pm\frac{\pi}{2}\right)$ = 0]

$\Rightarrow \pi\cos\theta+\pi\sin\theta=\pm\frac{\pi}{2}$ $\Rightarrow$cosθ + sinθ = $\pm\frac{1}{2}$

On multiplying both sides by $\frac{1}{\sqrt{2}}$, we get

$\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=\pm\frac{1}{2\sqrt{2}}$

$\Rightarrow$ cosθ $\times$ cos $\frac{\pi}{4}$ + sinθ $\times$ $\frac{\pi}{4}$ $=\pm\frac{1}{2\sqrt{2}}$

cos $\left(\theta-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}$

$[\because cos\ x \times\ \ cos\ y + sin\ x\ \times\ \ sin\ y = cos(x-y)]$

Hence proved.

34. Given that, S_p = a, S_q = b and S_r = c

Let A be the first term and d be the common difference. Then,

S_p = $\frac{p}{2}$ [2 A + (p - 1)d] = a

$\Rightarrow$ 2 A + (p - 1)d = $\frac{2a}{p}$ ...(i)

S_q = $\frac{q}{2}$ [2 A + (q - 1)d] = b

$\Rightarrow$ 2 A + (q - 1)d = $\frac{2b}{q}$ ...(ii)

and S_r = $\frac{r}{2}$ [2 A + (r - 1)d] = c

$\Rightarrow$ 2 A + (r - 1)d = $\frac{2c}{r}$ ...(iii)

On multiplying Eq. (i) by (q - r), Eq. (ii) by (r - p) and Eq. (iii) by (p - q), we get

[2 A + (p - 1)d] (q - r) = $\frac{2a}{p}$ (q - r) ...(iv)

[2 A + (q - 1)d] (r - p) = $\frac{2b}{q}$ (r - p) ...(v)

and [2 A + (r - 1)d] (p - q) = $\frac{2c}{r}$ (p - q) ...(vi

On adding Eq. (iv), Eq. (v) and Eqs. (vi), we get

$\frac{2a}{p}(q-r)+\frac{2b}{q}(r-p)+\frac{2c}{r}(p-q)$

= [2 A + (p - 1)d] (q - r) + [2 A + (q - 1)d] (r - p) + [2 A + (r - 1)d] (p - q)

= 2 A (q - r + r - p + p - q) + d[(p - 1) (q - r) + (q - 1) (r - p) + (r - 1) (p - q)]

= 2 A (0) + d[(pq - pr - q + r + qr - qp - r + p + rp - rq - p + q]

= 0 + d (0) = 0

$\therefore \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)$= 0

Hence proved.

35. Given equation is

$y^2 - 8y - x + 19 = 0$

$\Rightarrow y^2 - 8y + 16 = x - 19 + 16$

$\Rightarrow (y - 4)^2 = x - 3$ ...(i)

Let y - 4 = Y and x - 3 = X

Then, Eq. (i) becomes,

$Y^2 = X$ ...(ii)

Now, from Eq. (ii), coordinates of vertex are,

X = 0 and Y = 0

$\Rightarrow$ x - 3 = 0 and y - 4 = 0

$\Rightarrow$ x = 3 and y = 4

On comparing Eq. (ii) with $Y^2 = 4aX$, we get

$4a = 1 \Rightarrow a = \frac{1}{4}$

Coordinates of focus of parabola (ii) are,

X = a, Y = 0

$\Rightarrow x - 3 = \frac{1}{4}, y - 4 = 0$

$\Rightarrow x = \frac{1}{4} + 3, y = 4 \Rightarrow x = \frac{13}{4}, y = 4$

Equation of directrix of parabola (ii) is,

X = - a

$\Rightarrow x - 3 = -\frac{1}{4}$

$\Rightarrow x = -\frac{1}{4} + 3 \Rightarrow x = \frac{11}{4}$

Length of latusrectum = $|4a| = \left|4 \cdot \frac{1}{4}\right| = 1$

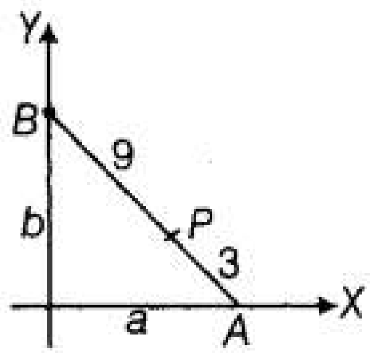
Hence, for given parabola vertex = (3, 4), axis, y = 4, focus $= \left(\frac{13}{4}, 4\right)$, directrix, x $= \frac{11}{4}$ and the length of latusrectum = 1.

OR

Let l be the length of the rod and which at any position meet X-axis at A (a, 0) and also meets the Y-axis at B (0, b), therefore we have

$l^2 = a^2 + b^2$

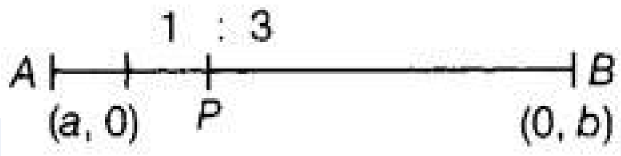
$\Rightarrow (12)^2 = a^2 + b^2$ ...(i) [$\because$ l = 12]



Let P be the point on AB which is 3 cm from A and hence 9 cm from B.

This means that the point P divides AB in ratio 3 : 9 i.e., 1 : 3.

If P = (x, y), then by section formula, we have



(x, y) = $\left(\frac{1\times0+3\times a}{1+3}, \frac{1\times b+3\times0}{1+3}\right)$

$\Rightarrow$ (x, y) = $\left(\frac{3a}{4}, \frac{b}{4}\right)$

$\Rightarrow$ x = $\frac{3a}{4}$, y = $\frac{b}{4}$ $\Rightarrow$ a = $\frac{4x}{3}$ and b = 4y

On putting the values of a and b in Equation (i), we get

144 = $\left(\frac{4x}{3}\right)^2$ + $(4y)^2$

$\Rightarrow \frac{x^2}{81} + \frac{y^2}{9}$ = 1

which is required equation.

36. For group A,

Let assumed mean $a = 45$

Class interval	Mid-point (x_i)	$u_i = \frac{x_i - 45}{10}$	u_i^2	f_i	f_iu_i	$f_iu_i^2$
$10-20$	15	-3	9	9	-27	81
$20-30$	25	-2	4	17	-34	68
$30-40$	35	-1	1	32	-32	32
$40-50$	45	0	0	33	0	0
$50-60$	55	1	1	40	40	40

$60-70$	65	2	4	10	20	40
$70-80$	75	3	9	9	27	81
Total				150	-6	342

Here,

$\sum f_i = N = 150, \sum f_i u_i = -6$, and $\sum f_i u_i^2 = 342$, b = 10

$\therefore \overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times b$ = 45 + $\frac{(-6)}{150} \times 10 = 45 - 0.4 = 44.6$

Variance, $\sigma_A = \frac{b^2}{N^2}\left[N\sum f_i u_i^2 - (\sum f_i u_i)^2\right]$

= $\frac{100}{22500}\left[150 \times 342 - (-6)^2\right]$

= $\frac{1}{225}\ (51300 - 36) = \frac{51264}{225} = 227.84$

$\therefore$ Standard deviation, $\sigma_A = \sqrt{227.84} = 15.09$

Coefficient of variation (CV) = $\frac{\sigma_A}{\overline{x}} \times 100$ = $\frac{15.09}{44.6} \times 100 = 33.83$

For group B,

Let the assumed mean $a = 45$

Class interval	Mid-point (x_i)	$u_i = \frac{x_i - 45}{10}$	f_i	u_i^2	$f_i u_i$	$f_i u_i^2$
$10-20$	15	-3	10	9	-30	90
$20-30$	25	-2	20	4	-40	80
$30-40$	35	-1	30	1	-30	30
$40-50$	45	0	25	0	0	0
$50-60$	55	1	43	1	43	43
$60-70$	65	2	15	4	30	60
$70-80$	75	3	7	9	21	63
Total			150		-6	366

$\sum F_i = N = 150,$

$\sum f_i u_i$ = -6 and $\sum f_i u_i^2$ = 366

$\therefore \overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times b$

= 45 + $\frac{(-6)}{150} \times 10 = 45 - 0.4 = 44.6$

Variance, $\sigma_B^2 = \frac{b^2}{N^2}\left[N\sum f_i u_i^2 - \left(\sum f_i u_i\right)^2\right]$

= $\frac{100}{22500}\left[150 \times 366 - (-6)^2\right]$

= $\frac{1}{225}(54900 - 36)$ = $\frac{1}{225}$ $\times 54864 = 243.84$

$\therefore$ Standard deviation, σ_B = $\sqrt{243.84}$ = 15.61

$\therefore$ Coefficient of variation (CV) = $\frac{\sigma_B}{\overline{x}}$ $\times 100$ = $\frac{15.61}{44.6}$ $\times 100 = 35$

Since, CV(B) > CV(A)

So, group B is more variable than group A.