CBSE Class 11 Mathematics
Sample Papers 03 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. A condition for a function y = f (x) to have an inverse is that it should be,

   a. strictly monotone and continuous in the domain

   b. continuous everywhere

   c. defined for all x

   d. an even function

2. The number of ways in which a necklace can be formed by using 5 identical red beads and 6 identical white beads is:

   a. $\frac{11!}{2(6!5!)}$

b. none of these.

c. $\frac{10!}{2(6!5!)}$

d. $\frac{10!}{(6!5!)}$

3. If the rth term in the expansion of $\left(\frac{x^3}{3}-\frac{2}{x^2}\right)^{10}$ contains x^{20}, then r =

a. 5

b. 3

c. 4

d. 2

4. The number of ways in which 4 red, 3 yellow and 2 green discs be arranged if the discs of the same colour and indistinguishable

a. 1260

b. 999

c. 1512

d. 2260

5. The function f (x) = log $\left(x+\sqrt{x^2+1}\right)$ is

a. a periodic function

b. neither an even nor an odd function

c. an odd function

d. an even function

6. 1 + 2 + 3 + ... n = $1/2(n(n+1))$, is trure

a. for n = 2 only

b. none of these

c. for all natural numbers n

d. only for $n > 2$

7. A dice is rolled 6 times. The probability of obtaining 2 and 4 exactly three times each is

a. 1/ 5184

b. 1/ 46656

c. none of these

d. 5/ 11664

8. The foot of perpendicular from (α, β, γ)on Y-axis is

a. (0 , β , 0)

b. (0 , 0 , 0)

c. (0 , α , 0)

d. none of these

9. The total area under the standard normal curve is

a. 2

b. none of these

c. 1/ 2

d. 1

10. If the coefficients of (2r +4)th term and (r -2)th term in the expansion of $(1+x)^{18}$ be equal then find the value of r

a. 6

b. 5

c. 7

d. 8

11. Fill in the blanks:

If A = {1, 2} and B = {3, 4}, and then no. of subsets of A $\times$ B is ________.

12. Fill in the blanks:

In the binomial expansion of $(a + b)^9$, the middle terms is ________ term.

13. Fill in the blanks:

Two events in succession can be performed in ________ ways.

14. Fill in the blanks:

The length of the foot of perpendicular from the point P(a, b, c) on z-axis is ________.

OR

Fill in the blanks:

The three planes determine a rectangular parallelopiped which has ________ pairs of rectangular faces.

15. Fill in the blanks:

The derivative of 5 secx + 4 cosx is ________.

OR

Fill in the blanks:

The value of the limit $\lim\limits_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$ is ________.

16. Justify whether the given information is a 'Set' or 'Not'? A collection of novels written by the writer Munshi Prem Chand.

17. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

18. Find the conjugate of complex number 3 + i.

OR

Evaluate i^{103}.

19. If X = {0, ± 2, 4} and Y = {0, 4, 5, 16}, then represent the rule $f : X \rightarrow Y$ given by $f(x) = x^2$ by an arrow diagram.

20. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that repetition of the digits is allowed

21. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

OR

Let A, B and C be three sets such that $A \cup B = C$ and $A \cap B = \phi$. Then prove that A = C - B.

22. If a letter is chosen at random from the English alphabet, find the probability that the letter is

    i. a vowel
    ii. a consonant

23. Expand the given expression $(1 - 2x)^5$

24. Find the equations of the altitudes of the triangle whose vertices are A (7, -1), B (- 2, 8) and C (1,2).

OR

Find the equation of a line that cuts off equal intercepts on the coordinate axis and passes through the point (2, 3).

25. Write the negation of the following statements.

i. Paris is in France and London is in England.
ii. 2 + 3 = 5 and 8 < 10.

26. Solve: $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$.

27. In a group of 850 persons, 600 like 'Sushmita Sen' and 340 like Aishwarya Rai'. Find

i. how many like both 'Sushmita Sen' and Aishwarya Rai'.
ii. how many like 'Sushmita Sen' only.
iii. how many like Aishwarya Rai' only.

28. The relation f is defined by $f(x) = \begin{cases} x^2, 0 \leq x \leq 3 \\ 3x, 3 \leq x \leq 10 \end{cases}$ and the relation g is defined by $g(x) = \begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$. Show that f is a function and g is not a function.

OR

If f (x) = $\frac{1}{1-x}$, show that f [f {f(x)}] = x.

29. Evaluate $\lim\limits_{x \to 0} \frac{\log(6+x) - \log(6-x)}{x}$.

30. If z_1 = 3 + i and z_2 = 1 + 4i, then verify that $|z_1 + z_2| \leq |z_1| + |z_2|$.

31. Solve: $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geqslant \frac{1}{3}(x - 6)$

OR

Solve the inequalities represent the solution graphically on number line:
$5(2x - 7) - 3(2x + 3) \leqslant 0, 2x + 19 \leqslant 6x + 47$

32. Prove the following by using the principle of mathematical induction for all $n \in N$:
$1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3 + \ldots + n \cdot 3^n = \frac{(2n-1)3^{n+1} + 3}{4}$

33. Prove that: 4 sin A sin (60^o - A) sin (60^o + A) = sin 3A.

Hence deduce that: sin $20^o \times$ sin $40^o \times$ sin $60^o \times$ sin 80^o = $\frac{3}{16}$

OR

Prove that: sin 10° sin 30° sin 50° sin 70° = $\frac{1}{16}$.

34. If p, q, r are in G. P and the equation $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, that show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A. P.

35. A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis.

OR

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

$$\frac{x^2}{100} + \frac{y^2}{400} = 1$$

36. Find the variance and standard deviation for the following distribution.

x_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i	1	5	12	22	17	9	4

Solution

Section A

1. (a) strictly monotone and continuous in the domain

   Explanation: By theorem, A continous and strictly monotomic function si invertible and the inverse function is also continuous.

2. (c) $\frac{10!}{2(6!5!)}$ **Explanation:** Total number of beads =11

   We have 6 beads are alike and next 5 beads are also alike , also since it is a necklace it can be observed from both the sides .

   Therefore required number of ways= $\frac{(11-1)!}{2!.5!.6!}=\frac{10!}{2!.5!.6!}$

3. (b) 3

   Explanation: We have the general term of $(x+a)^n$ is $T_{r+1}={}^n C_r \quad (x)^{n-r}a^r$

   Now consider $\left(\frac{x^3}{3}-\frac{2}{x^2}\right)^{10}$

   Here $T_{r+1}={}^{10} C_r \quad \left(\frac{x^3}{3}\right)^{10-r}\left(-\frac{2}{x^2}\right)^r$

   Hence rth term $=T_r={}^{10} C_{r-1} \quad \left(\frac{x^3}{3}\right)^{11-r}\left(-\frac{2}{x^2}\right)^{r-1}$

   Comparing the indices of x in x^{20} and in T_r, we get

   $\Rightarrow 33-3r-2r+2=20$

   $\Rightarrow$ 5r = 15

   $\Rightarrow$ r = 3

4. (a) 1260

   Explanation:

   Total number of discs =9

   Out of which red-4 ,yellow -3 and green -2 are of the same kind.

Hence required number of permutations= $\frac{9!}{4!.3!.2!} = \frac{5\times6\times7\times8\times9}{1\times2\times3\times1\times2} = 1260$

5. (c) an odd function

   Explanation:

   $$f(-x) = \log\left(-x + \sqrt{(-x)^2 + 1}\right) = \log\left(-x + \sqrt{x^2 + 1}\right.$$

   $$= \log\left(\sqrt{x^2 + 1} - x\right) = \log\left(\frac{(\sqrt{x^2 +1} - x)(\sqrt{x^2 +1} + x)}{(\sqrt{x^2 +1} + x)}\right)$$

   $$= \log\left(\frac{1}{(\sqrt{x^2 +1} + x)}\right) = \log(1) - \log\left(x + \sqrt{x^2 + 1}\right)$$

   $$= 0 - \log\left(x + \sqrt{x^2 + 1}\right)$$

   $$\Rightarrow f(-x) = -f(x)$$

   $\Rightarrow$ f is an odd fucntion

6. (c) for all natural numbers n

   Explanation:

   BY the process of mathematical induction , the given statement is true for all natural numbers n.

7. (d) 5/ 11664

   Explanation:

   Total ways of getting 2 and 4 exactly 3 times is 6! / (3! 3!) = 20

   Total number of ways in throwing 6 dice is 6^6

   Therefore probability is 20/ 6^6 = 5/11664

8. (a) (0 , β , 0)

   Explanation:

   Let P(α, β, γ) be the point and Q (0,b,0) be any point on Y axis.

drs of PQ=$(\alpha, \beta - b, \gamma)$

drs of y axis (0,b,0)

Since PQ perpendicular to y axis.hence $a_1a_2+b_1b_2+c_1c_2=0$

$0 + b(\beta - b) + 0 = 0$
$b(\beta - b) = 0$
$\beta = b$

hence the foot of perpendicular will be $(0, \beta, 0)$

9. (c) 1/ 2

   (d) 1

   Explanation:

   Since a standard normal curve represents a pdf and integral of pdf is same as area under a curve and integral of pdf is always 1

10. (a) 6

    Explanation: We have the general term of $(x+a)^n$ is $T_{r+1} = ^n C_r \quad (x)^{n-r}a^r$
    Now consider $(1+x)^{18}$
    Here $T_{r+1} = ^{18} C_r \quad (1)^{18-r}(x)^r$
    $T_{2r+4} = T_{(2r+3)+1} = ^{18} C_{2r+3} \quad (x)^{2r+3}$
    $T_{r-2} = T_{(r-3)+1} = ^{18} C_{r-3} \quad x^{r-3}$
    Given coe. of $T_{2r+4} =$ coe. $\quad$ of $\quad T_{r-2}$
    $\Rightarrow ^{18} C_{2r+3} = ^{18} C_{r-3}$ [$\because \quad ^nC_p = ^n C_q$ then either p = q or p + q = n]
    $\Rightarrow$ 2r + 3+ r - 3 = 18
    $\Rightarrow$ 3r = 18
    $\Rightarrow$ r = 6

11. 16

12. 5th and 6th

13. m $\times$ n

14. $\sqrt{a^2+b^2}$

OR

three

15. 5 secx tanx - 4 sinx

OR

$-\frac{1}{2}$

16. A collection of novels written by the writer Munshi Prem Chand is well defined and hence it forms a set.

17. In this problem, we have to find the number of words starting with E. Here in EXAMINATION we have two I's and two N's and all other letters are different.
$\therefore$ Number of ways of arrangement $=\frac{10!}{2!2!}$
$=\frac{10\times9\times8\times7\times6\times5\times4\times3\times2!}{2\times1\times2!}$= 907200

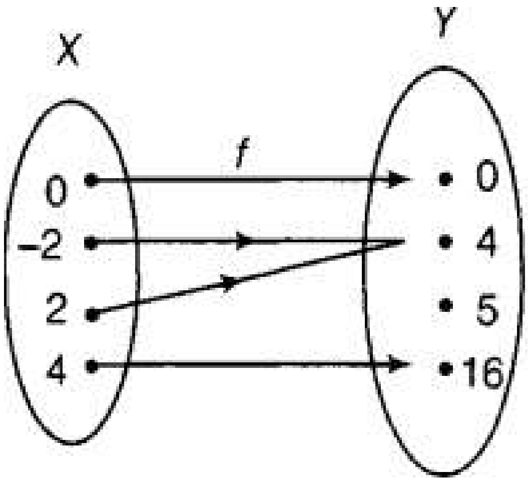
18. Suppose, $z=3+i$
$\therefore \overline{z}=3-i$

[the conjugate of complex number z, is the complex number, obtained by changing the sign of imaginary parts of z]

OR

Given,$i^{103}=i^{100}.i^{3}=(i^{4})^{25}.i^{2}.i$
= $(1)^{25}$. (- 1) i [$\because$ i^2 = - 1, i^4 = 1]
= 1 $\times$ (- 1) $\times$ i = - i

19. Domain of f = X = $\{0,\pm2,4\}$
Range of f = $\{0,4,16\}$



20. The unit place can be filled by anyone of the digits 1, 2, 3, 4 and 5. So the unit place can be filled in 5 ways. Similarly, the tens place and hundreds place can be filled in 5 ways each because the repetition of digits is allowed.
    $\therefore$Total number of 3-digits numbers = $5 \times 5 \times 5$= 125

21. Let F be the set of people who speak French and 'S' be the set of people who speak Spanish.
    Here n(F) = 50, n(S) = 20 and $n(F \cap S) = 10$
    We know that $n(F \cup S) = n(F) + n(S) - n(F \cap S)$
    $\therefore n(F \cup S) = 50 + 20 - 10 = 60$
    Number of people who speak at least one of these two languages = 60

OR

According to the question,
$A \cup B = C$
$\therefore \quad C - B = (A \cup B) - B$
We know that X - Y means the elements of X which are not present in Y
$[\because X - Y = X \cap Y']$
$= (A \cup B) \cap B'$
$= (A \cap B') \cup (B \cap B')$
$= (A \cap B') \cup \phi$
$= A \cap B'$
= A - B
= A

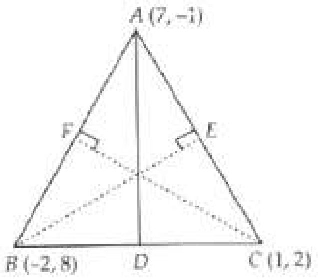
22. $n(S) = 26$ [∵ there are 26 letters in English alphabet]

i. Let E be the event that a vowel has been chosen.
$\therefore n(E) = {}^5C_1$ [∵ there are 5 vowels in English alphabet]
$\therefore P(E) = \frac{5}{26}$

ii. The probability that a consonant is chosen,
$\Rightarrow P(\overline{E}) = 1 - P(E)$
$= 1 - \frac{5}{26}$
$= \frac{21}{26}$

23. Using binomial theorem for the expansion of $(1 - 2x)^5$ we have
$(1-2x)^5 = {}^5C_0 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + {}^5C_3(-2x)^3 + {}^5C_4(-2x)^4$
$+ {}^5C_5(-2x)^5$
$= 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$
$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

24. Let AD, BE and CF be three altitudes of triangle ABC. Let m_1, m_2 and m_3 be the slopes of AD, BE and CF respectively. Then,



AD ⊥ BC ⇒ Slope of AD × Slope of BC = -1
$\Rightarrow m_1 \times \left(\frac{2-8}{1+2}\right) = -1 \Rightarrow m_1 = \frac{1}{2}$
BE ⊥ AC ⇒ Slope of BE × Slope of AC = -1
$\Rightarrow m_2 \times \left(\frac{-1-2}{7-1}\right) = -1 \Rightarrow m_2 = 2$
and, CF ⊥ AB ⇒ Slope of CF × Slope of AB = -1
$\Rightarrow m_3 \times \frac{-1-8}{7+2} = -1 \Rightarrow m_3 = 1.$
Since AD passes through A (7, -1) and has slope $m_1 = \frac{1}{2}$
So, its equation is
$y + 1 = \frac{1}{2}(x - 7) \Rightarrow x - 2y - 9 = 0$

Similarly, equation of BE is

y - 8 = 2 (x + 2) $\Rightarrow$ 2x - y + 12 = 0

Equation of CF is y - 2 = 1 (x - 1) $\Rightarrow$ x - y + 1 = 0

OR

Let equal intercepts on the coordinate axis be a and the line passes through point (2, 3).

$\therefore \frac{2}{a}+\frac{3}{a}=1 \Rightarrow$ a = 5

Thus equation of required line is

$\frac{x}{5}+\frac{y}{5}=1 \Rightarrow$ x + y = 5

25. i. Let p : Paris is in France and q : London is in England.
    Then, the conjunction is $p \wedge q$.
    Now, $\sim$ p : Paris is not in France,
    and, $\sim$ q : London is not in England.
    So, negation of $p \wedge q$ is given by
    $\sim(p \wedge q)=\sim p \vee \sim q$
    = Paris is not in France or London is not in England.
    ii. Let p : 2 + 3 = 5, q : 8 < 10
    Then, the conjunction is $p \wedge q$ is
    $\sim(p \wedge q)=\sim p \vee-q=(2+3 \neq 5)$ or $(8 \not< 10)$

26. We have,

$\cos \theta+\sin \theta=\cos 2 \theta+\sin 2 \theta$

$\Rightarrow \quad \cos \theta-\cos 2 \theta=\sin 2 \theta-\sin \theta$

$\Rightarrow \quad 2 \sin \frac{3 \theta}{2} \sin \frac{\theta}{2}=2 \cos \frac{3 \theta}{2} \cdot \sin \frac{\theta}{2}$ [since,

$\cos\ a-\cos\ b=2\ \sin\ \frac{a+b}{2} \sin \frac{b-a}{2}\ and\ \sin\ a-\sin\ b=2\ \sin\ \frac{a-b}{2} \cos \frac{a+b}{2}$]

$\Rightarrow \quad 2 \sin \frac{\theta}{2}\left(\sin \frac{3 \theta}{2}-\cos \frac{3 \theta}{2}\right)=0$

Either

$\sin \frac{\theta}{2}=0 \quad$ or $\quad \sin \frac{3 \theta}{2}-\cos \frac{3 \theta}{2}=0$

$\Rightarrow \quad \frac{\theta}{2}=n \pi, n \in Z$ or $\quad \tan \frac{3 \theta}{2}=1=\tan \frac{\pi}{4}$

$\Rightarrow \quad \theta=2 n \pi, n \in Z$ or $\quad \frac{3 \theta}{2}=n \pi+\frac{\pi}{4}, n \in Z$

$\Rightarrow \quad \theta=2 n \pi, n \in Z$ or $\theta=2 n \frac{\pi}{3}+\frac{\pi}{3 \times 2}, n \in Z$

Thus,

$\Rightarrow \quad \theta = 2n\pi \quad$ or $\quad 2n\frac{\pi}{3} + \frac{\pi}{6}, n \in Z$

27. Let S denotes the set of persons liking ‘Sushmita Sen' and A denotes the set of persons liking ‘Aishwarya Rai'.

Then, given, n(S) = 600, n(A) = 340 and n(S ∪ A) = 850

i. We know that, n(S ∪ A) = n (S) + n (A) - n (S ∩ A)

∴n(S ∩ A) = n(S) + n(A) - n(S ∪ A)

= 600 + 340 - 850

= 90

Thus, 90 persons like both ‘Sushmita Sen' and ‘Aishwarya Rai'.

ii. We know that,

n(S - A) + n (S ∩ A) = n(S)

n(S - A) = n(S) - n(S ∩ A)

= 600 - 90

= 510

Thus, 510 persons like ‘Sushmita Sen' only.

iii. We know that,

n(A - S) + n(A ∩ S) = n(A)

∴ n(A - S) = n(A) - n(A ∩ S)

= 340 - 90

= 250

Thus, 250 persons like ‘Aishwarya Rai' only.

28. $f(x) = \begin{cases} x^2, 0 \leq x \leq 3 \\ 3x, 3 \leq x \leq 10 \end{cases}$

At $x = 3, f(x) = x^2$

$\therefore f(3) = 3^2 = 9$

Also, at $x = 3, f(x) = 3x$

$\Rightarrow$ f(3) = $3 \times 3 = 9$

Since, f is defined at x = 3. Hence, f is a function.

Now, $g(x) = \begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$

At $x = 2, g(x) = x^2$

$\Rightarrow g(2) = 2^2 = 4$

Also, at x = 2, g(x) = 3x $\Rightarrow g(2) = 3 \times\ 2 = 6$

At x = 2, relation g has two values.

$\therefore$ The relation g is not a function.

OR

We have,

f (x) = $\frac{1}{1-x}$

Now,

f {f(x)} = f$\left\{\frac{1}{1-x}\right\}$

= $\frac{1}{1-\frac{1}{1-x}}$

= $\frac{1}{\frac{1-x-1}{1-x}}$

= $\frac{1-x}{-x}$

= $\frac{x-1}{x}$

$\therefore$ f[f {x}] = f $\left\{\frac{x-1}{x}\right\}$

= $\frac{1}{1-\left(\frac{x-1}{x}\right)}$

= $\frac{\frac{1}{x-x+1}}{x}$

= $\frac{x}{1}$

= x

$\therefore$ f[f(x)] = x Hence, proved.

29. We have,

$$\lim_{x \to 0} \frac{\log(6+x)-\log(6-x)}{x}$$

$$= \lim_{x \to 0} \frac{\log 6\left(1+\frac{x}{6}\right)-\log 6\left(1-\frac{x}{6}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\left[\log 6+\log\left(1+\frac{x}{6}\right)\right]-\left[\log 6+\log\left(1-\frac{x}{6}\right)\right]}{x}$$

$$= \lim_{x \to 0}\left[\frac{\log\left(1+\frac{x}{6}\right)}{x} - \frac{\log\left(1-\frac{x}{6}\right)}{x}\right]$$

$$= \lim_{x \to 0} \frac{1}{6} \frac{\log\left(1+\frac{x}{6}\right)}{\frac{x}{6}} + \lim_{x \to 0} \frac{1}{6} \frac{\log\left(1-\frac{x}{6}\right)}{-\frac{x}{6}} \quad \left[\because \lim_{x \to 0} \frac{\log(1+x)}{x} = \lim_{x \to 0} \frac{\log(1-x)}{-x} = 1\right]$$

$= \frac{1}{6} \times 1 + \frac{1}{6} \times 1 = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

30. We have, $z_1 = 3 + i$ and $z_2 = 1 + 4i$

$\therefore |z_1| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

and $|z_2| = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$

Now, $z_1 + z_2 = 3 + i + 1 + 4i$

$= 4 + 5i$

$\therefore |z_1 + z_2| = \sqrt{4^2 + 5^2} = \sqrt{16 + 25}$

$= \sqrt{41} = 6.40$ ...(i)

and $|z_1| + |z_2| = \sqrt{10} + \sqrt{17}$

= 3.16 + 4.12

= 7.28 ...(ii)

From Eqs. (i) and (ii), we get

$|z_1 + z_2| \leq |z_1| + |z_2|$

Hence verified.

31. Here $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geqslant \frac{1}{3}(x - 6)$

$\Rightarrow \frac{3x}{10} + 2 \geqslant \frac{x}{3} - 2$

$\Rightarrow \frac{3x}{10} - \frac{x}{3} \geqslant -2 - 2$

$\Rightarrow \frac{9x - 10x}{30} \geqslant -4$

$\Rightarrow \frac{-x}{30} \geqslant -4$

Multiplying both sides by 30, we have

$-x \geqslant -120$

Dividing both sides by -1, we have

$x \leqslant 120$

Thus the solution set is $[-\infty, 120]$

OR

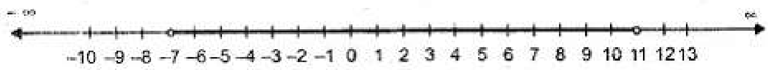
We have $5(2x - 7) - 3(2x + 3) \leqslant 0$ and $2x + 19 \leqslant 6x + 47$

$\Rightarrow 10x - 35 - 6x - 9 \leqslant 0$ and $-4x \leqslant 28$

$\Rightarrow 4x - 44 \leqslant 0$ and $x \geqslant -7$

$\Rightarrow 4x \leqslant 44$ and $x \geqslant -7$

$\Rightarrow x \leqslant 11$ and $x \geqslant -7$



32. Let P(n) = $1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3 + \ldots + n \cdot 3^n = \frac{(2n-1)3^{n+1}+3}{4}$ ... (i)

For n = 1

$P(1) = 1 \cdot 3 = \frac{(2\times1-1)3^{1+1}+3}{4} \Rightarrow 3 = \frac{9+3}{4} \Rightarrow 3 = 3$

$\therefore$ P (1) is true

Let P(n) be true for n = k

$\therefore P(k) = 1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \ldots + k \cdot 3^k = \frac{(2k-1)3^{k+1}+3}{4}$ ....(1)

For p(k+1), L.H.S=$1.3 + 2.3^2 + 3.3^3 + \ldots\ldots\ldots\ldots\ldots\ldots + k3^k + (k+1)3^{k+1}$

$= \frac{(2k-1)3^{k+1}+3}{4} + (k+1)3^{k+1}[using(1)]$

$= \frac{(2k-1)\cdot3^{k+1}}{4} + \frac{3}{4} + (k+1)3^{k+1}$

$= 3^{k+1}\left[\frac{2k-1}{4} + k + 1\right] + \frac{3}{4} = 3^{k+1}\left[\frac{2k-1+4k+4}{4}\right] + \frac{3}{4}$

$= 3^{k+1}\left[\frac{6k+3}{4}\right] + \frac{3}{4} = \frac{3^{k+1}\cdot3(2k+1)}{4} + \frac{3}{4}$

$= \frac{(2k+1)3^{k+2}+3}{4}$ = R.H.S of P(k+1)

$\therefore$ P(k+1) is true

Thus P(k) is true $\Rightarrow$ P (k + 1) is true

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.

33. $LHS = 4sinA \times \ sin(60^o - A) \times \ sin(60^o + A)$

$= 2sinA[2sin(60^o - A)sin(60^o + A)]$

= 2 sin A [cos {(60^o - A) - (60^o + A)} - cos {(60^o - A) + (60^o + A)}]

[$\because$ 2 sin A $\times$ sin B = cos (A - B) - cos (A + B)]

$= 2sinA[cos(-2A) - cos120^o]$

$= 2sinA[cos2A - cos120^o]$ [$\because$ cos (-θ) = cos θ]

$= 2sinA \times \ cos2A - 2sinA \ \times \ cos120^o$

= [sin (A + 2A) + sin (A - 2A)] - 2 sin A $\left(-\frac{1}{2}\right)$

[$\because 2sinA \times \ cosB = sin(A+B) + sin(A-B)$ and cos 120^o = $-\frac{1}{2}$]

$= sin3A + sin(-A) + sinA$

$= sin3A - sinA + sinA = sin3A =$ RHS [$\because$ sin (-θ) = - sin θ]

$\therefore$ LHS = RHS

Hence proved.

Now, 4 sin A sin $(60^o - A) \times$ sin $(60^o + A)$ = sin 3A

On putting A = 20^o, we get

4 sin $20^o \times$ sin $(60^o - 20^o)$ sin $(60^o + 20^o)$ = sin 3 $\times$ (20^o)

$\Rightarrow$ 4 sin $20^o \times$ sin $40^o \times$ sin 80^o = sin 60^o = $\frac{\sqrt{3}}{2}$

$\Rightarrow sin20^o \times sin40^o \times sin80^o = \frac{\sqrt{3}}{8}$

$\Rightarrow$ sin $20^o \times$ sin $40^o \times \frac{\sqrt{3}}{2} \times$ sin 80^o = $\frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

[multiplying both sides by $\frac{\sqrt{3}}{2}$]

$\therefore sin20^o \times sin40^o \times sin60^o \times sin80^o = \frac{3}{16}$ [$\because \frac{\sqrt{3}}{2}$ = sin 60^o]

OR

LHS = sin 30° (sin 10° sin 50°) sin 70°

$\Rightarrow$ LHS = $\frac{1}{2}$ (sin 50° sin 10°) sin 70°

$\Rightarrow$ LHS = $\frac{1}{2} \times \frac{1}{2}$ (2 sin 50° sin 10°) sin 70°

$\Rightarrow$ LHS = $\frac{1}{4}$ {(2 sin 50° sin 10°) sin 70°}

$\Rightarrow$ LHS = $\frac{1}{4}$ [{cos (50° - 10°) - cos (50° + 10°)} sin 70°] [$\because$ 2 sin A sin B = cos (A - B) - cos (A + B)]

$\Rightarrow$ LHS = $\frac{1}{4}$ [(cos 40° - cos 60°) sin 70°]

$\Rightarrow$ LHS = $\frac{1}{4}$ {sin 70° cos 40° - sin 70° cos 60°}

$\Rightarrow$ LHS = $\frac{1}{4}$ {sin 70° cos 40° - $\frac{1}{2}$ sin 70°}

$\Rightarrow$ LHS = $\frac{1}{8}$ {2 sin 70° cos 40° - sin 70°}

$\Rightarrow$ LHS = $\frac{1}{8}$ {sin (70° + 40°) + sin (70° - 40°) - sin 70°} [$\because$ 2 sin A cos B = sin (A + B) + sin (A - B)]

$\Rightarrow$ LHS = $\frac{1}{8}$ {sin 110° + sin 30° - sin 70°}

$\Rightarrow$ LHS = $\frac{1}{8}$ {sin (180° - 70°) + sin 30° - sin 70°}

$\Rightarrow$ LHS = $\frac{1}{8}$ {sin 70° + $\frac{1}{2}$ - sin 70°} [$\because$ sin (180 - x) = sin x]

$\Rightarrow$ LHS = $= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ = RHS

34. $px^2 + 2qx + r = 0$ has root
given by

$$x = \frac{-2q \pm \sqrt{4q^2 - 4rp}}{2p}$$

since p, q, r in G.P.

$q^2 = pr$

$x = \frac{-q}{p}$

but $\frac{-q}{p}$ is also root of

$dx^2 + 2ex + f = 0$

$d\left(\frac{-q}{p}\right)^2 + 2e\left(\frac{-q}{p}\right) + f = 0$

$dq^2 - 2eqp + fp^2 = 0$

$\div$ by pq^2

and using $q^2 = pr$

$\frac{d}{p} - \frac{2e}{q} + \frac{fp}{pr} = 0$

$\frac{2e}{q} = \frac{d}{p} + \frac{f}{r}$

Hence $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P

35. Let AB be a rod of length 12 cm and P (x, y) be any point on the rod such that PA = 3 cm and PB = 9 cm

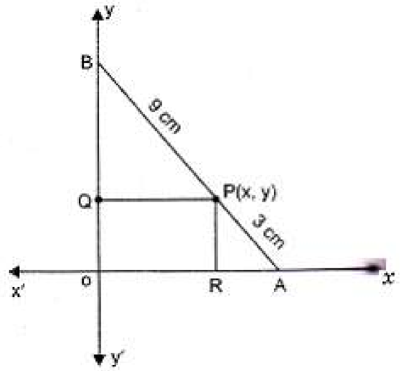
Let AR = a and BQ = b

Then triangle ARP ~ trianglePQB

$\therefore \frac{AR}{PQ} = \frac{AP}{PB}$

$\therefore \frac{a}{x} = \frac{3}{9} \Rightarrow 9a = 3x \Rightarrow a = \frac{x}{3}$

and $\frac{BQ}{BP} = \frac{PR}{PA}$



$\therefore \frac{b}{9} = \frac{y}{3} \Rightarrow 3b = 9y \Rightarrow b = 3y$

Now OA = OR + AR = x + a $= x + \frac{x}{3} = \frac{4x}{3}$

OB = OQ + BQ = y + b = y + 3 y = 4y

In right angled ΔAOB

$AB^2 = OA^2 + OB^2$

$\therefore (12)^2 = \left(\frac{4x}{3}\right)^2 + (4y)^2 \Rightarrow 144 = \frac{16x^2}{9} + 16y^2$

$\therefore \frac{x^2}{81} + \frac{y^2}{9} = 1$

Which is required locus of point P and which represents an ellipse.

OR

The equation of given ellipse is $\frac{x^2}{100} + \frac{y^2}{400} = 1$

Now 400 > 100 $\Rightarrow$ $a^2 = 400$ and $b^2 = 100$

So the equation of ellipse in standard form is $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

$\therefore a^2 = 400 \Rightarrow a = 20$ and $b^2 = 100 \Rightarrow b = 10$

We know that $c = \sqrt{a^2 - b^2}$

$\therefore c = \sqrt{400 - 100} = \sqrt{300} = 10\sqrt{3}$

$\therefore$ Coordinates of foci are $(0, \pm c)$ i.e. $(0, \pm 10\sqrt{3})$

Coordinates of vertices are $(0, \pm a)$ i.e. $(0, \pm 20)$

Length of major axis = 2 a $= 2 \times 20 = 40$

Length of minor axis = 2 b = 2×10 = 20

Eccentricity (e) $= \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 100}{20} = 10$

36. We need to make the following table from the given data:

x_i	f_i	$d_i = x_i - 34.5$	$u_i = \frac{x_i - 34.5}{10}$	u_i^2	f_iu_i	$f_iu_i^2$
4.5	1	-30	-3	9	-3	9
14.5	5	-20	-2	4	-10	20
24.5	12	-10	-1	1	-12	12
34.5	22	0	0	0	0	0
44.5	17	10	1	1	17	17
54.5	9	20	2	4	18	36
64.5	4	30	3	9	12	36

Total	N = 70				22	130

The formula to calculate the Variance is given as,

$$\sigma^2 = \left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right] \times h^2$$

h = difference between x_i - x_{i-1} = 10

Substituting values from the table, variance is,

= $\left[\frac{130}{70} - \left(\frac{22}{70}\right)^2\right] \times 100$ = $\left[\frac{13}{7} - \left(\frac{11}{35}\right)^2\right] \times 100$

= [1.857 - 0.099] $\times$ 100 = 175.8

and standard deviation = $\sqrt{Variance}$ = $\sqrt{175.8}$ = 13.259.