CBSE Class 11 Mathematics

Sample Papers 02 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. A class has 175 students. The following data shows the number of students opting one or more subject. Maths – 100, Physics – 70, Chemistry – 40, Maths and Physics – 30, Maths and Chemistry – 38, Physics and Chemistry – 23, Maths, Physics and Chemistry – 18. How many have opted for Mathematics alone ?

    a. 50

    b. 35

    c. 30

    d. 48

2. Number of ways in which 10 different things can be divided into two groups containing 6 and 4 things respectively is

   a. P(10,4)

   b. P(10,2)

   c. P(10,6)

   d. C(10 , 4)

3. The greatest term in the expansion of $(3+2x)^9$,when x = 1, is

   a. 5^{th}

   b. 7^{th}

   c. 4^{th}

   d. 6^{th}

4. In a multiple-choice question, there are 4 alternatives, of which one or more are correct. The number of ways in which a candidate can attempt this question is

   a. 15

   b. 2^5

   c. 4

   d. 16

5. If A = [a, b], B = [c,d], C = [d, e] then {(a, c), (a, d), (a,e), (b,c), (b, d), (b, e)} is equal to

   a. $A \cap (B \cup C)$

   b. $A \times (B \cap C)$

   c. $A \times (B \cup C)$

   d. $A \cup (B \cap C)$

6. If n is a +ve integer, then $3^{3n} - 26n - 1$ is divisible by

    a. 676

    b. 547

    c. 627

    d. 239

7. Three identical dice are rolled. The probability that the same number will appear on each of them is

    a. $\frac{1}{6}$

    b. $\frac{1}{18}$

    c. $\frac{1}{36}$

    d. $\frac{3}{28}$

8. The radius of the sphere through the points (4 ,3 , 0) , (0 , 4 , 3) ,(0 , 5 , 0) and (4 , 0 , 3) is

    a. 7/ 5

    b. 5/ 7

    c. 7

    d. 5

9. Both A and B throw a dice. The chance that B throws a number higher than that thrown by A is

    a. 1/ 2

    b. 13/36

    c. 21/ 36

d. 15/ 36

10. Find the number of terms in the expansion of $\left[\left(\sqrt{x}+\sqrt{y}\right)^{11}+\left(\sqrt{x}-\sqrt{y}\right)^{11}\right]$

a. 6

b. 4

c. 5

d. 12

11. Fill in the blanks:

If a pair of elements written in the small bracket and grouped together in a particular order, then such pair is called an ________.

12. Fill in the blanks:

The coefficients of 2nd, 3rd, and 4th terms in the binomial expansion of $(1 + x)^n$ are ${}^nC_1, {}^nC_2$ and ${}^nC_3$, respectively.

13. Fill in the blanks:

If ${}^nC_8 = {}^nC_6$, then ${}^nC_2$ is ________.

14. Fill in the blanks:

A line is parallel to x-axis if all the points on the line have equal ________.

OR

Fill in the blanks:

If the distance between the point (a, 2, 1) and (1, -1, 1) is 5, then a = ________.

15. Fill in the blanks:

The value of limit $\lim\limits_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$ is ________.

OR

Fill in the blanks:

For some constants a and b, the derivative of x^2 - (a + b)x + ab is ________.

16. List all the elements of set {x: x is a vowel in the word EQUATION}.

17. A man has 7 friends. In how many ways, can he invite one or more of them to a party?

18. Find the value of $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$

OR

Solve $x^2 + 3 = 0$

19. Find the range of $f(x) = x^2 + 2, x \in R$.

20. Evaluate $4!-3!$

21. Write the set A = $\{a_n : n \in N, a_{n+1} = 3a_n \text{ and } a_1 = 1\}$ in the roster form.

OR

If X = {a, b, c, d, e, f} and Y = { f , b, d, g, h, k}, then find the sets X - Y and Y - X.

22. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P (not A) (ii) P (not B) and (iii) P (A or B)

23. Find the coefficient of x^6y^3 in the expansion of $(x + 2y)^9$.

24. Prove that the line joining the mid-points of the two sides of a triangle is parallel to the third side.

OR

Obtain the equations of the lines passing through the intersection of lines 4x - 3y - 1 = 0 and 2x - 5y + 3 = 0 and equally inclined to the axes.

25. Check the validity of the statement:

r: 60 is a multiple of 3 or 5.

26. Prove that: tan 3A - tan 2A - tan A = $\tan 3A \cdot \tan 2A \cdot \tan A$.

27. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify

28. Let R be a relation on N defined by R = {(a, b) : a,b $\in$ N and a = b^2}
Are the following true?

    i. (a, a) $\in$ R for all a $\in$ N
    ii. (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R
    iii. (a, b) $\in$ R, (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$ R

    Justify your answer in each ease.

OR

Find the domain of each of the following function:

i. $\frac{1}{\sqrt{x-2}}$
ii. $\sqrt{4-x^2}$

29. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): sin (x + a)

30. Convert the following in the polar form: $\frac{1+3i}{1-2i}$

31. Solve the following system of inequalities graphically: $x-2y \leqslant 3, 3x+4y \geqslant 12$, $x \geqslant 0, y \geqslant 1$

OR

Solve the inequalities graphically in two dimensional plane: x + y < 5

32. Prove by Mathematical Induction that the sum of first n odd natural numbers is n^2.

33. If A + B + C = π, then prove that $\frac{\cos A}{\sin B \cdot \sin C} + \frac{\cos B}{\sin C \cdot \sin A} + \frac{\cos C}{\sin A \cdot \sin B}$ = 2.

OR

Prove that: $\cos 10° \cos 30° \cos 50° \cos 70° = \frac{3}{16}$.

34. The difference between any two consecutive interior angles of a polygon is 5^0. If the smallest angle is 120^0, find the no. of the sides of the polygon.

35. Find the coordinates of the point of intersection of the axis and the directrix of the parabola whose focus is (3, 3) and directrix is $3x - 4y = 2$. Find also the length of the latus-rectum.

OR

Find the equation to the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (- 3,1) and has an eccentricity $\sqrt{\frac{2}{5}}$.

36. Suppose that samples of polythene bags from two manufacturers, A and B, are tested by a prospective buyer for bursting pressure, with the following results:

Bursting Pressure in kg	**Number of bags manufactured by the manufacturer**	
	A	B
5-10	2	9
10-15	9	11
15-20	29	18
20-25	54	32
20-30	11	27
30-35	5	13

Which set of the bags has the highest average bursting pressure? Which has more uniform pressure?

Solution
Section A

1. (a) 50

   Explanation: Let M denote the set of students who had taken mathematics, P the set of students who had taken physics and C the set of students who had taken chemistry. Then we have

   $\mathrm{n}\ (\mathrm{U})\ =\ 175,\ \mathrm{n}(\mathrm{M})\ =\ 100,\ \mathrm{n}(\mathrm{P})\ =\ 70,\ \mathrm{n}(\mathrm{C})\ =\ 40,\ \mathrm{n}(\mathrm{M}\ \cap\ \mathrm{P})\ =\ 30,$ $\mathrm{n}(\mathrm{M}\cap\mathrm{C})\ =\ 38,\ \mathrm{n}(\mathrm{P}\cap\mathrm{C})\ =23,\ \mathrm{n}(\mathrm{M}\cap\mathrm{P}\cap\mathrm{C})\ =18$

   $\therefore$ no. of students who opted for mathematics alone $= n(M \cap P' \cap C')$

   $= \mathrm{n}(\mathrm{M}\cap\mathrm{P'}\cap\mathrm{C'})$

   $= \mathrm{n}\,[\mathrm{M}\cap(\mathrm{P}\cap\mathrm{C})']\ =\ \mathrm{n}(\mathrm{M}) - \mathrm{n}\ (\mathrm{M}\cap(\mathrm{P}\cup\mathrm{C}))$

   $= \mathrm{n}(\mathrm{M})\ - \mathrm{n}\ [(\mathrm{M}\cap\mathrm{P}\ )\cup(\mathrm{M}\cap\mathrm{C})]$

   $= \mathrm{n}(\mathrm{M})\ - [\ \mathrm{n}\ \ (\mathrm{M}\cap\mathrm{P}) + \mathrm{n}((\mathrm{M}\cap\mathrm{C}) - \mathrm{n}\ (\ \mathrm{M}\cap\mathrm{P}\cap\mathrm{C})]$

   $= 100\ -\ [30\ +38-18]$

   $= 100\ -[50]\ =\ 50$

2. (d) C(10, 4)

   Explanation: If there are 10 things and we have to make them into two groups containing 6 things and 4 things respectively, you have to select 6 to form first group, then automatically another group would have formed of 4 remaining things.

   Now 6 things can be selected from 10 things in ${}^{10}C_6$ different ways

   Also we have ${}^{10}C_6 = {}^{10}C_4 \quad [\because {}^nC_r = {}^nC_{n-r}\ ]$

3. (a) 5^{th}

Explanation: We have the general term in the expansion of $(3+2x)^9$ is given by

$T_{r+1} = {}^9C_r \quad (3)^{9-r}(2x)^r$

Now $\frac{T_{r+1}}{T_r} = \frac{9_{C_r}(3)^{9-r}(2x)^r}{{}^9C_{r-1}(3)^{10-r}(2x)^{r-1}}$

$= \frac{9!(3)^{9-r}(2x)^r}{(9-r)!r!} \times \frac{(10-r)!(r-1)!}{9!(3)^{10-r}(2x)^{r-1}}$

$= \frac{(10-r)\cdot 2x}{3r}$

$= \frac{2(10-r)}{3r}$ since gevin x = 1

Now $\frac{T_{r+1}}{T_r} \geq 1$

$\Rightarrow \frac{(20-2r)}{3r} \geq 1$

$\Rightarrow 20 - 2r \geq 3r$

$\Rightarrow r \leq \frac{20}{5}$

$\Rightarrow r \leq 4$

Hence the maximum value of r is 4

Now $T_5 = T_{4+1} = {}^9C_4 \quad (3)^{9-4}(2x)^4 = 126 \times 243 \times 16(x)^4 = 489888$

4. (a) 15

Explanation: Since given there are four alternatives in which one or more are correct,we have to consider the following four cases

The candidate choose 1 correct answer, 2 correct answers,3 correct answers or 4 correct answers.

1 correct answer can be chosen in ${}^4C_1$ ways = 4 ways

2 correct answer can be chosen in ${}^4C_2$ ways= 6 ways

3 correct answers can be chosen in ${}^4C_3$ways = 4 ways

4 correct answers can be chosen in ${}^4C_4$ ways = 1 way

Hence the totalnumber of ways = 4 + 6 + 4+ 1=15ways

5. (c) $A \times (B \cup C)$

Explanation:

$$A \times (B \cup C) = (A \times B) \cup A \times C$$

$$= \{a, b\} \times \{c, d\} \cup \{a, b\} \times \{d, c\}$$

$$= \{(a, c), (a, d), (b, c), (b, d)\} \cup \{(a, d), (a, c), (b, d), (b, c)\}$$

$$= \{(a, c), (a, d), (a, c), (b, c), (b, d), (b, e)\}$$

6. (a) 676

   Explanation: For n = 1 the value is 0. But when n = 2 the value is 676.
7. (c) $\frac{1}{36}$

   Explanation: Since throwing a single die three times is equivalent to throw three dice at a time

   ∴ Sample space = {(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6), ...}

   Here, n(5) = 6^3

   ∴ Required Probability $= \frac{6}{6^3} = \frac{1}{6^2} = \frac{1}{36}$
8. (d) 5

   Explanation:

   we know that general equation of sphere is
   $x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + c = 0$

   since sphere passing through the points (4 ,3 , 0) , (0 , 4 , 3) ,(0 , 5 , 0) and (4 , 0 , 3) so putting these values one by one in given equation , we get

   16+ 9+ 8g + 6f + c=0 =>8g +6f +c = -25 -------------1)

   16 +9 +8f +6h +c=0 =>8f + 6h +c= -25 -----------2)

   25 + 10f +c = 0 -----------------------3)

   16+ 9+ 8g + 6h +c=0 =>8g +6h +c= -25 -------------4)

   by 2) - 4) we have f = g

using this in 1) we have 16f +c = -25 -------5)

Now solving 3) and 5) we get f=0

using value of f we have g= 0 , h= 0 , c= -25

Now radius = $\sqrt{(-g)^2 + (-f) + (-h)^2 - c}$ = $\sqrt{25}$ =5

9. (d) 15/ 36

   Explanation:

   Total possible ways of getting number in B's dice throw greater that A's dice throw are

   {(1,2) , (1,3) , (1,4) , (1,5) , (1,6) , (2,3) , (2,4) , (2,5) , (2,6) , (3,4) , (3,5) , (3,6) , (4,5) , (4,6) , (5,6)} = 15

   Total number of elements in the sample space = 6X6 = 36

   Therefore P(probability of B is greater than A) = 15/36

10. (a) 6

    Explanation: We have

    $(a+b)^n + (a-b)^n$

    = [$^nC_0\, a^n$ + $^nC_1\, a^{n-1}b$ + $^nC_2\, a^{n-2}b^2$ + $^nC_3\, a^{n-3}b_3$ + ... + $^nC_n\, b^n$] + [$^nC_0\, a^n$-$^nC_1\, a^{n-1}b$+$^nC_2\, a^{n-2}b^2$-$^nC_3\, a^{n-3}b^3$+...$(-1)^n$.$^nC_n\, b^n$]

    = 2 [$^nC_0\, a^n$+$^nC_2\, a^{n-2}b^2$ + ...]

    If n is odd we have $(a+b)^n + (a-b)^n$ has number of terms = $\frac{n+1}{2}$
    Here since n = 11 we have number of terms $= \frac{n+1}{2} = \frac{11+1}{2} = 6$

11. ordered pair

12. True

13. 91

14. y, z-coordinates

OR

a = 5 or -3

15. 2

OR

2x - a - b

16. The vowels in the word EQUATION are E, U, A, I, O
Since, the order in which the elements of a set are written doesn't matter, Hence the set is {A, E, I, O, U}

17. A man may invite one of them, two of them, three of them,..., all of them and this can be done in ${}^7c_1$ ${}^7c_2$, ${}^7c_3$,..., ${}^7c_7$ ways.
$\therefore$ Total number of ways
= ${}^7C_1$ + ${}^7C_2$ + ${}^7C_3$ + ${}^7C_4$ + ${}^7C_5$ + ${}^7C_6$ + ${}^7C_7$
= 7 + 21 + 35 + 35 + 21 + 7 + 1 = 127

18. $\sqrt{-25}+3\sqrt{-4}+2\sqrt{-9}$
= $\sqrt{25}\sqrt{-1}+3\sqrt{4}\sqrt{-1}+2\sqrt{9}\sqrt{-1}$
= 5 $\times$ i + 3 $\times$ 2 $\times$ i + 2 $\times$ 3 $\times$ i [$\because$ $\sqrt{-1}$ = i]
= 5i + 6i + 6i = 17i

OR

Here x^2 + 3 = 0 $\Rightarrow x^2=-3 \Rightarrow x=\pm\sqrt{-3}=\pm\sqrt{3}i$

19. Let $f(x)=y=x^2+2 \Rightarrow x^2=y-2$
$\Rightarrow \quad x=\pm\sqrt{y-2} \Rightarrow y \geq 2$
$\therefore$ Range (f) = {y: y $\in$ R and y $\geq$ 2} or $[2,\infty)$

20. $4!-3!=4\times3\times2\times1-3\times2\times1=24-6=18$

21. Given, $a_1=1$ and $a_{n+1}\ =3a_n$, for all n$\in$N
Putting n = 1 in a_{n+1} = $3a_n$, we get

$a_2 = 3a_1 = 3 \times 1 = 3$ [$\because$ $a_1 = 1$]

Putting n = 2 in $a_{n+1} = 3a_n$, we get

$a_3 = 3a_2 = 3 \times 3 = 3^2$ [$\because$ $a_2 = 3$]

Putting n = 3 in $a_{n+1} = 3a_n$, we get

$a_4 = 3a_3 = 3 \times 3^2 = 3^3$ [$\because$ $a_3 = 3$]

Similarly, we obtain

$a_5 = 3a_4 = 3 \times 3^3 = 3^4$,

$a_6 = 3a_5 = 3 \times 3^4 = 3^5$ and so on.

Hence, A = $\{a_1, a_2, a_3, a_4, ...\} = \{1, 3, 3^2, 3^3, 3^4, 3^5, ...\}$

OR

According to the Question,

X = {a, b, c, d, e, f] and Y = {f, b, d, g, h, k)

Now X-Y represents the set of only those elements of Set X which do not belong to Set Y,

$\therefore$ X - Y = {a, b, c, d, e, f} - {f, b, d, g, h, k} = {a, c, e}

Similarly, Y-X represents the set of only those elements of Set Y which do not belong to Set X

$\therefore$ Y - X = {f, b, d, g, h, k} - {a, b, c, d, e, f } = {g, h, k}

22. Given,P(A) = 0.42, P(B) = 0.48 and P(A and B) =$P(A \cap B) = 0.16$

(i) Now, P (not A)$= P(\overline{A}) = 1 - P(A) = 1 - 0.42 = 0.58$

(ii) Now, P (not B) $= P(\overline{B}) = 1 - P(B) = 1 - 0.48 = 0.52$

= 1 - 0.48 = 0.52

(iii) Now,P(A or B) =$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16 = 0.74$

23. In the expansion of $(x + 2y)^9$, the general term is

$T_{r+1} = {}^9C_r x^{9-r} (2y)^r$ [$\because$ for $(x + a)^n$, $T_{r+1} = {}^nC_r x^{n-r} a^r$]

$= {}^9C_r x^{9-r} \times 2^r \times y^r = {}^9C_r \times 2^r \times x^{9-r} \times y^r$ ... (i)

This will contain x^6y^3, if 9 - r = 6 or r = 3.

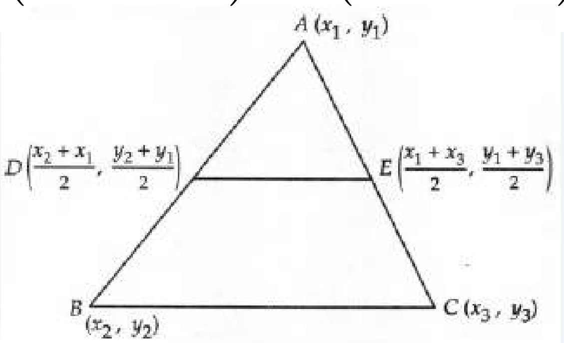
On putting r = 3 in Eq. (i), we get

$T_{3+1} = {}^9C_3 2^3 \times x^{9-3} \times y^3$

$= {}^9C_3 \times 8 \times x^6 \times y^3 = \frac{9!}{3!6!} \times 8 \times x^6 \times y^3$

$\therefore$ Coefficient of $x^6y^3 = \frac{9\times8\times7}{3\times2\times1} \times 8 = 672$

24. Let A (x_1, y_1), B(x_2, y_2) and C (x_3, y_3) be the vertices of a $\triangle$ABC and D and E be the mid-points of sides AB and AC respectively. Then, the coordinates of D and E are $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$ and $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$ respectively.



$\therefore m_1$ = Slope of DE = $\frac{\frac{y_1+y_3}{2} - \frac{y_2+y_1}{2}}{\frac{x_1+x_3}{2} - \frac{x_2+x_1}{2}} = \frac{y_3-y_2}{x_3-x_2}$ and m_2 = Slope of BC = $\frac{y_3-y_2}{x_3-x_2}$

Clearly, $m_1 = m_2$. Hence, DE is parallel to BC.

OR

Let the equation through the intersection of the given lines is

$(4x - 3y - 1) + \lambda(2x - 5y + 3) = 0$

$\Rightarrow \quad x(2\lambda+4) - y(5\lambda+3) + 3\lambda - 1 = 0$ ...(i)

Let m be the slope of this line. Then,

$m = \frac{2\lambda+4}{5\lambda+3}$

As the line is equally inclined with the axes.

$\therefore \quad m = \pm 1$

$\Rightarrow \quad \frac{2\lambda+4}{5\lambda+3} = \pm 1 \Rightarrow \lambda = -1 \text{ or } \frac{1}{3}$

On putting the values of λ in Eq. (i), we get

x + y - 2 = 0 and x = y are the equation of the required lines.

25. The statement is:
"60 is multiple of 3 or 5" is a compound statement of the following statements:
p: 60 is multiple of 3
q: 60 is multiple of 5
Suppose q is false. That is, 60 is not a multiple of 5. Clearly, p is true.
Thus, if we assume that q is false, then p is true.
Hence, the statement is true i.e. the statement "r" is a valid statement.

26. We have, $3A = 2A + A$

Taking tan both sides
$\Rightarrow tan3A = tan(2A + A)$
$\Rightarrow tan3A = \frac{\tan 2A + \tan A}{1 - \tan 2A \cdot \tan A}$
$[\because tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}]$
$\Rightarrow$ tan3 A (1 - $\tan 2A \cdot \tan A) = tan2A + tanA$
$\Rightarrow$ tan3 A - $\tan 3A \cdot \tan 2A \cdot \tan A$ = tan2 A + tan A
$\Rightarrow$ tan3 A - tan2 A - tan A = $\tan 3A \cdot \tan 2A \cdot \tan A$
Hence proved.

27. No, it is not true.
Take A = {1, 2} ad B = {2, 3}
Then $A \cup B = \{1, 2, 3\}$
$P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$
$P(B) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$
$\therefore P(A) \cup P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}\}$ . . (i)
$A \cup B = \{1, 2, 3\}$

$P(A \cup B) = \{\phi\}$,{1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}} . . . (ii)
From (i) and (ii), we have
$P(A \cup B) \neq P(A) \cup P(B)$

28. Given, R = {$(a, b) : a, b \in N$ and $a = b^2$}

   i. If $(a, a) \in$ R, then $a = a^2$. It is true for a = 1 $\in$ N only.
   $\therefore (1, 1) \in$ R but $(2, 2)(3, 3)(4, 4)\ldots$ does not belong to R.
   So, $(a, a) \in$ R for all a $\in$ is not true.

ii. If $(a, b) \in R$, then $a = b^2$ and if $(b, a) \in R$, then $b = a^2$.
Both conditions are not true for all a and b∈N.
[$\because (4, 2) \notin R$ as $4 \neq 2^2$, but (2, 4) $\notin$ R as $2 \neq 4^2$]
So, (a, b) ∈ R ⇒ (b, a) ∈ R is not true.

iii. If $(a, b) \in R$, then $a = b^2$,
if $(b, c) \in R$, then $b = c^2$,
and if $(a, c) \in R$, then $a = c^2$.
These three conditions are not possible at a time.
Since, $(16, 4) \in$ R, then 16 = 4^2
$(4, 2) \in$ R, then 4 = 2^2
whereas (16, 2) $\notin$ R, since, 16 $\notin$ 2^2.
So,$(a, b) \in$ R, (b, c) ∈ R ⇒ (a, c) ∈ R is not true.

OR

i. Given, $f(x) = \frac{1}{\sqrt{x-2}}$
f (x) assumes real values if x - 2 > 0 [here we take only greater than sign, since it is in rational form, so denominator should not be equal to zero]
$\Rightarrow x > 2 \Rightarrow x \in (2, \infty)$.
Hence, the domain of f is (2, ∞).

ii. Given, $f(x) = \sqrt{4 - x^2}$
Clearly, $f(x)$ assumes real values if,
$4 - x^2 \geq 0$
$\Rightarrow -(x^2 - 4) \geq 0$
$\Rightarrow x^2 - 4 \leq 0$ [multiplying by -1 on both sides]
$\Rightarrow (x - 2)(x + 2) \leq 0$ [$\therefore a^2 - b^2 = (a - b)(a + b)$]
⇒ x ∈ $[-2, 2]$
$\therefore$ domain of $f = [-2, 2]$

29. Here f (x) = sin (x + a)
$\therefore$ f'(x) = $\frac{d}{dx}$ [sin (x + a)]
= cos (x + a). $\frac{d}{dx}$ (x + a)
= cos (x + a)

30. $\frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i} = \frac{1+2i+3i+6i^2}{1-4i^2} = \frac{-5+5i}{5} = -1+i$

Let $z = -1 + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -1$ and $r\sin\theta = 1$

Squaring both sides of (i) and adding

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1 \Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

Since $\sin\theta$ is positive and $\cos\theta$ is negative

$\therefore \theta$ lies in second quadrant.

$\therefore \theta = \left(\pi - \frac{\pi}{4}\right) = \frac{3\pi}{4}$

Hence polar form of z is $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

31. The given inequality is $x - 2y \leqslant 3$

Draw the graph of the line x - 2y = 3

Table of values satisfying the equation x - 2y = 3

X	1	5
Y	-1	1

Putting (0, 0) in the given inequation, we have

$0 - 2 \times 0 \leqslant 3 \Rightarrow 0 \leqslant 3$, which is true.

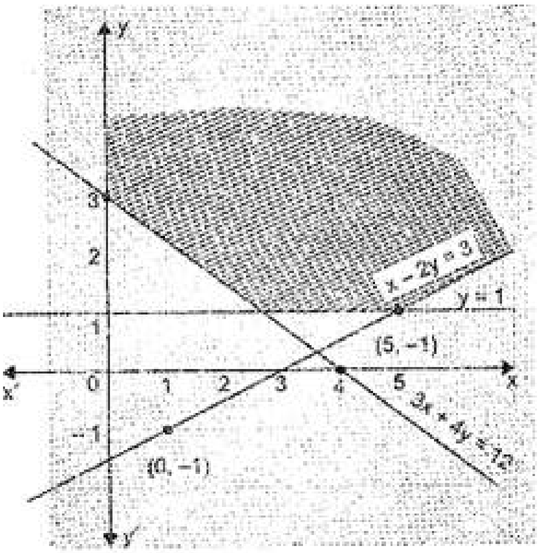
$\therefore$ Half plane of $x - 2y \leqslant 3$ is towards origin.

Also the given inequality is $3x + 4y \geqslant 12$

Draw the graph of the line 3x + 4y = 12

Table of values satisfying the equation 3x + 4y = 12

X	4	0
Y	0	3



Putting (0, 0) in the given inequation, we have

$3 \times 0 + 4 \times 0 \geqslant 12 \Rightarrow 0 \geqslant 12$, which is false.

$\therefore$ Half plane of $3x + 4y \geqslant 2$ is away from origin.

The given inequality is $y \geqslant 1$.

Draw the graph of the line y = 1.

Putting (0, 0) in the given inequation, we have

$0 \geqslant 1$, which is false.

$\therefore$ Half plane of $y \geqslant 1$ is away from origin.

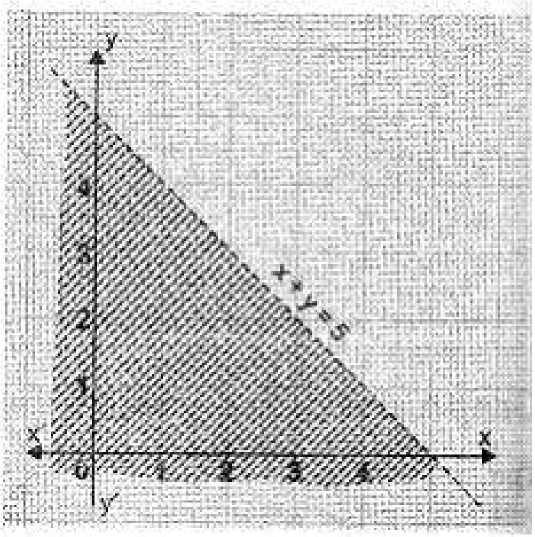
OR

The given inequality is x + y < 5.

Draw the graph of the line x + y = 5

Table of values satisfying the equation x + y = 5

x	1	2
y	4	3



Putting (0, 0) in the given in equation we have 0 + 0 < 5 $\Rightarrow$ 0 < 5 which is true.

$\therefore$ Half plane of x + y < 5 is towards origin.

32. **Step I** Let P(n) denotes the given statement, i.e.,

$P(n): 1+3+5+\ldots n(terms) = n^2$

i.e.,$P(n): 1+3+5+\ldots+(2n-1) = n^2$

Since,

First term $= 2 \times 1 \ -1 \ = 1$

Second term $= 2 \times \ 2 - \ 1 = 3$

Third term $= 2 \times \ 3 - 1 \ = 5$ ......

$\therefore$ n^{th} term$= 2n - 1$

Step II For n = 1, we have

LHS$= 2.1 - 1 = 1$

RHS $= 1^2 = 1 =$LHS

Thus, P(1) is true.

Step III For n = k, let us assume that P(k) is true,

i.e., $P(k): 1+3+5+\ldots+(2k-1) = k^2$ ...(i)

Step IV For n = k + 1, we have to show that P(k + 1) is true, whenever P(k) is true i.e.,

P(k + 1) :$1+3+5+\ldots+(2k-1)+[2(k+1)-1] = (k+1)^2$

LHS $= 1+3+5+\ldots+(2k-1)+[2(k+1)-1]$

$= k^2 + 2(k+1) - 1$ [from Eq. (i)]

$= k^2 \ + 2k + 1 = (k+1)^2 \ =$ RHS

So, P(k + 1) is true, whenever, P(k) is true.

Hence, by Principle of Mathematical Induction, P(n) is true for all $n \in N$.

33. Given, $A+B+C=\pi$

$\Rightarrow A=\pi-(B+C)$ ...(i)

Now, $\frac{\cos A}{sin B \cdot \sin C}=\frac{\cos [\pi-(B+C)]}{\sin B \cdot \sin C}$

= $\frac{-\cos (B+C)}{\sin B \cdot \sin C}$ [$\because \cos(\pi-\theta)$ = - cos θ]

= $\frac{-[\cos B \cdot \cos C-\sin B \cdot \sin C]}{\sin B \cdot \sin C}$

= - [cot B cot C - 1]

$\therefore \frac{\cos A}{\sin B \cdot \sin C}=1-cot\ B\ \times\ cot\ C$ ... (ii)

Similarly, $\frac{\cos B}{\sin C \cdot \sin A}=1-cot\ A\ \times\ cot\ C$ ...(iii)

and $\frac{\cos C}{\sin A \cdot \sin B}=1-cot\ A\ \times\ cot\ B$ ...(iv)

On adding Eqs. (ii), (iii) and (iv), we get

$\frac{\cos A}{\sin B \cdot \sin C}+\frac{\cos B}{\sin C \cdot \sin A}+\frac{\cos C}{\sin A \cdot \sin B}$

$=3-(cot\ B\ \times\ cot\ C+cot\ A\ \times\ cot\ C+cot\ A\ \times\ cot\ B)$ ...(v)

But cot (A + B) = $\frac{\cot A \cdot \cot B-1}{\cot B+\cot A}$

$\Rightarrow$ cot (π - C) = $\frac{\cot A \cdot \cot B-1}{\cot B+\cot A}$

[$\because A+B+C=\pi$]

[$\therefore$ A + B = π - C]

$\Rightarrow$ - cot C = $\frac{\cot A \cdot \cot B-1}{\cot B+\cot A}$ [$\because \cot(\pi-\theta)$ = - cot θ]

$\Rightarrow$ - cot C [cot B + cot A] = cot A cot B - 1

$\Rightarrow$ - (cot B cot C + cot C cot A) - cot A cot B = - 1

$\Rightarrow cot\ B\ \times\ cot\ C+cot\ C\ \times\ cot\ A+cot\ A\ \times\ cot\ B=1$ ...(vi)

On putting (vi) value in Eq. (v), we get,

$\frac{\cos A}{\sin B \cdot \sin C}+\frac{\cos B}{\sin C \cdot \sin A}+\frac{\cos C}{\sin A \cdot \sin B}=3-1=2$

Hence proved.

OR

cos 10° cos 30° cos 50° cos 70° = $\frac{3}{16}$

LHS = cos 10° cos 30° cos 50° cos 70°

= cos 30° cos 10° cos 50° cos 70°

$=\frac{\sqrt{3}}{2}$ (cos 10° cos 50° cos 70°)

$=\frac{\sqrt{3}}{2}$ (cos 10° cos 50°) cos 70°

$=\frac{\sqrt{3}}{4}$ (2 cos 10° cos 50°) cos 70° [Multiplying and dividing by 2]

$= \frac{\sqrt{3}}{4}$ cos 70° {cos (50° + 10°) + cos (10° - 50°)} [Using 2 cos A cos B = cos (A + B) + cos (A - B)]

$= \frac{\sqrt{3}}{4}$ cos 70° {cos 60° + cos (-40°)}

$= \frac{\sqrt{3}}{4}$ cos 70° [$\frac{1}{2}$ + cos 40°] [∵ cos 60° = $\frac{1}{2}$ and cos (-x) = cos x]

$= \frac{\sqrt{3}}{8}$ cos 70° + $\frac{\sqrt{3}}{4}$ cos 70° cos 40°

$= \frac{\sqrt{3}}{8}$ cos 70° + $\frac{\sqrt{3}}{8}$ (2 cos 70° cos 40°)

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos (70° + 40°) + cos (70° - 40°)]

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos 110° + cos 30°]

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos (180° - 70°) + $\frac{\sqrt{3}}{2}$] [∵ cos 30° = $\frac{\sqrt{3}}{2}$]

$= \frac{\sqrt{3}}{8}$ [cos 70° - cos 70° + $\frac{\sqrt{3}}{2}$] [∵ cos (180° - x) = - cos x]

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

= RHS

Hence proved.

34. a = 120^0, d = 5^0

$s_n = \frac{n}{2}[2a + (n-1)d]$

$s_n = \frac{n}{2}[2 \times 120 + (n-1)5]$

$= \frac{n}{2}[240 + 5n - 5]$

$= \frac{n}{2}[235 + 5n]$

Also

sum of interior angles of a polygon with n sides = (2n - 4) $\times$ 90

ATQ $\frac{n}{2}[235 + 5n] = (2n - 4) \times 90$

n = 9, 16

but n = 16 not possible

n = 9

$s_n = \frac{n}{2}[2a + (n-1)d]$

$s_n = \frac{n}{2}[2 \times 120 + (n-1)5]$

$= \frac{n}{2}[240 + 5n - 5]$

$= \frac{n}{2}[235 + 5n]$

Also

Sum of interior angle of polygon with n sides $= (2\mathrm{n} - 4) \times 90$

35. The axis of the parabola is a line $\perp$ to the directrix and passing through focus. The equation of a line $\perp$ to 3x - 4y - 2 = 0 is

y = $\frac{-4}{3}$ + λ [$\because$ m_1m_2= −1 $\Rightarrow$ m_2 = $\frac{-1}{m_1}$ and y = m_2x + λ]

$\Rightarrow$ 3y + 4x = 3λ

This will pass through focus (3, 3) if,

3 $\times$ 3 + 4 $\times$ 3 = 3λ

$\Rightarrow$ 9 + 12 = 3λ

$\Rightarrow$ 21 = 3λ

$\Rightarrow$ λ= $\frac{21}{3}$ = 7

So, the eqaution of axis is 3y + 4x = 3 $\times$ 7 = 21

$\Rightarrow$ 3y + 4x = 21 ....(i)

And the eqaution of directrix is

3x - 4y = 2 ....(ii)

Mutiplying equation (i) by 4 and equation (ii) by 3, we get

16x + 12y = 84 ....(iii)

9x - 12y = 6 ....(iv)

Adding equation (iii) and (iv), we get

16x + 9x = 84 + 6

$\Rightarrow$ 25x = 90

$\Rightarrow$ x = $\frac{90}{25} = \frac{18}{5}$

Putting x = $\frac{18}{5}$ in equation (i), we get

3y + 4 $\times$ $\frac{18}{5}$ = 21

$\Rightarrow$ 3y + $\frac{72}{5}$ = 21

$\Rightarrow$ 3y = 21 - $\frac{72}{5}$

$\Rightarrow$ 3y = $\frac{105-72}{5}$

$\Rightarrow$ 3y = $\frac{33}{5}$

$\Rightarrow$ y = $\frac{11}{5}$

Hence, the required point of intersection is $\left(\frac{18}{5}, \frac{11}{5}\right)$.

OR

Let the equation of the required ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = 1 ....(i)

$\therefore$ e = $\sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow \sqrt{\frac{2}{5}} = \sqrt{1-\frac{b^2}{a^2}}$ [$\because$ eccentricity = $\sqrt{\frac{2}{5}}$]

$\Rightarrow \frac{2}{5} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = 1 - \frac{2}{5}$

$\Rightarrow \frac{b^2}{a^2} = \frac{3}{5}$

$\Rightarrow$ 5b^2 = 3a^2

$\Rightarrow$ b^2 = $\frac{3a^2}{5}$ ....(ii)

Putting the value of b^2 = $\frac{3a^2}{5}$ in equation (i), and as it passes through the point (- 3,1), we get

$\frac{9}{a^2} + \frac{1}{\frac{3a^2}{5}}$ = 1

$\Rightarrow \frac{9}{a^2} + \frac{5}{3a^2}$ = 1

$\Rightarrow \frac{1}{a^2}\left[9+\frac{5}{3}\right]$ = 1

$\Rightarrow$ 9 + $\frac{5}{3}$ = a^2

$\Rightarrow$ a^2 = $\frac{32}{3}$ .....(iii)

Putting a^2 = $\frac{32}{3}$ in equation (ii), we get

b^2 = $\frac{3}{5} \times \frac{32}{3} = \frac{32}{5}$ ....(iv)

$\therefore$ The required equation of ellipse is

$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}}$ = 1

$\Rightarrow \frac{3x^2}{32} + \frac{5y^2}{32}$ = 1

$\Rightarrow$ 3x^2 + 5y^2 = 32.

This is the required equation of ellipse.

36. For determining the set of bags having higher average bursting pressure, we compute mean and for finding outset of bags having more uniform pressure we compute the coefficient of variation.

Manufacturer A:

For Computation of mean and standard deviation we need to make the following table:

Bursting pressure	Mid-values x_i	f_i	$u_i = \frac{x_i - 17.5}{5}$	u_i^2	$f_i u_i$	$f_i u_i^2$
5−10	7.5	2	−2	4	−4	8
10−15	12.5	9	-1	1	-9	9
15−20	17.5	29	0	0	0	0
20−25	22.5	54	1	1	54	54
25−30	27.5	11	2	4	22	44
30−35	32.5	5	3	9	15	45
		$N = \Sigma f_i$ =110	$\Sigma u_i = 3$	$\sum u_i^2 = 19$	$\Sigma f_i u_i$ =78	$\Sigma f_i u_i^2$ =160

$N = \sum f_i = 110$, $\sum f_i u_i = 78$, $\sum f_i u_i^2 = 160$

$$\overline{X}_A = a + h\left(\frac{\Sigma f_i u_i}{N}\right)$$

$\Rightarrow \overline{X}_A = 17.5 + 5 \times \frac{78}{110} = 17.5 + 3.5 = 21$

$\sigma_A^2 = h^2 \left\{\left(\frac{1}{N}\Sigma f_i u_i^2\right) - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right\} = 25 \times \left\{\frac{160}{110} - \left(\frac{78}{110}\right)^2\right\} = 25\left(\frac{17600-6084}{110\times110}\right) =$ 23.79

$\Rightarrow \sigma_A = \sqrt{23.79} = 4.87$

$\therefore$ Coefficient of variation(in percentage) $= \frac{\sigma_A}{\overline{X}_A} \times 100 = \frac{4.87}{21} \times 100 = 23.19\%$

Manufacturer B: To calculate mean and coefficient of variation we need to make the following table,

Bursting pressure	Mid-values x_i	f_i	$u_i = \frac{x_i - 17.5}{5}$	u_i^2	$f_i u_i$	$f_i u_i^2$
5−10	7.5	9	−2	4	−18	36
10−15	12.5	11	−1	1	−11	11
15−20	17.5	18	0	0	0	0

20−25	22.5	32	1	1	32	32
25−30	27.5	27	2	4	54	108
30−35	32.5	13	3	9	39	117
		N = Σ f_i = 110	$\sum u_i$= 3	$\sum u_i^2$ = 19	**Σ f_i u_i = 96**	**Σ f_i u_i^2 = 304**

Here, N = Σf_i = 110, $\Sigma f_i u_i$ = 96, $\Sigma f_i u_i^2$ = 304

$\therefore \overline{X}_B = a + h\left(\frac{\Sigma f_i u_i}{N}\right)$ = 17.5 + 5 $\times \frac{96}{110}$ = 17.5 + 4.36 = 21.86

and $\sigma_B^2 = h^2\left\{\left(\frac{1}{N}\Sigma f_i u_i^2\right) - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right\}$ = 25 $\left\{\frac{304}{110} - \left(\frac{96}{110}\right)^2\right\}$ = 25 $\left(\frac{33440-9216}{110\times110}\right)$ = 50.04

$\Rightarrow \quad \sigma_B = \sqrt{50.04}$ = 7.07

$\therefore$ Coefficient of variation(in percentage) = $\frac{\sigma_B}{\overline{X}_B} \times 100$ = $\frac{7.07}{21.86} \times 100$ = 32.41%

We observe that the average bursting pressure is higher for manufacturer B as manufacturer A has mean of 21 whereas manufacturer B has mean of 21.86. So, bags manufactured by B have higher bursting pressure.

The coefficient of variation is less for manufacturer A, as manufacturer A has C.V of 23.19% whereas the manufacturer B has C.V. of 32.41% . So, bags manufactured by A have more uniform pressure.