CBSE Class 11 Mathematics

Sample Papers 10 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. Let S be the set of all real numbers. Then the relation $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) : 1 + \mathrm{ab} > 0\}$ on S is

    a. reflexive symmetric and transitive

    b. reflexive and symmetric but not transitive

    c. reflexive and transitive but not symmetric

    d. symmetric and transitive but not reflexive

2. In an examination, a candidate is required to pass at least four different subjects out of 6. Then the number of ways in which he can fail is

a. 12

b. 16

c. 24

d. 42

3. Find a if the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal

a. $\frac{8}{5}$

b. $\frac{9}{5}$

c. $\frac{8}{7}$

d. $\frac{9}{7}$

4. Number of divisors of n = 38808 (except 1 and n) is

a. 74

b. 70

c. 68

d. 72

5. In Z, the set of all integers, inverse of – 7 w.r.t. '∗' defined by a∗b = a + b + 7 for all a, b ∈ Z, is

a. - 14

b. 14

c. - 7

d. 7

6. If $x^n - 1$ is divisible by x – k for all n belongs to natural numbers N, then the least positive integral value of k is :

a. 4

b. 3

c. 1

d. 2

7. A sum of money is rounded off to the nearest rupee. The probability that the error occurred in rounding off is at least 15 paise is

a. $\frac{29}{101}$

b. $\frac{29}{100}$

c. $\frac{71}{101}$

d. $\frac{71}{100}$

8. The points A (5 , - 1 , 1) , B (7 , - 4 , 7) , C (1 , - 6 , 10) and D (- 1 , - 3 , 4) are the vertices of

a. square

b. rhombus

c. none of these

d. rectangle

9. Five letters are sent to different persons and addresses on the five envelopes are written at random. The probability that all the letters reach correct destiny is

a. none of these

b. $\frac{44}{120}$

c. $\frac{1}{5}$

d. $\frac{1}{120}$

10. In the expansion of $(1+x)^{60}$, the sum of coefficients of odd powers of x is

a. 2^{58}

b. 2^{60}

c. 2^{61}

d. 2^{59}

11. If f(1 + x) = x^2 + 1, then f(2 - h) is ________.

12. Fill in the blanks:

    The coefficient of x^5 on the expansion $(x + 3)^8$ is ________.

13. Fill in the blanks:

    The continued product of first n natural numbers, is called the ________.

14. Fill in the blanks:

    The plane parallel to yz-plane is perpendicular to ________.

    OR

    Fill in the blanks:

    If the point P lies on z-axis, then coordinates of P are of the form ________.

15. Fill in the blanks:

    The value of the limit: $\lim\limits_{x \to 3} x + 3$ is ________.

    OR

    Fill in the blanks:

    The value of limit $\lim\limits_{x \to 0} \frac{\sin ax}{bx}$ is ________.

16. If A = {3, 5, 7, 9, 11} , B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17} find: $(A \cap B) \cap (B \cup C)$

17. How many natural numbers less than 1000 can be formed with the digits 1,2,3,4 and 5, if repetition of digits is allowed?

18. Find the product of complex numbers (2 + 9i), (11 + 3i).

OR

Express (sin 135^{0} - i cos 135^{0}) in polar form.

19. If U = {1, 2, 3, 4} and R = {(x, y): y > x for all x, y $\in$ U}, then find the domain and range of R.

20. In how many ways, can a cricket team of 11 players be selected out of 16 players, If two particular players are always to be included?

21. In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach physics and mathematics. How many teach physics?

OR

Find AΔB, if A = {1, 3, 4} and B = {2, 5, 9, 11}.

22. Two die are thrown together. What is the probability that the sum of the number on the two faces is either divisible by 3 or by 4?

23. Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$

24. Without using Pythagoras theorem, show that (12, 8), (-2, 6) and (6, 0) are the vertices of right-angled triangle.

OR

Find the slope of a line, which passes through the origin and mid-point of the line segment joining the points P(0, - 4) and B(8, 0).

25. Given below are two statements
p: 25 is a multiple of 5
q: 25 is a multiple of 8
Write the compound. statements connecting these two statements with "and" and

"or". In both cases check the validity of the compound statement.

26. Solve: sin 2x + cos x = 0.

27. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had none of the subjects.

28. Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and f = {(1, 5), (2, 9), (3, 1), (4, 5), (2,11)}. Are the following true?
(i) f is a relation from A to B
(ii) f is a function from A to B. Justify.

OR

Find the domain and the range of the real function f defined by f(x) = |x-1|.

29. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax + b)(cx + d)^2$

30. Solve the equation $25x^2 - 30x + 11 = 0$ by using the general expression for the roots of a quadratic equation and show that the roots are complex conjugate.

31. Solve the following system of inequalities graphically: $x + y \leqslant 9, y > x, x \geqslant 0$

OR

Solve the inequalities graphically in two-dimensional plane: $-3x + 2y \geqslant -6$

32. Use the Principle of Mathematical Induction to prove that $n^3 + 3n^2 + 5n + 3$ is divisible by 3, for all-natural number n.

33. Prove that $\cos^3 A + \cos^3 (120^o + A) + \cos^3 (240^o + A) = \frac{3}{4} \cos 3A$.

OR

If $x \cos\theta = y \cos(\theta + \frac{2\pi}{3}) = z \cos\left(\theta + \frac{4\pi}{3}\right)$, then show that xy + yz + zx = 0.

34. Find the sum to n terms in each of the series $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$

35. Find the equation of the hyperbola whose foci are (6,4) and (- 4,4) and eccentricity is 2.

OR

Find the equation of the ellipse, whose foci are(± 3, 0) and passing through (4, 1).

36. An original frequency table with mean 11 and variance 9.9 was lost but the following table derived from it was found. Construct the original table.

Value of deviation (d)	-2	-1	0	1	2
Frequency (f)	1	6	7	4	2

Solution
Section A

1. (b) reflexive and symmetric but not transitive
   Explanation: Reflexive: since $1 + a^2 > 0 \ \forall \ a \in S$

   $\Rightarrow 1 + a.\ a > 0 \forall\ a \in S \Rightarrow (a,\ a) \in R\ \forall\ a \in$

   $\Rightarrow$ R is reflexive

   Symmetric: Ket a, b $\in$ S Such that (a, b) $\in$ R, then,

   $1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow (b,\ a) \in R$

   $\Rightarrow$ R is symmetric

   Transitivity: Here 1 + 1.2 = 1 + 2 = 3 > 0

   $\Rightarrow 1 + (-2)(-3) = 1 + 6 = 7 > 0$

   $\Rightarrow (1, 2),\ (-2,\ -3) \in R$

   Now, $1 + 1.(-3) = 1 - 3 = -2 < 0$

   $\therefore\ (1,\ -3) \notin R$

   $\therefore$ R is not transition

2. (d) 42
   Explanation: Given that to pass in the exam one has to pass atleast 4 subjects.So he can fail in any of these cases

   He can fail all 6 subjects = ${}^6C_6$ = 1.

   He can fail in 5 subjects = ${}^6C_5$ = 6.

He can fail in 4 subjects = ${}^6C_4$ = 15.

He can fail in 3 subjects = ${}^6C_3$ = 20.

total ways to fail = 42 ways

3. (d) $\frac{9}{7}$

Explanation:
$(3+ax)^9 = {}^9C_0 \quad 3^9 + {}^9C_1 \quad 3^8(ax) + {}^9C_2 \quad 3^7(ax)^2 + {}^9C_3 \quad 3^6(ax)^3 + \ldots\ldots.$
that the coefficient of x^2 = coefficients of x^3
$\Rightarrow {}^9C_2 \quad 3^7a^2 = {}^9C_3 \quad 3^6a^3$
$\Rightarrow \frac{9!}{2!\cdot 7!} \cdot 3 = \frac{9!}{6!\cdot 3!} \cdot a$
$\Rightarrow \frac{3}{7} = \frac{a}{3}$
$\Rightarrow a = \frac{9}{7}$

4. (b) 70 **Explanation:** We have $38808 = 2^3 3^2 7^2 11^1$

To form factors we have to do selections from a lot of 2's,3's,7's and 11's and multiply them together.

Number of ways of selecting any number of 2's from a lot of 3 identical 2's=4(select 0,select 1,select 2,select 3)

Number of ways of selecting any number of 3's from a lot of 2 identical 3's=3(select 0, select 1, select 2)

Number of ways of selecting any number of 7's from a lot of 2 identical 7's=3(select 0, select 1, select 2)

Number of ways of selecting any number of 11's from a lot of 1 identical 11's=2(select 0, select 1)

Hence we get the total number of ways of selecting factors=$4 \times 3 \times 3 \times 2 = 72$

Hence number of factors other than 1 and n=$72 - 2 = 70$

5. (c) - 7

   Explanation: We have, $\mathrm{a} \times \mathrm{b} = \mathrm{a} + \mathrm{b} + 7$

   Let e be an identity element then,

   $\mathrm{a} \times \mathrm{e} = \mathrm{a} = \mathrm{e} \times \mathrm{a}$

   $\Rightarrow \mathrm{a} \times \mathrm{e} = \mathrm{a}$ and $\mathrm{e} \times \mathrm{a} = a$

   $\Rightarrow \mathrm{e} = -7 \in \mathrm{z}$

   $\therefore -7$ is identity elements

   Since inverse of identity elements e is itself

   ∴ inverse of - 7 = - 7

6. (c) 1

   Explanation: since we have x - 1 as a factor of $x^n - 1^n$.

7. (d) $\frac{71}{100}$

   Explanation: The sample space is,

   s = {-0.50, -0.49, -0.48, ..., -0.01, 0.00, 0.01, ... 0.49}

   Let A be the event that the round off eror is at least 15 paise.

   Then A^c is the event that the round off error is at most 14 paise.

   The sample space of A^c is,

   A^c = {-0.14, -0.15, ..., -0.01, 0.00, 0.01, ..., 0.14}

   $\therefore P(A^c) = \frac{29}{100}$

   $\therefore P(A) = 1 - p(A^c) = 1 - \frac{29}{100} = \frac{71}{100}$

8. (b) rhombus

   Explanation: distance between AB=7, BC =7 CD= 7 AND DA=7

   whereas distance betweeen AC and BD IS NOT EQUAL

9. (d) $\frac{1}{120}$ **Explanation:** Let A denotes the event that all the letters reach correct destiny. Then,

n(A) = 1

Also, five letters can be sent of different persons in 5! ways

$\therefore$ Required probability $= P(A) = \frac{1}{5!} = \frac{1}{120}$

10. (d) 2^{59}

Explanation: We have
$(1+x)^n = ^n C_0 + ^n C_1 \quad (x) + ^n C_2 \quad (x)^2 + \ldots\ldots\ldots + (x)^n \quad \ldots\ldots (i)$
Also
$(1-x)^n = ^n C_0 \quad - ^n C_1 \quad (x) + ^n C_2 \quad (x)^2 - \ldots\ldots\ldots + (-1)^n (x)^n \quad \ldots . (ii)$N
$(1-x)^n = [^nc_0 + {^nC_2} (x)^2 + {^nC_4} (x)^4 + \ldots..] - [^nC_1 (x) + {^nC_3} (x)^3 + {^nC_5} (x)^5 + \ldots..]$

Let x = 1 and n = 60
From equation (i), we get $2^{60} = ^{60} C_0 + ^{60} C_1 + ^{60} C_2 + \ldots\ldots\ldots\ldots + ^{60} C_{60}$
$\Rightarrow 2^{60}$ = (sum of coefficients of even powers of x) + (sum of coefficients of odd powers of x) ....(iii)
From equation (ii), we get $0 = \left[^{60}C_0 + ^{60} C_2 + ^{60} C_4 + \ldots\ldots.\right] - \left[^{60}C_1 + ^{60} C_3 + ^{60} C_5 + \ldots\ldots\ldots\ldots\right]$
$\Rightarrow$ 0 = (sum of coefficients of even power of x) - (sum of coefficients of odd powers of x) ...(iv)
Now subtract equation (iv) from equation iii), we get

2^{60} - 0 = 2 (Sum of coefficients of odd powers of x)

$\Rightarrow$ sum of coefficients of odd powers of x = 2^{59}

11. h^2 - 2h + 2

12. 1512

13. 'n factorial'

14. x-axis

OR

(0, 0, z)

15. 6

OR

$\frac{a}{b}$

16. Here A = {3,5,7,9,11}, B = {7,9,11,13}, C = {11,13,15} and D = {15,17}

$$(A \cap B) \cap (B \cup C) =$$
$$(\{3,5,7,9,11\} \cap \{7,9,11,13\}) \cap (\{7,9,11,13\} \cup \{11,13,15\})$$
$$= \{7,9,11\} \cap \{7,9,11,13,15\}$$

= {7, 9, 11}

17. The number less than 1000 means, it will be either 1-digit or 2-digit or 3-digit numbers. When digits are repeated, then

Number of 1-digit numbers = 5^1 = 5

Number of 2-digit numbers = 5^2 = 25

Number of 3-digit numbers = 5^3 =125

Hence, the required number of numbers

= 5 + 25 +125

= 155

18. (2 + 9i).(11 + 3i) = 2 $\times$ 11 + 2 $\times$ 3i +11 $\times$ 9i + 9 $\times$ 3i^2

= 22 + 6i + 99i - 27 [$\because$ i^2 = - 1]

= - 5 + 105i

OR

$$sin135^o - i\ cos135^o$$
$$= sin(90^o\ + 45^o) - i\ cos(90^o\ + 45^o)$$

= cos 45^o + i sin 45^o

= 1 (cos 45^o + i sin 45^o)

19. The elements of R will be the pairs (x, y) satisfying y > x and x, y $\in$ U.
$\therefore$ R = $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$
Then, domain of R = {1, 2, 3} and range of R = $\{2,3,4\}$

20. When two players are always to be included, then 9 more players are to be selected out of the remaining 14 players, which can be done in ${}^{14}C_9$ ways
$= \frac{14!}{9!5!} = \frac{14\times13\times12\times11\times10}{5\times4\times3\times2\times1}$
= 2002 ways

21. Let n(P) denote the number of teachers who teach Physics and n(M) denote the number of teachers who teach mathematics.
We have,
n(P$\cup$M) = 20, n(M) = 12 and n(P$\cap$M) = 4
To find : n(P)
We know that
n(P$\cup$M) = n(P) + n(M) - n(P$\cap$M)
$\Rightarrow$ 20 = n(P) + 12 - 4
$\Rightarrow$ 20 = n(P) + 8
$\Rightarrow$ n(P) = 20 - 8
= 12
$\therefore$ 12 teachers teach Physics.

OR

We know that AΔB represents the symmetric difference between sets A and B.

That is, AΔB = (A - B) $\cup$ (B -A)

According to the Question,
A = {1, 3, 4} and B = {2, 5, 9, 11}

Then, (A - B) = {1, 3, 4} - {2, 5, 9, 11} = {1, 3, 4}

and (B - A) = {2, 5, 9, 11} - {1, 3, 4} = {2, 5, 9, 11}

Hence,

AΔB = (A - B) $\cup$ (B - A)

= {1, 3, 4} $\cup$ {2, 5, 9, 11}

= {1, 2, 3, 4, 5, 9, 11}

22. Total number of events = 36

A = the sum of the numbers on two faces is divisible by 3

= {(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)}

B = the sum of the number on two faces is divisible by 4

= {(2, 2), (2, 6), (6, 2), (1, 3), (3, 1), (4, 4), (3, 5), (5, 3), (6, 6)}

$\therefore\ P(A) = \frac{12}{36} = \frac{1}{3},\ P(B) = \frac{9}{36} = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{4} - \frac{1}{36} = \frac{12+9-1}{36} = \frac{20}{36} = \frac{5}{9}$

23. The general term of $\left(3x - \frac{2}{x^2}\right)^{15}$ is

$T_{r+1} = {}^{15}C_r\, (3x)^{15-r} \left(\frac{-2}{x^2}\right)^r$

$= {}^{15}C_r\, (3)^{15-r}\, (-2)^r\, x^{15-3r}$

For term independent term of x, put 15 - 3r = 0 $\Rightarrow$ r = 5

$\therefore$ The term independent of x = ${}^{15}C_5\, (3)^{15-5}\, (-2)^5 = -3003\, (3^{10})\, (2^5)$

24. Let A(12, 8), B(-2, 6) and C(6, 0) are vertices of ΔABC

Slope of $AB = \frac{6-8}{-2-12} = \frac{-2}{-14} = \frac{1}{7}$ $\left[\text{slope } = \frac{y_2-y_1}{x_2-x_1}\right]$

Slope of $BC = \frac{0-6}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Sope of $AC = \frac{0-8}{6-12} = \frac{-8}{-6} = \frac{4}{3}$

Now, slope of BC $\times$ slope of $AC = \frac{-3}{4} \times \frac{4}{3} = -1$

$\therefore \quad BC \perp AC$

Hence, ABC is right-angle triangle, right-angled at C.

OR

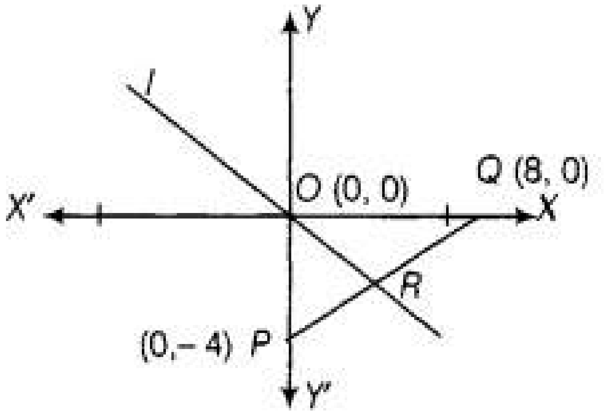
If two points are given, then slope m = $\frac{y_2-y_1}{x_2-x_1}$

Given points are P (0, -4) and Q (8, 0).

$\therefore$ $x_1 = 0$, $y_1 = -4$, $x_2 = 8$, $y_2 = 0$

These points plotted in XY - plane are given below.

Mid-point of PQ is R



$R=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)=(4,-2)$

$\therefore$ Slope of $OR=\frac{y_2-y_1}{x_2-x_1}=\frac{-2-0}{4-0}=\frac{-2}{4}=-\frac{1}{2}$

$[\because x_1=0, y_1=0,\ x_2=4, y_2=-2]$

25. The compound statement with "and " is

25 is a multiple of 5 and 8.

Since p is true and q is false, so the compound statement with "and' is not true

Thus the statement "p and q" is not valid.

Now the compound statement with 'or' is

25 is a multiple of 5 or 8.

Since p is true and q is false, so the compound statement with "or" is true

Thus the statement "p or q" is valid.

26. $sin\ 2x+cos\ x=0$

$\Rightarrow cos\ x=-sin\ 2x$

$\Rightarrow \quad \cos x=\cos\left(\frac{\pi}{2}+2x\right)$

$\Rightarrow \quad x=2n\pi\pm\left(\frac{\pi}{2}+2x\right), n\in Z$

Taking positive sign, we have

$x=2n\pi+\frac{\pi}{2}+2x$

$\Rightarrow \quad -x=2n\pi+\frac{\pi}{2}, n\in Z$

$\Rightarrow \quad x=-2n\pi-\frac{\pi}{2}, n\in Z$

$\Rightarrow \quad x=2m\pi-\frac{\pi}{2}$, where $m=-n\in Z.$

Taking negative sign, we have

$x=2n\pi-\left(\frac{\pi}{2}+2x\right)\Rightarrow 3x=2n\pi-\frac{\pi}{2}\Rightarrow x=\frac{2n\pi}{3}-\frac{\pi}{6}, n\in Z.$

Hence, $x=2m\pi-\frac{\pi}{2}, or, x=\frac{2n\pi}{3}-\frac{\pi}{6}$, where $m,n\in Z.$

27. Let M be the set of students who had taken mathematics, P be the set of students who had taken physics and C be the set of students who had taken chemistry.

Here n(U) = 25, n(M) = 15, n(P) = 12, n(C) = 11, $n(M \cap C) = 5$,

$n(M \cap P) = 9$, $n(P \cap C) = 4$, $n(M \cap P \cap C) = 3$,

From the Venn diagram, we have

n(M) = a + b + d + e = 15

n(P) = b + c + e + f = 12

n(C) = d + e + f + g = 11

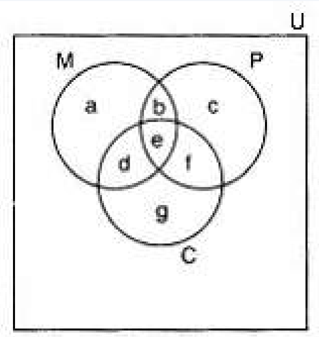
$n(M \cap C) = d + e = 5,$

$n(M \cap P) = b + e = 9,$

$n(P \cap C) = e + f = 4$

$n(M \cap P \cap C) = e = 3$

Now e = 3



$d + e = 5 \Rightarrow d + 3 = 5 \Rightarrow d = 5 - 3 \Rightarrow d = 2$

b +e = 9$\Rightarrow$b + 3 = 9 $\Rightarrow$b = 9 - 3 $\Rightarrow$ b = 6

e + f = 4 $\Rightarrow$ 3 + f = 4 $\Rightarrow$ f = 4 - 3 $\Rightarrow$ f = 1

a +b + d + e = 15 $\Rightarrow$ a + 6 + 2 + 3 = 15 $\Rightarrow$ a = 15 - 14 = 4

b + c + e + f = 12 $\Rightarrow$ 6 + c + 3 + 1 = 12 $\Rightarrow$ c = 12 - 10 = 2

d + e + f + g = 11 $\Rightarrow$ 2 + 3+ 1+ g = 11 $\Rightarrow$g = 11 - 6 = 5

$\therefore$ 25 - (a + b + c + d + e + f + g) = 25 -23 = 2

28. (i) Here A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

$\therefore A \times B$= {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}

f = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

Now (1, 5), (2, 9), (3, 1), (4, 5), (2, 11) $\in A \times B$

$\therefore$ f is a relation from A to B.

(ii) Here f (2) = 9 and f (2) = 11

∴ f is not a function from A to B.

OR

Here f (x) = |x - 1|

The function f (x) is defined for all values of x

∴ Domain of f(x) = R

when x > 1

|x - 1| = x - 1>0

When x = 1

|x - 1| = 0

When x < 1

|x - 1| = - x + 1 > 0

Range of f(x) = all real numbers ≥ 0

$$= [0, \infty)$$

29. Here f(x) = (ax + b) (cx + d)^2

$\therefore \; f'(x) = \frac{d}{dx}[(ax+b)(cx+d)^2]$

= (ax + b) $\frac{d}{dx}(cx+d)^2 + (cx+d)^2 \cdot \frac{d}{dx}(ax+b)$

= (ax + b) × 2(cx + d) × c + (cx + d)^2 × a

= 2c(ax + b)(cx + d) + a(cx + d)^2

30. Give, 25x^2 - 30x + 11 = 0 ...(i)

On comparing Eq. (i) with ax^2 + bx + c = 0, we get

a = 25, b = - 30 and c = 11. Therefore the two roots are,

$\because \alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$

and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

$\therefore \alpha = \frac{30+\sqrt{(-30)^2 - 4\times25\times11}}{2\times25}$

$\Rightarrow \alpha = \frac{30+\sqrt{900-1100}}{50}$

$\Rightarrow \alpha = \frac{30+\sqrt{-200}}{50} \Rightarrow \alpha = \frac{30+10i\sqrt{2}}{50}$

and $\beta = \frac{30-\sqrt{(-30)^2-4\times25\times11}}{2\times25}$

$\Rightarrow \beta = \frac{30-\sqrt{900-1100}}{50}$

$\Rightarrow \beta = \frac{30-\sqrt{-200}}{50} \Rightarrow \beta = \frac{30-10i\sqrt{2}}{50}$

$\therefore \alpha = \frac{3}{5} + \frac{\sqrt{2}}{5}i, \beta = \frac{3}{5} - \frac{\sqrt{2}}{5}i$

Hence, the roots are complex conjugate.

31. The given inequality is $x + y \leqslant 9$

Draw the graph of the line x + y = 9

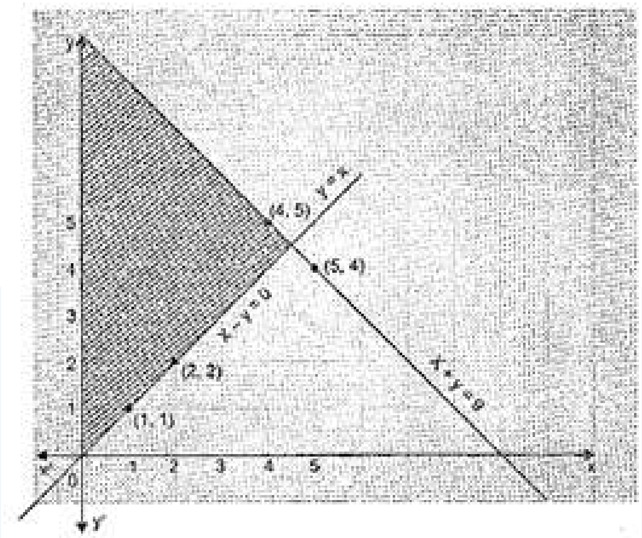


Table of values satisfying the equation x + y = 9

X	5	4
Y	4	5

Putting (0, 0) in the given inequation, we have

$0 + 0 \leqslant 9 \Rightarrow 0 \leqslant 9$, which is true.

$\therefore$ Half plane of $x + y \leqslant 9$ is towards origin.

Also the given inequality is x - y < 0

Draw the graph of the line x - y = 0

Table of values satisfying the equation x - y = 0

X	1	2
Y	1	2

Putting (0, 3) in the given inequation we have

$0 - 3 < 0 \Rightarrow -3 < 0$, which is true.

$\therefore$ Half plane of x - y < 0 containing the point (0, 3).

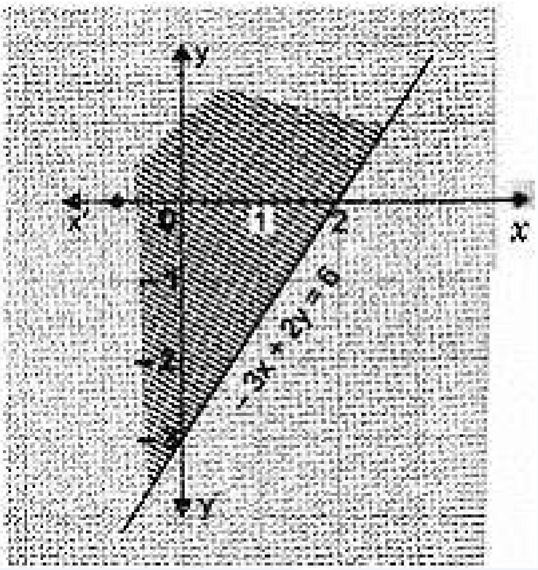
OR

The given inequality is $-3x + 2y \geqslant -6$.

Draw the graph of the line -3x + 2y = -6

Table of values satisfying the equation -3x + 2y = -6

X	2	0
Y	0	-3



Putting (0, 0) in the given in equation, we have

$-3 \times 0 + 2 \times 0 \geqslant -6 \Rightarrow 0 \geqslant -6$ which is true.

$\therefore$ Half plane of $-3x + 2y \geqslant -6$ is towards origin.

32. **Step I** Let P(n) : $n^3 + 3n^2 + 5n + 3$ is divisible by 3, be the given statement.

Step II For n = 1, we have

$P(1) : 1^3 + 3.1^2 + 5.1 + 3 = 1 + 3 + 5 + 3 = 12$

which is divisible by 3. Thus, P(1) is true.

Step III For n = k, let us assume that P(k) is true, i.e., P(k): $k^3 + 3k^2 + 5k + 3$ is divisible by 3.

Then, $k^3 + 3k^2 + 5k + 3 = 3m$ for some $m \in N$ ...(i)

Step IV For n = k + 1, we have to show that P(k + 1) is true, whenever P(k) is true, i.e.,

$P(k+1) : (k+1)^3 + 3(k+1)^2 + 5(k+1) + 3$ is divisible by 3.

Now, consider $(k+1)^3 + 3(k+1)^2 + 5(k+1) + 3$

$= (k^3 + 3k^2 + 3k + 1) + 3(k^2 + 2k + 1) + 5k + 5 + 3$

$= (k^3 + 3k^2 + 3k + 1) + (3k^2 + 6k + 3) + 5k + 8$

$= (k^3 + 3k^2 + 5k + 3) + (3k^2 + 9k + 9)$

= 3m + 3(k^2 + 3k + 3) [from Eq. (i)]

= 3 [m + (k^2 + 3k + 3)]

which is a multiple of 3.

So P(k + 1) is true, whenever P(k) is true.

Hence, by Principle of Mathematical Induction, P(n) is true for all $n \in N$.

33. $cos^3 A + cos^3(120^o + A) + cos^3(240^o + A) = \frac{3}{4}\ cos3A$

We know, $cos3A = 4cos^3A - 3cosA$

$\Rightarrow cos^3$A = $\frac{3}{4}\ cosA +\ \frac{1}{4}\ cos3A$

Using this, we get

LHS $=\ cos^3 A +\ cos^3\ (120^o + A) + cos^3(240^o + A)$

= [$\frac{3}{4}\ cosA+\ \frac{1}{4}$ cos 3A] + [$\frac{3}{4}$ cos (120^0 + A) + $\frac{1}{4}$ cos 3 (120^0 + A)] + [$\frac{3}{4}$ cos (240^0 + A) + $\frac{1}{4}$ cos 3 (240^0 + A)]

= [$\frac{3}{4}$ cos A + $\frac{1}{4}$ cos 3A] + [$\frac{3}{4}$ cos (120^0 + A) + $\frac{1}{4}$ cos (360^0 + 3A)] + [$\frac{3}{4}$ $cos(240^o + A) +\ \frac{1}{4}\ cos(720^o + 3A)$]

= $\frac{3}{4}$ [cos A + cos (120^0 + A) + cos (240^0 + A)] + $\frac{1}{4}$ [cos 3A + cos (2π + 3A) + cos (2 $\times$ 2π + 3A)]

= $\frac{3}{4}$ [cos A + cos (120^0 + A) + cos (240^0 + A)] + $\frac{1}{4}$ [cos 3A + cos 3A + cos 3A]

[$\because$ cos (2n $\pi + \theta$) = cos θ]

= $\frac{3}{4}$ [cos A + cos (120^0 + A) + cos (240^0 + A)] + $\frac{1}{4}$ $\times$ 3 (cos 3A)]

= $\frac{3}{4}$ [cos A + 2 cos $\left(\frac{360^\circ + 2A}{2}\right)$ $\times$ cos $\left(-\frac{120^\circ}{2}\right)$] + $\frac{3}{4}$ cos 3A

[$\because cos\ x + cos\ y = 2cos\ \left(\frac{x+y}{2}\right) \times cos\ \left(\frac{x-y}{2}\right)$]

= $\frac{3}{4}$ [cos A + 2 (180^0 + A) $\times$ cos (- 60^0)] + $\frac{3}{4}$ cos 3A

= $\frac{3}{4}$ [cos A + 2 (- cos A) $\times$ cos 60^0] + $\frac{3}{4}$ cos 3A [using cos (180^0 + θ) = - cos θ]

= $\frac{3}{4}$ [cos A - 2 cos A $\times$ $\frac{1}{2}$] + $\frac{3}{4}$ cos 3A

= $\frac{3}{4}\ [cosA - cosA] +\ \frac{3}{4}\ cos3A$

= $\frac{3}{4}\ cos3A$

= **RHS**

$\therefore$ **LHS** = **RHS**

Hence proved.

OR

Suppose, $xcos\ \theta = ycos$ (θ + $\frac{2\pi}{3}$)$= zcos\ \left(\theta + \frac{4\pi}{3}\right) = k$

$\Rightarrow \frac{1}{x} = \frac{\cos\theta}{k}, \frac{1}{y} = \frac{\cos\left(\theta+\frac{2\pi}{3}\right)}{k}, \frac{1}{z} = \frac{\cos\left(\theta+\frac{4\pi}{3}\right)}{k}$ ... (i)

Now, **LHS** $= xy + yz + zx$

= xy $\left(\frac{z}{z}\right)$ + yz $\left(\frac{x}{x}\right)$ + zx $\left(\frac{y}{y}\right)$ = $\frac{xyz}{z} + \frac{xyz}{x} + \frac{xyz}{y}$

= xyz $\left[\frac{1}{z} + \frac{1}{x} + \frac{1}{y}\right]$

= xyz $\left[\frac{\cos\left(\theta+\frac{4\pi}{3}\right)}{k} + \frac{\cos\theta}{k} + \frac{\cos\left(\theta+\frac{2\pi}{3}\right)}{k}\right]$ [using Eq. (i)]

= $\frac{xyz}{k}\left[\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\theta\right]$

= $\frac{xzz}{k}\left[2\cos\left(\frac{\theta+\frac{4\pi}{3}+\theta+\frac{2\pi}{3}}{2}\right)\cos\left(\frac{\theta+\frac{4\pi}{3}-\theta-\frac{2\pi}{3}}{2}\right) + \cos\theta\right]$[$\because cos\ x + cos\ y = 2cos$ $\left(\frac{x+y}{2}\right)\ cos\ \left(\frac{x-y}{2}\right)$]

= $\frac{xyz}{k}\left[2\cos\left(\frac{2\theta+2\pi}{2}\right)\cdot\cos\frac{2\pi}{6} + \cos\theta\right]$

= $\frac{xyz}{k}\left[2\cos(\pi+\theta)\cdot\cos\frac{2\pi}{6} + \cos\theta\right]$

= $\frac{xyz}{k}\left[-2\cos\theta\cdot\cos\frac{\pi}{3} + \cos\theta\right]$

= $\frac{xyz}{k}\left[-2\cos\theta\cdot\left(\frac{1}{2}\right) + \cos\theta\right]$

= $\frac{xyz}{k}\left[-\cos\theta + \cos\theta\right]$

$= 0 =$ **RHS**

$\therefore$ **LHS** = **RHS**

Hence proved.

34. Given: $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$ to n terms

$\therefore$ a_n = [n^{th} term of 3, 5, 7, ...][n^{th} term of 1, 2, 3, 4, ...]^2

= (2n + 1) $(n)^2$ = $2n^3$ + n^2 = $2n^3 + n^2$

$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (2k^3 + k^2)$

= $[2.1^3 + 1^2] + [2.2^3 + 2^2] + [2.3^3 + 3^2] + \ldots\ldots + [2n^3 + n^2]$

= $2(1^3 + 2^3 + 3^3 + \ldots\ldots\ldots n^3) + (1^2 + 2^2 + 3^2 + \ldots\ldots n^2)$

$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$

$= 2\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$

$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{2n+1}{3}\right]$

$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+2n+1}{3}\right]$

$= \frac{n(n+1)(3n^2+3n+2n+1)}{6}$

= $\frac{n(n+1)(3n^2+5n+1)}{6}$

35. The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{6-4}{2}, \frac{4+4}{2}\right)$ i.e., (1, 4).

Let 2a and 2b be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is,

$\frac{(x-1)^2}{a^2} - \frac{(y-4)^2}{b^2}$ = 1 ....(i)

Now, the distance between two foci = 2ae

$\Rightarrow \sqrt{(6+4)^2 + (4-4)^2}$ = 2ae [$\because$ Foci = (6, 4) and (-4, 4)]

$\Rightarrow \sqrt{(10)^2}$ = 2ae

$\Rightarrow$ 10 = 2ae

$\Rightarrow$ 2ae = 10

$\Rightarrow$ 2a $\times$ 2 = 10 [$\because$ e = 2]

$\Rightarrow$ a = $\frac{10}{4}$

$\Rightarrow$ a = $\frac{5}{2}$

$\Rightarrow$ a^2 = $\frac{25}{4}$

Now,

$b^2 = a^2 (e^2 - 1)$

$\Rightarrow b^2 = \frac{25}{4} (2^2 - 1)$

$= \frac{25}{4} (4 - 1)$

$= \frac{25}{4} \times 3 = \frac{75}{4}$

Putting $a^2 = \frac{25}{4}$ and $b^2 = \frac{75}{4}$ in equation (i), we get

$\frac{(x-1)^2}{\frac{25}{4}} - \frac{(y-4)^2}{\frac{75}{4}} = 1$

$\Rightarrow \frac{4(x-1)^2}{25} - \frac{4(y-4)^2}{75} = 1$

$\Rightarrow \frac{4\times3(x-1)^2-4(y-4)^2}{75} = 1$

$\Rightarrow 12 (x - 1)^2 - 4(y - 4)^2 = 75$

$\Rightarrow 12[x^2 + 1 - 2x] -4 [y^2 + 16 - 8y] = 75$

$\Rightarrow 12x^2 + 12 - 24x - 4y^2 - 64 + 32y = 75$

$\Rightarrow 12x^2 - 4y^2 - 24x + 32y - 52 - 75 = 0$

$\Rightarrow 12x^2 - 4y^2 - 24x + 32y - 127 = 0$

This is the equation of the required hyperbola.

OR

We have, foci of ellipse at (± 3, 0) which are on X-axis.

Therefore, equation of the ellipse is of the form

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

Its foci are (± ae, 0) = (± 3, 0)

$\therefore$ ae = 3

Now, $b^2 = a^2 (1 - e^2)$

$\Rightarrow a^2 e^2 = a^2 - b^2$

$\Rightarrow 9 = a^2 - b^2$ [$\because$ ae = 3] ...(ii)

Since, Eq. (i) passes through (4, 1)

$\therefore \frac{16}{a^2} + \frac{1}{b^2} = 1$

$\Rightarrow \frac{16}{9+b^2} + \frac{1}{b^2} = 1$ [putting Eq. (ii)]

$\Rightarrow 16 b^2 + 9 + b^2 = b^2 (9 + b^2)$

$\Rightarrow 17 b^2 + 9 = 9 b^2 + b^4$

$\Rightarrow$ $b^4 - 8b^2 - 9 = 0$

$\Rightarrow$ $(b^2 - 9)(b^2 + 1) = 0$

$\Rightarrow$ $b^2 = 9, -1$

But $b^2 \neq -1$

$\therefore$ $b^2 = 9$

From Eq. (ii), we get

$a^2 = 9 + b^2$

$\Rightarrow$ $a^2 = 9 + 9$

$\Rightarrow$ $a^2 = 18$

On putting the values of a^2 and b^2 in Eq. (i), we get

$\frac{x^2}{18} + \frac{y^2}{9} = 1$

$\Rightarrow$ $x^2 + 2y^2 = 18$

This is the required equation of the ellipse

36.

d	f	d^2	fd	fd^2
-2	1	4	-2	4
-1	6	1	-6	6
0	7	0	0	0
1	4	1	4	4
2	2	4	4	8
Total	$\sum f = 20$		$\sum fd = 0$	$\sum fd^2 = 22$

As we know, $\overline{x} = a + h\frac{\sum fd}{\sum f}$

$\Rightarrow 11 = a + h \times \frac{0}{20} \Rightarrow a = 11$

Also, variance, $\sigma^2 = h^2\left[\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right]$

$\Rightarrow 9 = h^2\ [\frac{22}{20}] \Rightarrow h = 3$

Mid value is given by, $d = \frac{x-a}{h} \Rightarrow x = a + dh$

$\therefore$ Different values of x for different values of d are:

$11-2\times 3, 11-1\ \times\ 3, 11-0\times\ 3, 11+1\times\ 3, 11+2\times\ 3$

i.e., $5, 8, 11, 14, 17.$

$\therefore$ The original frequency table is as follows:

Class	3.5-6.5	6.5-9.5	9.5-12.5	12.5-15.5	15.5-18.5
Frequency	1	6	7	4	2