CBSE Class 11 Mathematics
Sample Papers 01 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. If f $f(x)=\frac{1}{\sqrt{x-|x|}}$, then D_f is equal to

   a. ϕ

   b. R

   c. $(0, \infty)$

   d. $(-\infty, 0)$

2. There are 10 true-false questions. The number of ways in which they can be answered is

a. 2^{10}

b. 10

c. none of these

d. $10!$

3. 5th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^{12}$ is

a. $7920x^4$

b. $7920x^{-4}$

c. $-7920x^{-4}$

d. $-7920x^4$

4. Numbers greater than 1000 but not greater than 5000 are to be formed with the digits 0, 1, 2, 3, 5, allowing repetitions, the number of possible numbers is

a. none of these.

b. 365

c. 370

d. 375

5. The domain of the function $f(x)=\sqrt{1-\cos x}$ is

a. none of these

b. R

c. $\{\mathrm{x}=2\mathrm{n}\pi : \mathrm{n}\in \mathrm{I}\}$

d. $[0,2\pi]$

6. The statement P (n) : "$(n+3)^2 > 2^{n+3}$ " is true for :

a. all n ≥ 2

b. no n $\in$ N ,

c. all n ≥ 3

d. all n .

7. The probability that the length of a randomly chosen chord of a circle lies between 2/3 and 5/6 of it's diameter is

a. 1/16

b. 1/4

c. 5/6

d. 5/12

8. The distance between the planes $\vec{r}.\hat{n} = p_1 and\ \vec{r}.(-\hat{n}) = p_2 is$

a. none of these

b. $p_2 - p_1$

c. $|p_1 - p_2|$

d. $p_1 + p_2$

9. A pair of dice is tossed once and a total of 8 has come up. The chance that both the dice show up same number is

a. 1/ 5

b. 1/ 6

c. none of these

d. 5/ 216

10. If the 21 st and 22nd terms in the expansion of $(1+x)^{44}$ are equal, find x

a. $\frac{7}{6}$

b. $\frac{5}{8}$

c. $\frac{7}{8}$

d. $\frac{6}{8}$

11. Fill in the blanks:

    The subset of B containing the images of elements of A is called the ________ of the function.

12. Fill in the blanks:

    The number of terms in the binomial expansion of $(1+\sqrt{5}x)^7 + (1-\sqrt{5}x)^7$ is ________.

13. Fill in the blanks:

    If the letters of the word RACHIT are arranged in all possible ways as listed in the dictionary, then the rank of the word RACHIT is ________

14. Fill in the blanks:

    x = a represent a plane parallel to ________.

OR

Fill in the blanks:

A line is parallel to xy-plane if all the points on the line have equal ________.

15. Fill in the blanks:

    The value of limit $\lim_{x\to 1} \frac{ax^2+bx+c}{cx^2+bx+a}$ where a + b + c $\neq$ 0 is ________.

OR

Fill in the blanks:

The value of limit $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$ is ________.

16. If A = {3, 5, 7, 9, 11} , B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17} find: $A \cap (B \cup D)$

17. Find the total number of 9-digit numbers which have all different digits.

18. Express (5 i) $\left(\frac{-3}{5} i\right)$ in the form of a + ib, where a, n $\in$ R.

OR

Find the difference of the complex numbers (- 4 + 7i), (- 11 - 23i)

19. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd, $\mathbf{x} \in A, y \in B\}$. Write R in roster form.

20. Compute. $\frac{8!}{4!}$, is $\frac{8!}{4!} = 2!$?

21. Show that $A \cap B = A \cap C$ need not imply B = C?

OR

If A and B are two sets such that n(A) = 35, n(B) = 30 and n(U) = 50, then find,

i. the greatest value of n(A ∪ B)
ii. the least value of (A ∩ B)

22. The probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty brakes as well as badly worn tires?

23. Find the 4th term from the end in the expansion of $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$.

24. Show that the lines x - y - 6 = 0, 4x - 3y - 20 = 0 and 6x + 5y + 8 = 0 are concurrent

OR

The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, then find the slope of the lines.

25. Show that the statement "For any real numbers a and, b, $a^2 = b^2$ implies that a = b" is not true by giving a counter example.

26. Prove that: tan 20° tan 40° tan 80° = tan 60°

27. For any two sets A and B prove that: $P(A \cap B) = P(A) \cap P(B)$.

28. Which of the following relations are functions? Give reasons. If it is a function determine its domain and range.

    (i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

    (ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

    (iii) {(1, 3), (1, 5), (2, 5)}

OR

The Cartesian product $A \times A$ has 9 elements among which are found to be (-1, 0) and (0, 1). Find the set A and the remaining element of $A \times A$.

29. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ at x = - 1. Also, prove that f'(0) + 3f'(-1) = 0.

30. Find the multiplicative inverse of the complex numbers $= \sqrt{5} + 3i$

31. Solve the inequalities graphically in two-dimensional plane: x - y $\leq$ 2

OR

Solve the following system of inequalities graphically: $x + y \geqslant 4, 2x - y < 0$

32. Prove the following by using the principle of mathematical induction for all $n \in N$:

    $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \ldots\ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

33. Prove that cot $7\frac{1^\circ}{2}$ = tan $82\frac{1^\circ}{2}$ = $(\sqrt{3} + \sqrt{2})(\sqrt{2} + 1)$

OR

Prove that cos 12^{o} + cos 60^{o} + cos 84^{o} = cos 24^{o} + cos 48^{o}

34. Find the sum of the following series up to n terms: $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots$

35. Find the equation of the ellipse whose foci are (4, 0) and (- 4, 0), eccentricity = 1/3.

OR

Draw the shape of the ellipse $4x^2 + 9y^2 = 36$ and find its major axis, minor axis, value of c, vertices, directrices, foci, eccentricity and length of latusrectum.

36. In a survey of 44 villages of a state, about the use of LPG as a cooking mode, the following information about the families using LPG was obtained.

Number of families	0-10	10-20	20-30	30-40	40-50	50-60
Number of villages	6	8	16	8	4	2

i. Find the mean deviation about median for the following data.
ii. Do you think more awareness was needed for the villagers to use LPG as a mode of cooking?

Solution
Section A

1. (a) ϕ **Explanation:**

   this function is defined only if

   $x-|x|>0$
   $\Rightarrow x>|x|$

   which is not possible because

   $|x| \geq x$

   so, $x \in \phi$

2. (a) 2^{10} **Explanation:**

   you can either choose true or false, therefore for 10 questions you will have 2^{10} possibilities.

3. (b) $7920 x^{-4}$ **Explanation:** We have the general term of $(x+a)^n$ is
   $T_{r+1}={ }^n C_r \quad(x)^{n-r} a^r$
   Now consider $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^{12}$
   Here $T_{r+1}={ }^{12} C_r \quad\left(\frac{x^3}{2}\right)^{12-r}\left(-\frac{2}{x^2}\right)^r$
   We have 5th term from the end = (12-5+2(th term from the beginning\
   Required term is $T_9=T_{8+1}={ }^{12} C_8 \quad\left(\frac{x^3}{2}\right)^{12-8}\left(-\frac{2}{x^2}\right)^8=495 \times 2^{8-4} x^{12-16}=7920 x^{-4}$

4. (d) 375

   Explanation:

th	h	t	o
3	5	5	5

One's place can be occupied by any of the 5 numbers, tens place by any of the 5 numbers and hundreds place in 5 ways since repetition is allowed. But thousands place can be occupied by 2,3,1, only since the required number should be greater than 1000 and less than 5000. Hence total number of arrangement=3x5x5x5=375

5. (b) R

   Explanation: this function exists only if

   $$1 - \cos x \geq 0$$
   $$\Rightarrow \cos x \leq 1$$

   it is possible $\forall x \in R$

6. (b) no n $\in$ N,

   Explanation:

   When n = 1 we get $16 > 16$, which is false. when n = 2 we get$25 > 32$, which is false as well. As n = 3,4,5....the inequalty does not hold correct.

7. (b) 1/4

   Explanation:

   If l is the length of a chord, r, the distance of the mid-point of the chord from the centre of the circle and a radius of the given circle, then

   $r = a \cos \theta, l = 2a \sin \theta$

   Given : $\frac{2}{3} 2a < 2a \sin \theta < \frac{2}{3} 2a$

   $\Rightarrow \frac{5}{6} a < a \cos \theta < \frac{\sqrt{5}}{3} a$

   $\Rightarrow \frac{\sqrt{11}}{6} < r < \frac{\sqrt{5}}{3} a$

   Thus the given condition is satisfied if the mid-point of the chord lies within the region between the concentric circles of radius

   $\frac{\sqrt{11}}{6}$ and $\frac{\sqrt{5}}{3}$

   Hence, required probability $= \frac{\text{The area of the circular annulus}}{\text{Area of the given circle}}$

   $= \left(\frac{5}{9} - \frac{11}{36}\right) = \frac{1}{4}$

8. (d) $p_1 + p_2$ **Explanation:**

P_1 is distance of first plane from origion in the direction of normal vector and P_2 is distance of second plane from origion opposite to normal vector. So distance between planes is $P_1 - (-P_2) \; = \; P_1 + P_2$

9. (a) 1/ 5

   Explanation:

   on tossing a pair of dice total outcomes are 36

   out of which getting a total of 8 have possiblities {(2,6),(6,2),(3,5),(5,3),(4,4)}=5

   and from these 5 outcomes getting same no. on both dice is (4,4)=1

   so, probability is 1/5

10. (c) $\frac{7}{8}$ **Explanation:** We have the general term of $(x+a)^n$ is $T_{r+1} = ^n C_r \quad (x)^{n-r} a^r$
    Now consider $(1+x)^{44}$
    Here $T_{r+1} = ^{44} C_r \quad (1)^{44-r}(x)^r$
    So $T_{21} = T_{20+1} = ^{44} C_{20}(x)^{20}$ and $T_{22} = T_{21+1} = ^{44} C_{21}(x)^{21}$
    Given $T_{21} = T_{22} \Rightarrow ^{44} C_{20}(x)^{20} = ^{44} C_{21}(x)^{21}$
    $\Rightarrow x = \frac{44\mathrm{C}_{20}}{44\mathrm{C}_{21}} = \frac{(44)!}{(20)!24!} \frac{(21)!23!}{(44)!} = \frac{21}{24} = \frac{7}{8}$

11. range

12. 4

13. 481

14. yz-plane

OR

z-coordinates

15. 1

OR

$\frac{11}{4}$

16. Here A = {3,5,7,9,11}, B = {7,9,11,13}, C = {11,13,15} and D = {15,17}

$A \cap (B \cup D) = A \cap (B \cup D) = \{3,5,7,9,11\} \cap (\{7,9,11,13\} \cup \{15,17\})$
$= \{3,5,7,9,11\} \cap \{7,9,11,13,15,17\}$
= {7,9, 11}

17. The total number of 9-digit numbers, having all digits are different = $9 \times {}^9P_8$
$= \frac{9 \times 9!}{1!} = 9 \times 9!$
[∵ in the first place, we select any one of 9 numbers except 0. In rest of the eight places, we select any eight numbers from the remaining 9 numbers]

18. We have, (5i) $\left(\frac{-3}{5}i\right) = [5 \times \frac{(-3)}{5}] \times (i \times i) = -3i^2$
$= -3(-1) = 3 = 3 + 0i$ [∵ i^2 = - 1]

OR

(- 4 + 7i) - (- 11 - 23i) = (- 4 + 7i) + (11 + 23i)
= (- 4 + 11) + (7 + 23) i
= 7 + 30 i

19. Given, A = $\{1, 2, 3, 5\}$ and B = $\{4, 6, 9\}$
R = {(x, y): the difference between x and y is odd, $x \in A, y \in B\}$
In roster form, R = $\{(1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\}$

20. We have,
$\frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$ $[\because n! = n(n-1)(n-2)\ldots 1]$
$= 8 \times 7 \times 6 \times 5$ = 1680
Again, $2! = 2 \times 1 = 2 \neq$ 1680
$\therefore \frac{8!}{4!} \neq 2!$

21. Let A = {1, 2, 3, 4} , B = {2, 3, 4, 5, 6}, C = {2, 3, 4, 9, 10}
$\therefore A \cap B$= {1, 2, 3, 4} ∩ {2, 3, 4, 5, 6}
= {2,3, 4}
$A \cap C$ = {1, 2, 3, 4}, B = {2, 3, 4, 5, 6}, C = {2, 3, 4, 9, 10}
= {2, 3, 4}
$A \cap C$ = {1, 2, 3, 4} ∩ {2, 3, 4, 9, 10}

= {2, 3, 4}

Now we have $A \cap B = A \cap C$

But $B \neq C$

OR

According to the question, n(A) = 35, n(B) = 30 and n(U) = 50

i. We know that
$A \cup B \subseteq U$
$\Rightarrow n(A \cup B) \leq n(U)$
$\Rightarrow n(A \cup B) \leq 50$
So, the greatest value of $n(A \cup B)$ is 50.

ii. From (i), we have
$n(A \cup B) \leq 50$
$\Rightarrow \quad n(A) + n(B) - n(A \cap B) \leq 50$
$\Rightarrow 35 + 30 - n(A \cap B) \leq 50$
$\Rightarrow 15 \leq (A \cap B) \Rightarrow n(A \cap B) \geq 15$
So, the least value of $n(A \cap B)$ is 15.

22. Suppose B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires.

Given, $P(B) = 0.23$, $P(T) = 0.24$ and $P(B \cup T) = 0.38$. We have to find P (B $\cap$ T).

As we know,

$P(B\cup\ T) = P(B) + P(T) - P(B\cap\ T)$ [By addition theorem]

$\Rightarrow P(B\ \cap\ T) = P(B) + P(T) - P(B\cup\ \ T) = 0.23 + 0.24 - 0.38 = 0.09$

23. We have, $\left(\frac{3}{x^2} - \frac{x^3}{6}\right)^7$

Clearly, the given expansion contains 8 terms. So, 4th term from the end = (7 - 4 + 2)th term from the beginning.

$\therefore$ Required term = $T_5 = T_{4+1} = {}^7C_4 \left(\frac{3}{x^2}\right)^{7-4}\left(\frac{-x^3}{6}\right)^4$

$= {}^7C_4 \left(\frac{3}{x^2}\right)^3 \frac{x^{12}}{6^4} = \frac{7!}{4!3!} \cdot \frac{3^3}{x^6} \cdot \frac{x^{12}}{6^4}$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \cdot \frac{3^3 \cdot x^{12-6}}{(3\times 2)^4}$

= $\frac{35 \cdot 3^3 \cdot x^6}{(3)^4 \cdot 2^4}$
= $\frac{35 \cdot x^6}{3 \cdot 2^4} = \frac{35}{48}x^6$

24. Given lines are x - y - 6 = 0 ...(i)
    4x - 3y - 20 = 0 ...(ii)
    and 6x + 5y + 8 = 0 ...(iii)
    On solving Eq. (i) and Eq. (ii) by cross-multiplication method,
    we get
    $\frac{x}{20-18} = \frac{y}{-24+20} = \frac{1}{-3+4} \Rightarrow x = 2 \text{ and } y = -4$
    Thus, intersection point of first two lines is (2,-4).
    Now, if given three lines are concurrent, then this point will satisfies the Eq. (iii).
    On putting x = 2 and y = -4 in LHS of Eq. (iii), we get
    LHS = 6(2) + 5(-4) + 8 = 12 - 20 + 8 = 0 = RHS
    Hence, given three lines are concurrent.

OR

If slope of one line is m. Then, the slope of the other line is 2m.
Let angle between these two lines be θ.
Then, $\tan\theta = \frac{1}{3}$ [given)
$\Rightarrow \quad \left|\frac{2m-m}{1+2m \cdot m}\right| = \frac{1}{3}\left[\because \tan\theta = \left|\frac{m_2-m_1}{1+m_1 \cdot m_2}\right|\right]$
$\Rightarrow \quad \frac{m}{1+2m^2} = \frac{1}{3}$
$\Rightarrow$ 2m^2 - 3m + 1 = 0
$\Rightarrow$ (2m - 1) (m - 1) = 0 $\Rightarrow m = \frac{1}{2}, m = 1$
Thus, the slope of these lines are $\frac{1}{2}$ and 1.

25. The given compound statement is of the form "if p then q".
    We assume that p is true then
    $a, b \in R$ such that a^2 = b^2
    Let us take a =-3 and b = 3
    So when p is true q is false.
    Thus the given compound statement is not true.

26. LHS = tan 20° tan 40° tan 80° $= \frac{\sin 20^\circ \sin 40^\circ \sin 80^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ}$

$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ}{(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$

$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ}$ [∵ 2 sin a sin b = cos (a-b) - cos (a+b), 2 cos a cos b = cos (a+b) + cos (a-b)]

$= \frac{\sin 80^\circ \cos 20^\circ - (1/2)\sin 80^\circ}{(1/2)\cos 80^\circ + \cos 80^\circ \cos 20^\circ}$ [$\because \cos\ 60^o = \frac{1}{2}$]

$= \frac{2\sin 80^\circ \cos 20^\circ - \sin 80^\circ}{\cos 80^\circ + 2\cos 80^\circ \cos 20^\circ}$

$= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\cos 80^\circ + \cos 100^\circ + \cos 60^\circ}$[∵ 2 sin a cos b = sin (a+b) + sin (a-b)]

$= \frac{\sin(180^\circ - 80^\circ) + \sin 60^\circ - \sin 80^\circ}{\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ}$

$= \frac{\sin 80^\circ + \sin 60^\circ - \sin 80^\circ}{\cos 80^\circ - \cos 80^\circ + \cos 60^\circ} = \frac{\sin 60^\circ}{\cos 60^\circ}$ = tan 60° = RHS

27. Let $x \in P(A \cap B)$

$\Rightarrow x \subset (A \cap B)$

$\Rightarrow x \subset A$ and $x \subset B$

$\Rightarrow x \in P(A)$ and $x \in P(B)$

$\Rightarrow x \in P(A) \cap P(B)$

$\Rightarrow x \subset P(A) \cap P(B)$

$\therefore P(A \cap B) \subset P(A) \cap P(B)...$ (i)

Let $x \in P(A) \cap P(B)$

$\Rightarrow x \in P(A)$ and $x \in P(B)$

$\Rightarrow x \subset A$ and $\Rightarrow x \subset B$

$\Rightarrow x \subset A \cap B$

$\Rightarrow x \subset P(A \cap B)$

$\therefore P(A) \cap P(B) \subset P(A \cap B)....$ (ii)

From (i) and (ii), we have

$P(A \cup B) = P(A) \cap P(B)$

28. (i) Here the relation is

{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

All values of x are distinct. Each value of x has a unique value of y.

So the relation is a function.

∴ Domain of function = {2,5, 8, 11, 14, 17}

Range of function = {1}

(ii) Here the relation is

{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

All values of x are distinct. Each value of x has a unique value of y.

So the relation is a function.

$\therefore$ Domain of function = {2, 4, 6, 8, 10, 12, 14}

Range of function = {1, 2, 3, 4, 5, 6, 7}

(iii) Here the relation is

{(1, 3), (1, 5), (2, 5)}

This relation is not a function because there is an element 1 which is associated to two elements 3 and 5.

OR

Here $(-1, 0) \in A \times A \Rightarrow -1 \in A$ and $0 \in A$

(0, 1) $\in A \times A \Rightarrow 0 \in A$ and $1 \in A$

$\therefore -1, 0, 1, \in A$

It is given that $n(A \times A) = 9$ which implies that n (A) = 3

$\therefore$ A = (-1, 0, 1)

$\therefore A \times A$ = {(-1, -1), (-1, 0), (-1, 1) (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)}

So the remaining elements of $A \times A$ are

(-1, 1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) and (1, 1)

29. First, we find the derivatives of f(x) at x = -1 and x = 0. We have,

$$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$\left[\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right]$$

$$= \lim_{h \to 0} \frac{\left[2(-1+h)^2 + 3(-1+h) - 5\right] - \left[2(-1)^2 + 3(-1) - 5\right]}{h}$$

$$= \lim_{h \to 0} \frac{\left[2\left(1+h^2-2h\right) - 3 + 3h - 5\right] - [2 - 3 - 5\}}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 - h}{h} = \lim_{h \to 0}(2h - 1) = 2(0) - 1 = -1$$

and $f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$$\left[\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right]$$

$$= \lim_{h \to 0} \frac{\left[2(0+h)^2 + 3(0+h) - 5\right] - \left[2(0)^2 + 3(0) - 5\right]}{h}$$

$= \lim_{h \to 0} \frac{2h^2+3h}{h}$

$= \lim_{h \to 0}(2h+3)$

= 2(0) + 3 = 3

Now, f'(0) + 3f'(-1) = 3 - 3 = 0.

Hence proved.

30. M.I. of $= \sqrt{5}+3i$

$= \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$

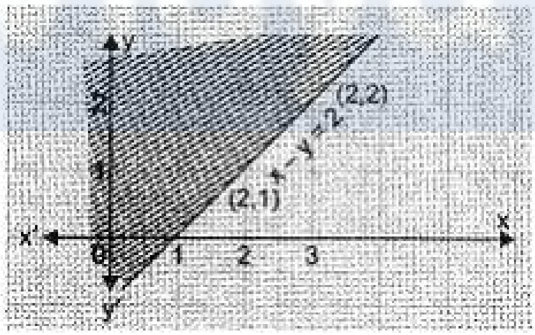
$= \frac{\sqrt{5}-3i}{5-9i^2} = \frac{\sqrt{5}-3i}{5+9} = \frac{1}{14}(\sqrt{5}-3i)$

31. The given inequality is x - y $\leq$ 2.

Draw the graph of the line x - y = 2

Table of values satisfying the equation x - y = 2

X	2	3
Y	1	2



Putting (0, 0) in the given inequation, we have

0 - 0 $\leq$ 2 $\Rightarrow 0 \leq 2$ which is true

$\therefore$ Half-plane of x - y $\leq$ 2 is towards origin

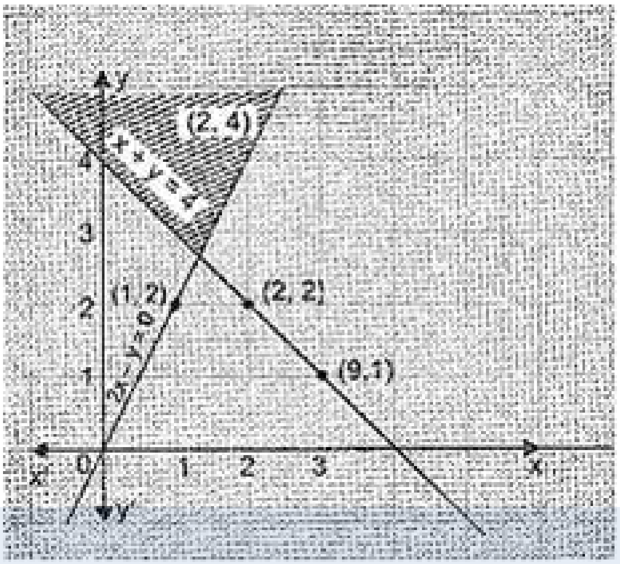
OR

The given inequality is $x + y \geqslant 4$

Draw the graph of the line x + y = 4.

Table of values satisfying the equation x + y = 4.

X	3	2
Y	1	2



Putting (0, 0) in the given inequation, we have

$0+0 \geqslant 4 \Rightarrow 0 \geqslant 4$, which is false.

$\therefore$ Half plane of $x+y \geqslant 4$ is away from origin.

Also the given inequality is 2x - y < 0

Draw the graph of the line 2x - y = 0

Table of values satisfying the equation 2x - y = 0

X	1	2
Y	2	4

Putting (3, 0) in the given inequation, we have

$2 \times 3-0<0 \Rightarrow 6<0$, which is false.

$\therefore$ Half plane of 2x - y = 0 does not contain (3, 0)

32. Let P (n) = $\frac{1}{1 \cdot 2 \cdot 3}+\frac{1}{2 \cdot 3 \cdot 4}+\frac{1}{3 \cdot 4 \cdot 5}+\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)}$

For n = 1

$P(1)=\frac{1}{1(1+1)(1+2)}=\frac{1(1+3)}{4(1+1)(1+2)} \Rightarrow \frac{1}{1 \times 2 \times 3}=\frac{4}{4 \times 2 \times 3} \Rightarrow \frac{1}{6}=\frac{1}{6}$

$\therefore$ P(1) is true

Let P (n) be true for n = k

$\therefore P(k)=\frac{1}{1 \cdot 2 \cdot 3}+\frac{1}{2 \cdot 3 \cdot 4}+\frac{1}{3 \cdot 4 \cdot 5}+\ldots+\frac{1}{k(k+1)(k+2)}+\frac{k(k+3)}{4(k+1)(k+2)}$ .... (i)

For P(k+1)

R.H.S. $= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$

L.H.S. $= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$ [Using (i)]

$= \frac{1}{(k+1)(k+2)}\left[\frac{k^2+3k}{4} + \frac{1}{k+3}\right]$

$= \frac{1}{(k+1)(k+2)}\left[\frac{k^3+6k^2+9k+4}{4(k+3)}\right] = \frac{1}{(k+1)(k+2)}\left[\frac{(k+1)^2(k+4)}{4(k+3)}\right]$

$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$

$\therefore$ P (k + 1) is true

Thus P(k) is true $\Rightarrow$ P(k + 1) is true

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.

33. **LHS** $= tan\, 82\frac{1^\circ}{2} = tan(90^o - 7\frac{1^\circ}{2}) = cot\, 7\frac{1^\circ}{2} = cotA$ [say]

where, $A = 7\frac{1^\circ}{2}$

Now, $cotA = \frac{\cos A}{\sin A} = \frac{\cos A(2\cos A)}{\sin A(2\cos A)}$

[multiplying numerator and denominator by $2cos\ A$]

= $\frac{2\cos^2 A}{2\sin A\cdot\cos A}$

= $\frac{1+\cos 2A}{\sin 2A}$ [$\because cos^2x = \frac{1+\cos 2x}{2}$ and $sin\ 2x = 2sin\ x\ \times\ cos\ x$]

$\Rightarrow \cot 7\frac{1^\circ}{2} = \frac{1+\cos 2\left(7\frac{1^\circ}{2}\right)}{\sin 2\left(7\frac{1^\circ}{2}\right)} = \frac{1+\cos 2\left(\frac{15}{2}\right)^\circ}{\sin 2\left(\frac{15}{2}\right)^\circ}$ [put A = $7\frac{1^\circ}{2}$]

$\Rightarrow \cot 7\frac{1}{2} = \frac{1+\cos 15^\circ}{\sin 15^\circ} = \frac{1+\cos(45^\circ-30^\circ)}{\sin(45^\circ-30^\circ)}$

= $\frac{1+(\cos 45^\circ\cdot\cos 30^\circ+\sin 45^\circ\cdot\sin 30^\circ)}{(\sin 45^\circ\cdot\cos 30^\circ-\cos 45^\circ\cdot\sin 30^\circ)}$

[$\because cos(x-y) = cos\ x\ cosy + sin\ x\ sin\ y$ and

$sin(x-y) = sin\ x\ cos\ y - cos\ x\ sin\ y$]

= $\frac{1+\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)}$

= $\frac{1+\left(\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)}$

= $\frac{2\sqrt{2}+\sqrt{3}+1}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

[multiplying numerator and denominator by $\sqrt{3}$ + 1]

= $\frac{2\sqrt{2}(\sqrt{3}+1)+(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

= $\frac{2\sqrt{6}+2\sqrt{2}+(\sqrt{3}+1)^2}{3-1} = \frac{2\sqrt{6}+2\sqrt{2}+3+1+2\sqrt{3}}{2}$

= $\frac{2\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4}{2}$

= $\frac{2(\sqrt{6}+\sqrt{2}+2+\sqrt{3})}{2} = \sqrt{6}+\sqrt{2}+2+\sqrt{3}$

= $\sqrt{2}\cdot\sqrt{3}+\sqrt{2}+\sqrt{2}\cdot\sqrt{2}+\sqrt{3}$

= $(\sqrt{2}\cdot\sqrt{3}+\sqrt{2}\cdot\sqrt{2})+(\sqrt{2}+\sqrt{3})$

= $\sqrt{2}(\sqrt{3}+\sqrt{2})+1(\sqrt{2}+\sqrt{3}) = (\sqrt{2}+1)(\sqrt{3}+\sqrt{2})$

= RHS

∴ **LHS = RHS**

Hence proved.

OR

LHS $= cos12^o + cos60^o + cos84^o$

$= cos12^o + (cos84^o + cos60^o)$

=$cos12^o + [2cos\left(\frac{84^\circ+60^\circ}{2}\right) \times cos\left(\frac{84^\circ-60^\circ}{2}\right)$]

[∵ cosx +cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$= cos12^o + [2cos\ \frac{144^\circ}{2}$ × cos $\frac{24^\circ}{2}$]

$= cos12^o + [2cos72^o \times\ \ cos12^o] = cos12^o[1 + 2cos72^o]$

$= cos12^o[1 + 2cos(90^o - 18^o)]$

= cos 12^o [1 + 2 sin 18^o] [∵ cos (90^o - θ) = sin θ]

= cos 12^o [1 + 2 $\left(\frac{\sqrt{5}-1}{4}\right)$] [∵ sin 18^o = $\frac{\sqrt{5}-1}{4}$]

= $\left(1+\frac{\sqrt{5}-1}{2}\right)$ cos 12^o = $\left(\frac{\sqrt{5}+1}{2}\right)$ cos 12^o

RHS = cos 24^o + cos 48^o

= 2 cos $\left(\frac{24^\circ+48^\circ}{2}\right)$ cos $\left(\frac{24^\circ-48^\circ}{2}\right)$ [∵ cosx + cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$= 2cos36^o cos(-12^o)$

= 2 cos 36^o × cos 12^o [∵ cos (- θ) = cos θ]

= 2 × $\frac{\sqrt{5}+1}{4}$ × cos 12^o = $\frac{\sqrt{5}+1}{2}$ × cos 12^o [∵ cos 36^o = $\frac{\sqrt{5}+1}{4}$]

∴ **LHS = RHS**

Hence proved.

34. Given: $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots$ up to n terms

$\therefore a_n=\frac{1^3+2^3+3^3+\ldots\ldots+n^3}{1+3+5+\ldots\ldots(2n-1)}$

$=\frac{\sum n^3}{\frac{n}{2}[2+(n-1)2]}=\frac{\sum n^3}{\frac{n}{2}(2n)}=\frac{\sum n^3}{n^2}=\frac{n^2(n+1)^2}{4n^2}$

$=\frac{1}{4}\left(n^2+2n+1\right)$

$\therefore S_n=\sum_{k=1}^{n}a_k=\sum_{k=1}^{n}\frac{k^2+2k+1}{4}$

$=\frac{1}{4}$ [(1^2 + 2.1 + 1) + (2^2 + 2.2 + 1) + (3^2 + 2.3 + 1) + ... +(n^2 + 2n + 1)]

$=\frac{1}{4}\left[\sum n^2+2\sum n+n\right]$

$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}+n\right]$

$=\frac{n}{4}\left[\frac{2n^2+3n+1+6n+6+6}{6}\right]$

$=\frac{n}{24}\left(2n^2+9n+13\right)$

35. Let the equation of the required ellipse be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 .....(i)

The coordinate of foci are (+ ae, 0) and (-ae, 0).

$\therefore$ae = 4 [$\because$ foci : ($\pm$4, 0)]

$\Rightarrow$ a $\times\frac{1}{3}$ = 4 $\left[\because e=\frac{1}{3}\right]$

$\Rightarrow$ a = 12

$\Rightarrow$ a^2 = 144

Now, b^2 = a^2 (1 -e^2)

$\Rightarrow$ b^2 = 144 $\left[1-\left(\frac{1}{3}\right)^2\right]$

$\Rightarrow$ b^2 = 144 $\left[1-\frac{1}{9}\right]$

$\Rightarrow$ b^2 = 144 $\times\frac{8}{9}$

$\Rightarrow$ b^2 = 16 $\times$ 8 = 128

Substituing a^2 = 144 and b^2 = 128 in equation (i), we get

= $\frac{x^2}{144} + \frac{y^2}{128}$ = 1

$\Rightarrow \frac{1}{16}\left[\frac{x^2}{9} + \frac{y^2}{8}\right]$ = 1

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{8}$ = 16

This is the required equation of the ellipse.

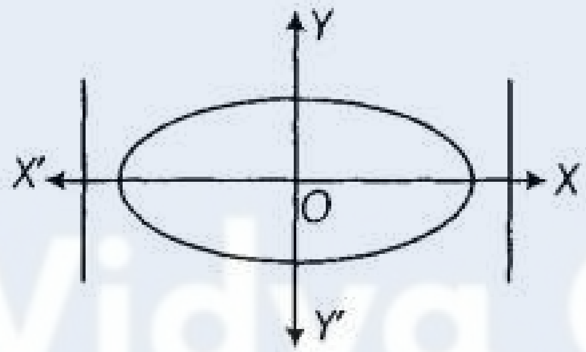
OR

We have, equation of ellipse is $4x^2 + 9y^2 = 36$

or $\frac{x^2}{9} + \frac{y^2}{4}$ = 1

Since, the denominator of $\frac{x^2}{9}$ is greater than denominator of $\frac{y^2}{4}$

So, the major axis lies along X-axis.



i. Shape is shown above.
ii. Major axis, 2a = 2 $\times$ 3 = 6
iii. Minor axis, 2b = 2 $\times$ 2 = 4
iv. Value of c = $\sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$
v. Vertices = (- a, 0) and (a, 0) i.e., (- 3, 0) and (3, 0)
vi. Directrices, x = $\pm\frac{a^2}{c} = \pm\frac{9}{\sqrt{5}}$
vii. Foci = (- c, 0) and (c, 0) i.e., (- $\sqrt{5}$, 0) and ($\sqrt{5}$, 0)
viii. Eccentricity, e = $\frac{c}{a} = \frac{\sqrt{5}}{3}$
ix. Length of latusrectum, 2l = $\frac{2b^2}{a} = \frac{2\times4}{3} = \frac{8}{3}$

36. i.

Number of families	Mid value (x_i)	Number of villages (f_i)	cf	$\lvert x_i$ - M$\rvert$	$f_i\lvert x_i$ - M$\rvert$

$0-10$	5	6	6	20	120
$10-20$	15	8	14	10	80
$20-30$	25	16	30	0	0
$30-40$	35	8	38	10	80
$40-50$	45	4	42	20	80
$50-60$	55	2	44	30	60
		44			420

Here, $N=44$

Now, $\frac{N}{2}$ = $\frac{44}{2}=22$, which, lies in the cumulative frequency of 30, therefore median class is 20-30.

$\therefore l=20, f=16, cf=14 \; and \; h=10$

$\therefore$ Median (M) = $l+\frac{\frac{N}{2}-cf}{f}\times b$

$=20+\frac{22-14}{16}\times 10$

$=20+\frac{8}{16}\times 10=20+5=25$

$\therefore$ Mean deviation about median = $\frac{\sum_{i=1}^{6} f_i|x_i-M|}{\sum f_i}$ = $\frac{420}{44}=9.55$

ii. There is a need for awareness among villagers for using LPG as a mode of cooking. Because it will help in keeping the environment clean and will also help in saving of forests.