CBSE Class 10th Mathematics
Standard Sample Paper - 03

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The difference of a rational and an irrational number is always

   a. a rational number

   b. an irrational number

   c. an integer

   d. None of these

2. If two positive integers ‘m’ and ‘n’ can be expressed as $m = x^2y^5$ and $n = x^3y^2$,where ‘x’ and ‘y’ are prime numbers, then HCF(m, n) =

   a. x^2y^2

b. x^2y^3

c. x^3y^2

d. x^3y^3

3. To represent 'the less than type' graphically, we plot the ___________ on the x – axis.

a. class marks

b. class size

c. lower limits

d. upper limits

4. Which of the following has two distinct roots?

a. $x^2 + x - 5 = 0$

b. $5x^2 - 3x + 1 = 0$

c. None of these

d. $x^2 + x + 5 = 0$

5. If two trees of height 'x' and 'y' standing on the two ends of a road subtend angles of 30° and 60° respectively at the midpoint of the road, then the ratio of x : y is

a. 1 : 3

b. 1 : 2

c. 3 : 1

d. 1 : 1

6. If $cos\ 9\alpha\ =\ sin\ \alpha$, then the value of α

a. 20°

b. 0°

c. 9°

d. 30°

7. If $x = a\sec\theta\cos\varphi$, $y = b\sec\theta\sin\varphi$ and $z = c\tan\theta$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is

a. $1 + \frac{z^2}{c^2}$

b. $-1 - \frac{z^2}{c^2}$

c. $\frac{z^2}{c^2} - 1$

d. $1 - \frac{z^2}{c^2}$

8. When a die is thrown, the probability of getting an odd number less than 3 is

a. $\frac{1}{3}$

b. $\frac{1}{6}$

c. $\frac{1}{4}$

d. $\frac{1}{2}$

9. The length of the median through A of ΔABC with vertices A(7, – 3), B(5, 3) and C(3, – 1) is

a. 5 units

b. 3 units

c. 7 units

d. 25 units

10. The co – ordinates of the mid – point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by

a. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

b. $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$

c. $\left(\frac{x_1 - y_1}{2}, \frac{x_2 - y_2}{2}\right)$

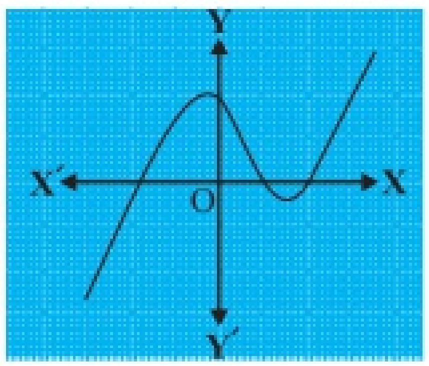
d. $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$

11. Fill in the blanks:

    Space occupied by an object/solid body is called the ________ of that particular object/solid.

12. Fill in the blanks:

    The graph of y = p(x) is given in the figure below, for some polynomial p(x). The number of zeroes of p(x) is ________.



OR

Fill in the blanks:

The remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by x - 1 is ________.

13. Fill in the blanks:

    The ________ is the line drawn from the eye of an observer to the point in the object viewed by the observer.

14. Fill in the blanks:

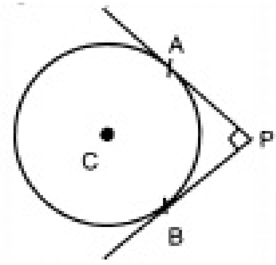
    The sum of the AP, 1 + 2 + 3 + 4 + 5 + 6 + ---- 10 is ________.

15. Fill in the blanks:

    The ________ is the name of the common point of the tangent to a circle and the circle.

16. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds if they first beep together at 12 noon, at what time will they beep again for the first time?

17. In fig., PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then find the length of each tangent.



18. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them?

19. Find the next term in AP: 3, 1, -1, -3.

    OR

    Two APs have same common difference. The first term of one of these is -1 and that of the other is -8 Find the difference between their 4th terms.

20. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

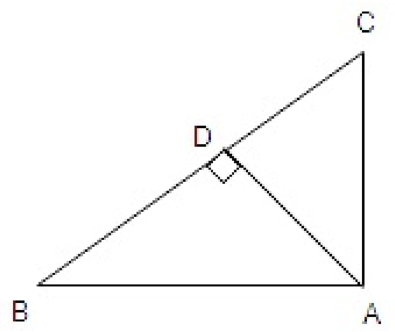
Section B

21. A coin is tossed 3 times. List the possible outcomes, find the probability of getting

    i. all heads
    ii. at least two heads

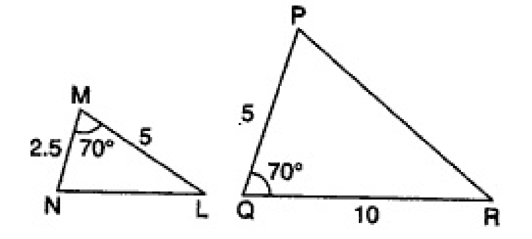
22. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

23. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



OR

State whether the pairs of triangles in the figure are similar or not. Write the similarity criterion used for answering the question and also write the pairs of similar triangles in the symbolic form.



24. Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. If α, β be the elevations of the top of the tower from these stations, prove that its inclination θ to the horizontal is given by $\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$

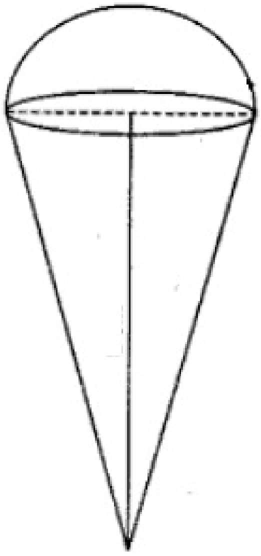
25. Find the value of k for which the given equation has equal roots. Also, find the roots, $2kx^2 - 40x + 25 = 0$

OR

If x = $\frac{2}{3}$ and x = - 3 are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

26. An 'ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9

cm and the radius of its base is 2.5 cm.



By reading the above-given information, find the following:

i. The volume of the ice-cream without hemispherical end.
ii. The volume of the ice-cream with a hemispherical end.

Section C

27. If p is a prime number, then prove that $\sqrt{p}$ is irrational.

OR

Define HCF of two positive integers and find the HCF of the pairs of numbers: 56 and 88.

28. The line segment joining the points (3, -4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, -2) and (5/3, q) respectively. Find the values of p and q.

29. Use elimination method to find all possible solutions of the following pair of linear equations
$ax + by - a + b = 0$ and $bx - ay - a - b = 0$

OR

Six years hence a man's age will be three times the age of his son and three years ago he was nine times as old as his son. Find their present ages.

30. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

31. Find the sum of all natural numbers between 100 and 500 which are divisible by 8.

32. If, $\cot B = \frac{12}{5}$ prove that $\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$

OR

If A, B, C, are the interior angles of a ΔABC, show that $\sin \frac{B+C}{2} = \cos \frac{A}{2}$.

33. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road.

34. An integer is chosen between 0 and 100.
What is the probability that it is

   i. divisible by 7?
   ii. not divisible by 7?

Section D

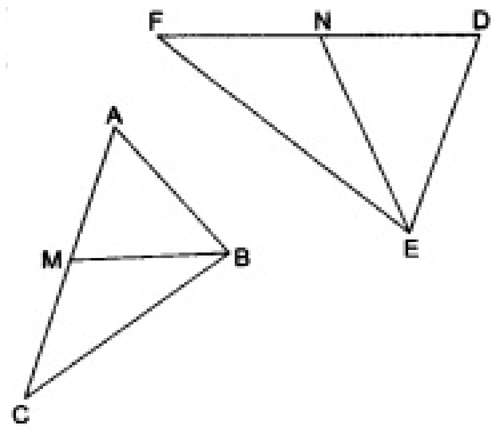
35. Draw a ΔABC in which BC = 6 cm, AB = 4 cm and AC = 5 cm. Draw a triangle similar to ΔABC with its sides equal to $(3/4)^{th}$ of the corresponding sides of ΔABC.

OR

Construct an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

36. In the given figure, BM and EN are respectively the medians of $\triangle ABC$ and $\triangle DEF$. If $\triangle ABC \sim \triangle DEF$, prove that:

   a. $\triangle AMB \sim \triangle DNE$
   b. $\triangle CMB \sim \triangle FNE$
   c. $\frac{BM}{EN} = \frac{AC}{DF}$



37. If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.

OR

Solve graphically the system of linear equation:

$4x - 3y + 4=0$

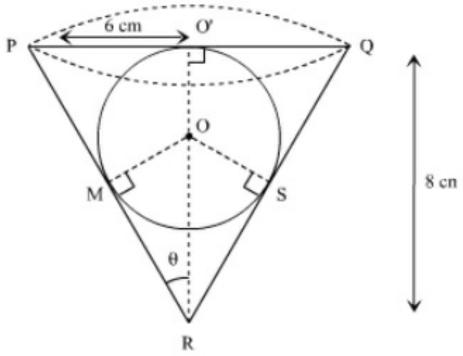
$4x + 3y -20=0$

Find the area bounded by these lines and x-axis.

38. Find the area of a rhombus each side of which measures 20 cm and one of whose diagonals is 24 cm.

OR

A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Figure. What fraction of water over flows?



39. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp-post CD are observed to be 30° and 60° respectively. Find the horizontal distance between AB and CD.

40. The table below shows the daily expenditure on food of 30 households in a locality:

Daily expenditure(in Rs)	Number of households
100 - 150	6
150 - 200	7
200 - 250	12
250 - 300	3
300 - 350	2

Find the mean and median daily expenditure on food.

Solution

Section A

1. (b) an irrational number
   Explanation:

   Rational Numbers say $\frac{4}{9}$, $\frac{p}{q}$, $\sqrt{4}$, fraction, whole numbers, terminating decimal, repeating decimal, perfect square, can be expressed as a ratio of two integers provided the denominator is not equal to zero

   Irrational Numbers $\sqrt{2}$,$\sqrt{5}$,$\sqrt{7}$, π not a fraction, decimal does not repeat, decimal does not end, non-perfect square, we cannot express as a ratio but both can be expressed as decimal numbers

   The difference between a rational and an irrational number is always an irrational number.

   e.g. rational - irrational = irrational   say 2 - $\sqrt{2}$ = irrational

2. (a) x^2y^2

   Explanation:

   $x^2y^5 = y^3(x^2y^2)$

   $x^3y^2 = x(x^2y^2)$

   Therefore HCF (m , n) is x^2y^2

3. (d) upper limits
   Explanation:

   To represent ‘the less than type’ graphically, we plot the upper limits on the x-axis.

   e.g marks obtained by students are represented in grouped data as (0 - 10) , (10 - 20), (20 - 30) , (30 - 40) ....

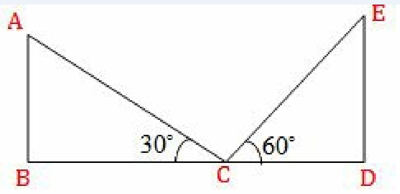
only upper limits such as 10, 20, 30, 40 .... are taken for the x-axis

4. (a) $x^2 + x - 5 = 0$

   Explanation:

   In equation $x^2 + x - 5 = 0$
   $a = 1, b = 1, c = -5$
   $\therefore b^2 - 4ac$ = $(1)^2 - 4 \times 1 \times (-5)$ = 1 + 20 = 21
   Since $b^2 - 4ac > 0$ therefore, $x^2 + x - 5 = 0$ has two distinct roots.

5. (a) 1 : 3
   Explanation:



   Here two trees AB and ED are of height x x and y y respectively. And BC = CD
   $\therefore \tan 30^\circ = \frac{x}{\text{BC}}$
   $\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{\text{BC}}$
   $\Rightarrow x = \frac{\text{BC}}{\sqrt{3}}$ And $\tan 60^\circ = \frac{y}{\text{CD}}$
   $\Rightarrow \sqrt{3} = \frac{y}{\text{CD}}$
   $\Rightarrow y = \text{CD}\sqrt{3} = \text{BC}\sqrt{3}$ [BC = CD]
   Now, $\frac{x}{y} = \frac{\text{BC}}{\sqrt{3} \times \text{BC}\sqrt{3}}$
   = $\frac{1}{3}$
   $\Rightarrow x : y = 1 : 3$

6. (c) 9°

   Explanation:

   Given: $\cos 9\alpha = \sin \alpha$

   L.H.S =$\cos 9\alpha = \sin(90^\circ - 9\alpha)$

   R.H.S = $sin\alpha$

on comparing L.H.S and R.H.S we get

$90 - 9\alpha$ = α

$90 = 10\alpha$

$\alpha = 9^o$

7. (a) $1 + \frac{z^2}{c^2}$

Explanation:

Given: $x = a \sec\theta \cos\varphi, y = b \sec\theta \sin\varphi$
and $z = c \tan\theta$,
$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2}$
= $\frac{a^2 \sec^2\theta \cos^2\phi}{a^2} + \frac{b^2 \sec^2\theta \sin^2\phi}{b^2}$
= $\sec^2\theta \left(\cos^2\phi + \sin^2\phi\right)$ = $\sec^2\theta$
$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}$
= $1 + \tan^2\theta$
= $1 + \frac{z^2}{c^2}$
[Given: $z = c \tan\theta$]

8. (b) $\frac{1}{6}$

Explanation:

Odd outcomes are 1 , 3 , 5 but 3 and 5 are not less than 3
Number of possible outcomes whtch are odd and less than 3 = 1
Number of possible outcomes = {1} = 1
Number of Total outcomes = 6
$\therefore$ Required Probability = $\frac{1}{6}$

9. (a) 5 units
Explanation:

ABC is a triangle with A(7, - 3), B(5, 3) and C(3, - 1)

Let median on BC bisects BC at D. (AD is given as the median)

$\therefore$ Coordinates of D are $\left(\frac{5+3}{2}, \frac{3-1}{2}\right)$ = $(4, 1)$

$\therefore$ AD = $\sqrt{(4-7)^2+(1+3)^2}$

= $\sqrt{9+16}$

= $\sqrt{25}$ = 5 units

10. (a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Explanation:

we know that the mid point formula = $\frac{x1+x2}{2}$, $\frac{y1+y2}{2}$

The co-ordinates of the mid-point of the line segment joining the points $(x_1, y_1)$and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

11. volume

12. 3 OR 2

13. line of sight

14. 55

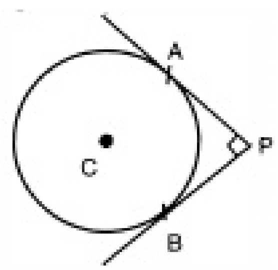
15. point of contact

16. 50 = $2 \times 5 \times 5$, 48 = $2 \times 2 \times 2 \times 2 \times 3$

LCM of 50 and 48 = $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5$ = 1200

$\because$ 1200 sec = 20 min

Hence at 12.20 pm they will beep again for the first time

17. 

Construction: Join AC and BC

Now, $AC \perp AP \text{ and } CB \perp BP$

$\angle APB = 90^0$

Therefore, CAPB will be a square

CA = AP = PB = BC = 4 cm

$\therefore$ Length of tangent = 4 cm.

18. We know , tangent and radius of a circle are perpendicular to each other at point of contact

    $\therefore$ Angle between the radii $= 360^0 - 90^0 - 90^0 - 30^0 = 150^0$

19. first term, a=3 and common difference,

    d = 1 - 3 = - 2

    Clearly, next term of given AP

    $a_5 = a + 4d = 3 + 4(-2) = 3 - 8 = -5$

    Aliter:

    $a_5 = a_4 + d = -3 + (-2) = -3 - 2 = -5$

    OR

    For 1st AP,

    a = -1, common difference = d

    $\therefore a_4 = -1 + 3d$

    For 2nd AP,

    1st term, A = -8, common difference = d

    $A_4 = -8 + 3d$

    Now, a_4 - A_4 = (-1 + 3d) - (-8 + 3d) = 7

20. Since the quadratic equation has equal roots, then

    $b^2 - 4ac = 0$

    $\Rightarrow 4ac = b^2$

    $\Rightarrow c = \frac{b^2}{4a}$

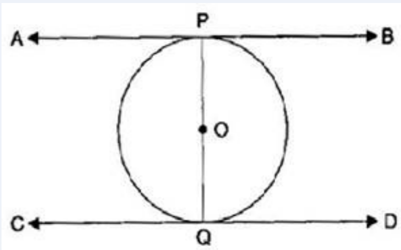
Section B

21. Total number of possible outcomes when a coin is tossed 3 times= $(HHH),(HHT),(HTH),(THH)(TTT)(TTH)(THT)(HTT)$

$\Rightarrow T(E) = 8$

Total probability= 1

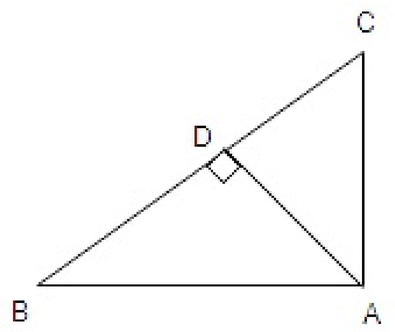
i. All head i.e., Favourable outcomes = 1
ii. We know that ,$P(E) = \frac{F(E)}{T(E)} = \frac{1}{8}$
iii. Also Number of favourable outcomes of getting at least 2 heads i.e Favourable outcomes = 4 therefore,
$P(E) = \frac{F(E)}{T(E)} = \frac{4}{8} = \frac{1}{2}$

22. 

Given: PQ is a diameter of a circle with centre O.
The lines AB and CD are the tangents at P and Q respectively.
To Prove: AB $\|$ CD
Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore$ $\angle$OPA = 90^o .... (i)
[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]
$\because$ CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore$ $\angle$OQD = 90^o .... (ii)
[The tangent at any point of a circle is $\perp$ to the radius through the point of contact]
From eq. (i) and (ii), $\angle$OPA = $\angle$OQD
But these form a pair of equal alternate angles also,
$\therefore$ AB $\|$ CD

23. 

In right angled $\triangle BDA$,

By pythagoras theorem

$AB^2 = AD^2 + BD^2$ ...(i)

And in right angled $\triangle CDA$,

By pythagoras theorem

$AC^2 = CD^2 + AD^2$ ...(ii)

On subtracting Eq(ii) from Eq(i) , we get

$AB^2 - AC^2 = [AD^2 + BD^2] - [CD^2 + AD^2]$

$AB^2 - AC^2 = AD^2 + BD^2 - CD^2 - AD^2$

$AB^2 - AC^2 = BD^2 - CD^2$

$\therefore AB^2 + CD^2 = BD^2 + AC^2$

OR

In $\triangle$MNL and $\triangle$QPR,

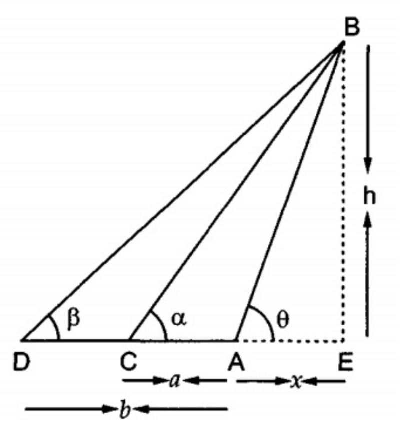
$\frac{ML}{QR} = \frac{MN}{QP}\left(= \frac{1}{2}\right)$ and $\angle NML = \angle PQR$

$\therefore \triangle MNL \sim \triangle QPR$........SAS similarity criterion

24. Let AB be the leaning tower and let C and D be two given stations at distances a and b respectively from the foot A of the tower.

Let AE = x and BE = h

In $\triangle AEB,$ we have



$\tan\theta = \frac{BE}{AE}$

$\Rightarrow \quad \tan\theta = \frac{h}{x}$

$\Rightarrow \quad x = h\cot\theta$ ....(i)

In ΔCEB, we have

$\tan\alpha = \frac{BE}{CE}$

$\Rightarrow \quad \tan\alpha = \frac{h}{a+x}$

$\Rightarrow \quad a + x = h\cot\alpha$

$\Rightarrow \quad x = h\cot\alpha - a$.........(ii)

In ΔDEB, we have

$\tan\beta = \frac{BE}{DE}$

$\Rightarrow \quad \tan\beta = \frac{h}{b+x}$

$\Rightarrow \quad b + x = h\cot\beta$

$\Rightarrow \quad x = h\cot\beta - b$ ....(iii)

On equating the values of x obtained from equations (i) and (ii), we have

$h\cot\theta = h\cot\alpha - a$

$\Rightarrow \quad h(\cot\alpha - \cot\theta) = a$

$\Rightarrow \quad h = \frac{a}{\cot\alpha - \cot\theta}$ ....(iv)

On equating the values of x obtained from equations (i) and (iii), we get

$h\cot\theta = h\cot\beta - b$

$\Rightarrow \quad h(\cot\beta - \cot\theta) = b$

$\Rightarrow \quad h = \frac{b}{\cot\beta - \cot\theta}$ ....(v)

Equating the values of h from equations (iv) and (v), we get

$\frac{a}{\cot\alpha - \cot\theta} = \frac{b}{\cot\beta - \cot\theta}$

$\Rightarrow \quad a(\cot\beta - a\cot\theta) = b(\cot\alpha - \cot\theta)$

$\Rightarrow \quad (b - a)\cot\theta = b\cot\alpha - a\cot\beta$

$\Rightarrow \quad \cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$

25. The given equation is $2kx^2 - 40x + 25 = 0$. Here, a = 2k, b = - 40 and c = 25

$\therefore$ D = $b^2 - 4ac = (-40)^2 - 4 \times 2k \times 25 = 1600 - 200k$

The equation will have equal roots, if

D = 0 $\Rightarrow$ 1600 - 200 k = 0 $\Rightarrow$ k = 8

Substituting k = 8 in the given equation, we get

$16x^2 - 40x + 25 = 0 \Rightarrow (4x - 5)^2 = 0 \Rightarrow x = 5/4$

Hence, the roots of the given equation are each equal to 5/4.

OR

Substituting x = $\frac{2}{3}$ in $ax^2 + 7x + b = 0$

$\therefore \frac{4}{9}a + \frac{14}{3} + b = 0$

$\Rightarrow 4a + 42 + 9b = 0$

$\Rightarrow 4a + 9b = -42$............(i)

and substituting x = - 3

$\Rightarrow 9a - 21 + b = 0$

$\Rightarrow 9a + b = 21$...............(ii)

Solving (i) and (ii), we get a = 3 and b = - 6

26. For cone, Radius of the base (r)

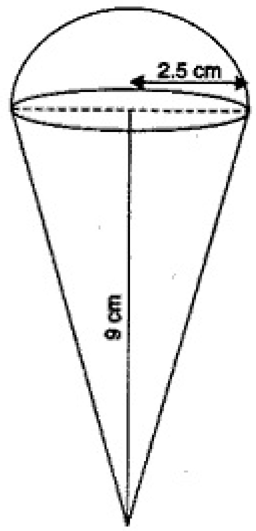
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. The volume of the ice-cream without hemispherical end = Volume of the cone
$= \frac{825}{14}\text{cm}^3$

ii. Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere
$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$
$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

Section C

27. Let us assume, to the contrary, that √p is rational.

So, we can find co-prime integers a and b(b ≠ 0)

$\sqrt{p} = \frac{a}{b}$

$a = b_{\sqrt{P}}$

on squaring both sides we get

$a^2 = pb^2$ ...... (1)

so a^2 is divisible by p

hence a is divisible by p .... (2)

So, we can write a = pc for some integer c.

Squaring both the sides we get

$a^2 = p^2 c^2$ ....

⇒ $pb^2 = p^2 c^2$ ....[From (1)]

⇒ $b^2 = pc^2$

⇒ b^2 is divisible by p

⇒ b is divisible by p .... (3)

From (2) and (3) we conclude that p divides both a and b.

∴ a and b have at least p as a common factor.

But this contradicts the fact that a and b are co-prime. (As per our assumption)

This contradiction arises because we have assumed that √p is rational.

∴ √p is irrational.

OR

HCF (highest common factor) : The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.

We need to find H.C.F. of 56 and 88.

By applying Euclid's Division lemma

88 = 56 × 1 + 32.

Since remainder ≠ 0, apply division lemma on 56 and remainder 32

56 = 32 × 1 + 24.

Since remainder ≠ 0, apply division lemma on 32 and remainder 24

32 = 24 × 1+ 8.

Since remainder ≠ 0, apply division lemma on 24 and remainder 8

24 = 8 × 3 + 0. Therefore, H.C.F. of 56 and 88 = 8

28. 

We have P(p, -2) and $Q\left(\frac{5}{3}, q\right)$ are the points of trisection of the line segment joining A(3, -4) and B(1, 2)

We know AP : PB = 1 : 2

By section formula $\left[\frac{mx_2+nx_1}{m+n}\right], \left[\frac{my_2+ny_1}{m+n}\right]$ coordinates of P are

$\left(\frac{1\times1+2\times3}{1+2}, \frac{1\times2+2\times(-4)}{1+2}\right)$

$= \left(\frac{7}{3}, -2\right)$

Hence, $P = \frac{7}{3}$

Again we know that AQ : QB = 2 : 1

Therefore, Coordinates of Q are (using section formula)

$\left(\frac{2\times1+1\times3}{2+1}, \frac{2\times2+1\times(-4)}{2+1}\right)$

$= \left(\frac{5}{3}, 0\right)$

Hence, q = 0

Therefore, value of p and q is $\frac{7}{3}$ and 0 respectively.

29. Given pair of linear equation is $ax + by - a + b = 0$ .....(i)

and $bx - ay - a - b = 0$ .... (ii)

Multiplying $ax + by - a + b = 0$ by a and $bx - ay - a - b = 0$ by b, and adding them, we get

$a^2x + aby - a^2 + ab = 0$ and $b^2x - aby - ab - b^2 = 0$

$(a^2x + aby - a^2 + ab) + (b^2x - aby - ab - b^2) = 0$

$a^2x + aby - a^2 + ab + b^2x - aby - ab - b^2 = 0$

$a^2x + b^2x - a^2 - b^2 = 0$

$\Rightarrow (a^2 + b^2)x = (a^2 + b^2)$

$\Rightarrow x = \frac{(a^2+b^2)}{(a^2+b^2)} = 1$

On putting x =1 in first equation, we get

$ax + by - a + b = 0$

$a + by = a - b$

$\Rightarrow y = -\frac{b}{b} = -1$

Hence, x=1 and y=-1, which is the required unique solution.

OR

Let us suppose that the present age of Man's be x years

Let us suppose that the present age of his son's be y years.

Six years hence, Man's age = (x + 6) years

Son's age = (y + 6) years

According to question after 6 years

$x + 6 = 3(y + 6)$

$\Rightarrow x + 6 = 3y + 18$

$\Rightarrow x - 3y = 18 - 6$

$\Rightarrow x - 3y = 12$ ....(i)

Three years ago, Man's age = (x - 3) years

Son's age = (y - 3) years

Using the given information, we get

$x - 3 = 9(y - 3)$

$\Rightarrow x - 3 = 9y - 27$

$\Rightarrow x - 9y = -27 + 3$

$\Rightarrow x - 9y = -24$ ....(ii)

Subtracting equation (ii) from equation (i), we get

x -3y - (x- 9y) = 12 + 24

x - 3y - x + 9y = 36

$\Rightarrow$ 6y = 36

$\Rightarrow y = \frac{36}{6} = 6$

Putting the value of y = 6 in equation (i), we get

x - 3 (6) = 12

$\Rightarrow x - 18 = 12$

$\Rightarrow x = 12 + 18$

x = 30

Therefore present age of man is 30 years

and present age of son is 6 years.

30. If α and β are the zeroes of polynomial $2x^2 - 3x + 1$,

a=2, b=-3, c=1

then $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

$\alpha\beta = \frac{c}{a} = \frac{1}{2}$

General equation of the quadratic polynomial = x^2 - (Sum of the roots) x + Product of the roots

$\therefore$The quadratic polynomial whose zeroes are 3α and 3β is :

$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$

$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$

$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$

$= x^2 - \frac{9}{2}x + \frac{9}{2}$

$= \frac{1}{2}\left(2x^2 - 9x + 9\right)$

$\therefore$ the required quadratic polynomial is $\frac{1}{2}\left(2x^2 - 9x + 9\right)$.

31. All the numbers between 100 and 500 which are divisible by 8 are

104, 112, 120, 128, ..., 496

Here, $a_1 = 104$

$a_2 = 112$

$a_3 = 120$

$a_4 = 128$

$\therefore a_2 - a_1 = 112 - 104 = 8$

$a_3 - a_2 = 120 - 112 = 8$

$a_4 - a_3 = 128 - 120 = 8$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ...$ (8 each)

$\therefore$ This sequence is an arithmetic progression whose difference is 8.

Here, a = 104

d = 8

l = 496

Let the number of term s be n . Then,

l = a + (n - 1)d

$\Rightarrow$ 496 = 104 + (n - 1)8

$\Rightarrow$ 392 = (n - 1)8

$\Rightarrow$ (n - 1)8 = 392

$\Rightarrow$ n - 1 = 49

$\Rightarrow$ n = 49 + 1

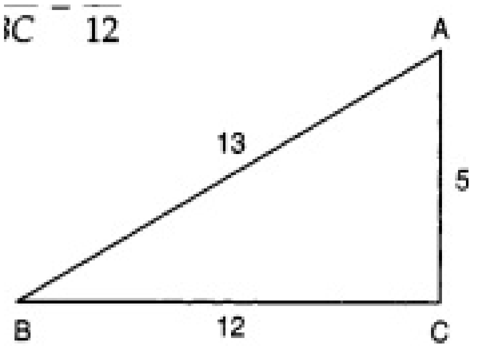
$\Rightarrow$ n = 50

$\therefore S_n = \frac{n}{2}(a+l)$

$= \left(\frac{50}{2}\right)(104+496)$

= (25) (600)

= 15000

32. 

We have

$\cot B = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$

Let us draw a right triangle ABC, in which $\angle C = 90^\circ$ such that

Base = BC = 12 units and, Perpendicular = AC = 5 units.

Applying Pythagoras Theorem in,ΔBCA we get

$AB^2 = BC^2 + AC^2$

$\Rightarrow \quad AB^2 = 12^2 + 5^2 = 169$

$\Rightarrow \quad AB = \sqrt{169} = 13$

$\therefore \quad \sin B = \frac{AC}{AB} = \frac{5}{13}, \tan B = \frac{AC}{BC} = \frac{5}{12}$ and, $\sec B = \frac{AB}{BC} = \frac{13}{12}$

Now, L.H.S$= \tan^2 B - \sin^2 B$

$\Rightarrow \quad \text{L.H.S} = (\tan B)^2 - (\sin B)^2$

$\Rightarrow \quad \text{L.H.S} = \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{25}{144} - \frac{25}{169}$

$\Rightarrow \quad \text{L.H.S} = 25\left(\frac{1}{144} - \frac{1}{169}\right) = 25\left(\frac{169-144}{144\times169}\right)$

$\Rightarrow \quad \text{L.H.S} = 25 \times \frac{25}{144\times169} = \frac{25\times25}{144\times169} = \frac{5^2\times5^2}{12^2\times13^2}$ ....(i)

and, $\text{R.H.S} = \sin^4 B \cdot \sec^2 B$

$\Rightarrow \quad \text{R.H.S} = (\sin B)^4 (\sec B)^2$

$= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2$

$= \frac{5^4\times13^2}{13^4\times12^2} = \frac{5^4}{13^2\times12^2}$

$= \frac{5^2\times5^2}{13^2\times12^2}$

.....(ii)

From (i) and (ii), we have

$\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$

Hence proved.

OR

In ΔABC, by angle sum property

A + B + C = 180°

$\Rightarrow$ B + C =180° - A........(1)

Now , L.H.S

$\sin \frac{B+C}{2}$
$= \sin\left(\frac{180^\circ - A}{2}\right)$ [From (1)]
$= \sin\left(\frac{180^\circ}{2} - \frac{A}{2}\right)$
$= \sin\left(90^\circ - \frac{A}{2}\right)$ $[\sin(90^\circ - \theta) = \cos\theta]$
$= \cos\frac{A}{2} = RHS$ Hence Proved.

33. Radius of a circular park = 105m

$\therefore$ Area of a circular park $= \pi(105)^2 sq.m$

Now, radius of outer circle cincluding road = 105 + 21 = 126m

$\Rightarrow$ Area of outer circle including road $= \pi(126)^2 sq.m$

$\therefore$ Area of road = Area of outer circle including road - Area of circular park

$= \pi(126)^2 - \pi(105)^2$
$= \pi\left[(126)^2 - (105)^2\right]$
$= \frac{22}{7}[(126 + 105)(126 - 105)]$
$= \frac{22}{7}[231 \times 21]$
= 15246 sq.m

34. The total numbers of integers between 0 and 100 = 99

Let E_1 be the event of getting a integer.

Numbers divisible by 7 are 7,14,21,28,35,42,49,56,63,70,77,84,91,98

i. $\therefore$ Number of favorable outcomes=14

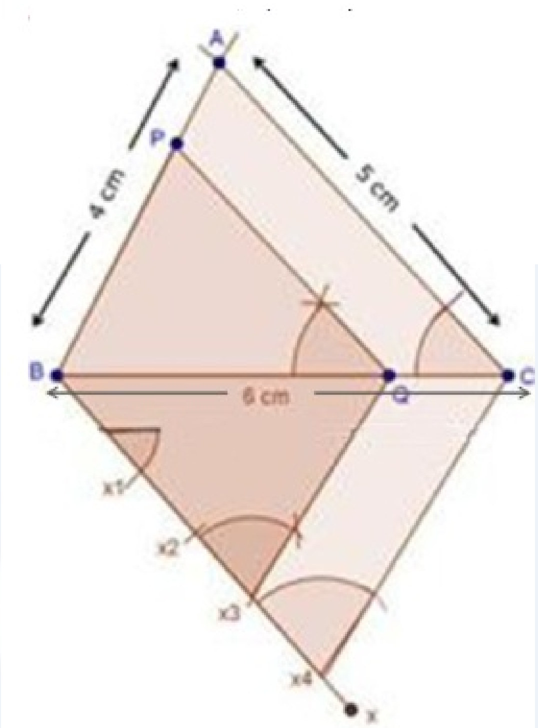
   $P(E_1) = \frac{14}{99}$

ii. Let E_2 be the event of getting a number which is not divisible by 7

   $\therefore P(E_2)=1-P(E_1)= 1 - \frac{14}{99} = \frac{85}{99}$

Section D

35. Steps of construction



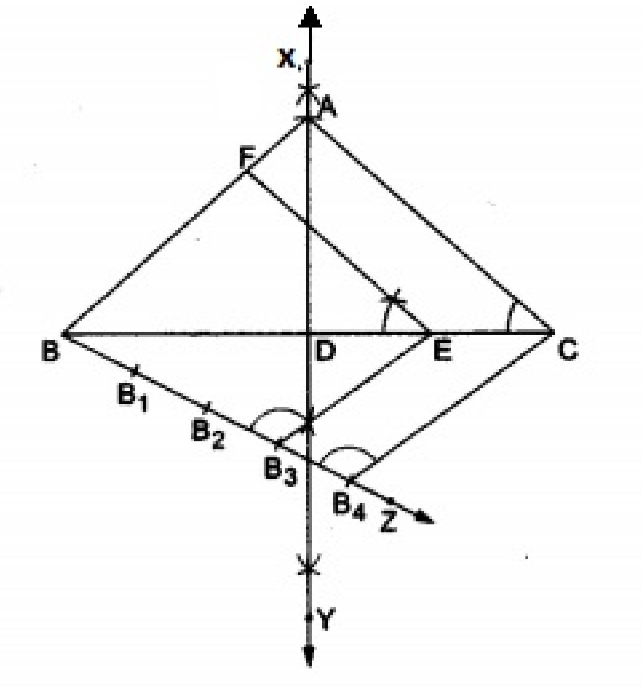
i. Draw a line segment BC of 6 cm.
ii. With centres B and C, and radii 4 cm and 6 cm respectively draw two arcs which intersect each other at A.
iii. Join AB and AC.
iv. At B, draw $\angle CBX$ of any measure.
v. Starting from B, cut 4 equal parts on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4$
vi. Join X_4C
vii. Through X_3, draw $X_3Q \| X_4C$
viii. Through Q, draw $QP \| CA$

   $\therefore \triangle PBQ \sim \triangle ABC$

OR

Steps of Constructions:

1. Draw a line segment BC = 6 cm.
2. Draw a perpendicular bisector XY of BC, cutting BC at D.
3. With D as centre and radius 4 cm, draw an arc cutting XY at A.
4. Join AB and AC.
   Thus, isosceles $\triangle$ABC having base 6 cm and altitude 4 cm is obtained.



5. Below BC, make an acute angle $\angle$CBZ.
6. Along BZ, mark off four points B_1, B_2, B_3 and B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
7. Join B_4C.
8. From B_3, draw $B_3E \parallel B_4C$, meeting BC at E.
9. From E, draw EF $\parallel$ CA, meeting BA at F.
   Then, $\triangle$FBE is the required triangle, each of whose sides is $\frac{3}{4}$ times the corresponding side of $\triangle$ABC.
   Proof Since EF $\parallel$ CA, we have $\triangle FBE \sim \triangle ABC$.
   $\therefore \quad \frac{FB}{AB} = \frac{EF}{CA} = \frac{BE}{BC} = \frac{3}{4}$

36. $\triangle ABC \sim \triangle DEF$ (given)

$\therefore \quad \angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$ ...(i)

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ ...(ii)

Since BM and EN are medians, we have

CA = 2AM = 2CM

and FD = 2DN = 2FN.

$\therefore$ from (ii), we have

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2AM}{2DN} = \frac{2CM}{2FN}$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{AM}{DN} = \frac{CM}{FN}$ ...(iii)

a. In $\triangle AMB$ and $\triangle DNE$, we have

   $\angle BAM = \angle EDN$ [$\because \angle A = \angle D$ from (i)]

   and $\frac{AB}{DE} = \frac{AM}{DN}$ [from (iii)].

   $\therefore \quad \triangle AMB \sim \triangle DNE$ [by SAS-similarity]

b. In $\triangle CMB$ and $\triangle FNE$, we have

   $\angle BCM = \angle EFN$ [$\because \angle C = \angle F$ from (i)]

   and $\frac{BC}{EF} = \frac{CM}{FN}$ [from (iii)]

   $\therefore \quad \triangle CMB \sim \triangle FNE$ [by SAS-similarity].

c. As proved above, $\triangle AMB \sim \triangle DNE$ and so

   $\frac{AB}{DE} = \frac{BM}{EN}$ ...(iv)

   From (ii) and (iv), we get

   $\frac{BM}{EN} = \frac{AC}{FD}$.

37. Let father's age (in years) be x and that of son's be y.

$\Rightarrow x + 2y = 70$ (by first condition)

$2x + y = 95$(by second condition)

This system of equations may be written as

$x + 2y - 70 = 0$

$2x + y - 95 = 0$

By cross-multiplication, we get

$\frac{x}{2\times-95-(-70)} = \frac{-y}{1\times-95-2\times-70} = \frac{1}{1\times1-2\times2}$

$\Rightarrow \frac{x}{-190+70} = \frac{-y}{-95+140} = \frac{1}{-3}$

$\Rightarrow \frac{x}{-120} = \frac{y}{-45} = \frac{1}{-3} \Rightarrow x = \frac{-120}{-3} = 40$ and $y = \frac{-45}{-3} = 15$

$\therefore$ father's age is 40 years and the son's age is 15 years.

OR

The given system of equation is $4x - 3y + 4 = 0$ and $4x + 3y - 20 = 0$

Now, $4x - 3y + 4 = 0$

$x = \frac{3y-4}{4}$

Solution table for $4x - 3y + 4 = 0$

x	2	-1
y	4	0

We have,

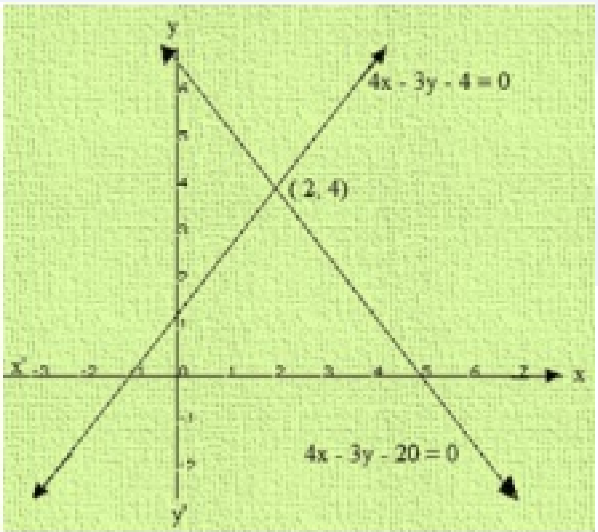
$4x + 3y - 20 = 0$

$x = \frac{20-3y}{4}$

Solution table for $4x + 3y - 20 = 0$

x	5	2
y	0	4

Graph of the given system is:



Clearly, the two lines intersect at A(2, 4)

We also observe that the lines meet x - axis B(-1, 0) and C(5, 0)

Thus x = 2 and y = 4 is the solution of the given system of equations.

AD is drawn perpendicular A on x - axis. Clearly we have,

AD = y - coordinate point A(2, 4)

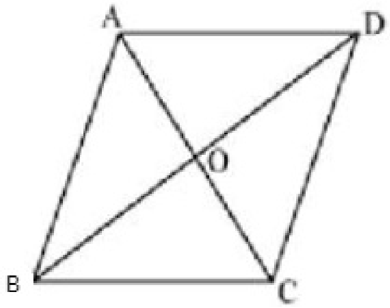
AD = 3 and BC = 5 - (-1) = 4 + 1 = 6

Area of the shaded region = $\frac{1}{2}$ × base × altitude

$= \frac{1}{2} \times 6 \times 4$

= 12 sq. units

38.



Let us suppose that ABCD be the given rhombus.

In this fig, AB = BC = CD = AD = 20 cm and BD = 24 cm

Since, we know that the diagonals of a rhombus bisect each other,

$\therefore$ OA = $\frac{1}{2}AC$

$\therefore$ AC = 2 $\times$ OA .... (i)

Also, OB = $\frac{1}{2}BD$

= $\frac{1}{2} \times 24$

= 12 cm

and $\angle$AOB = 90^o

Now, in right ΔAOB, using Pythagoras theorem, we get,

$\mathrm{OA}^2 + \mathrm{OB}^2 = \mathrm{AB}^2$

$\Rightarrow OA^2 + 12^2 = 20^2$

$\Rightarrow OA^2 + 144 = 400$

$\Rightarrow OA^2 = 256$

$\Rightarrow$ OA = 16

$\Rightarrow$ AC = 2 $\times$ OA = 32 (from(i))

Thus, the length of the other diagonal is 32 cm.

Area of the given rhombus = $\frac{1}{2} \times AC \times BD$

= $\frac{1}{2} \times 32 \times 24$

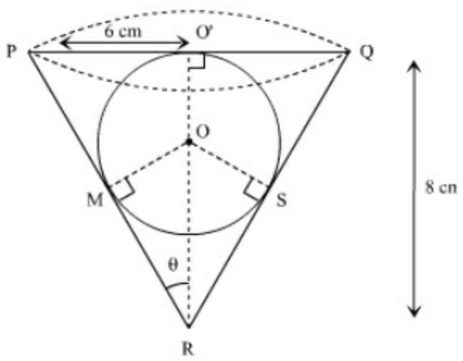
= 384 cm^2

Hence, the area of the given rhombus is 384 cm^2

OR

Radius (R) of conical vessel = 6 cm

Height (H) of conical vessel = 8 cm



Volume of conical vessel (V_c) = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \pi \times 6 \times 6 \times 8$

= 96 π cm^3

Let the radius of the sphere be r cm

In right Δ PO'R by pythagoras theorem We have

$l^2 = 6^2 + 8^2$

l = $\sqrt{36+64}$ = 10 cm

In right triangle MRO

$\sin\theta = \frac{OM}{OR}$

$\Rightarrow \quad \frac{3}{5} = \frac{r}{8-r}$

$\Rightarrow$ 24 - 3r = 5r

$\Rightarrow$ 8r = 24

$\Rightarrow$ r = 3 cm

$\therefore$ V_1 = Volume of the sphere ,= $\frac{4}{3}\pi \times 3^3 \text{cm}^3 = 36\pi\text{cm}^3$

V_2 = Volume of the water = Volume of the cone = $\frac{1}{3}\pi \times 6^2 \times 8\text{cm}^3 = 96\pi\text{cm}^3$

Clearly, volume of the water that flows out of the cone is same as the volume of the sphere i.e., V_1.

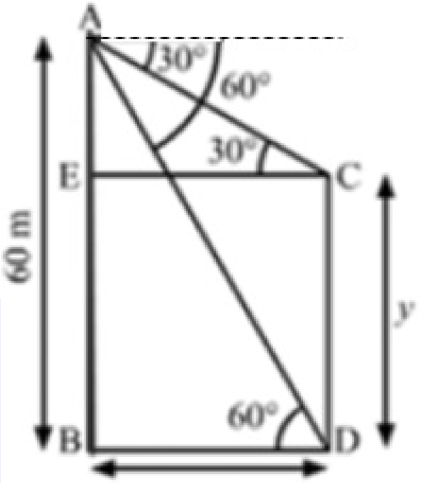
$\therefore$ Fraction of the water that flows out =$V_1 : V_2 = 36\pi : 96\pi = 3 : 8$

39. Let CD be the tower and AB be the building. Let height of the tower be y.

Let BD = CE = x and CD = BE = y

$\Rightarrow$ AE = AB - BE = 60 - y

It is given that the angle of depression of the top C and bottom D of the tower, observed from the top of the building be 30° and 60° respectively.



In right ΔABD,

$\tan 60^\circ = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{60}{x}$

$\Rightarrow x = \frac{60}{\sqrt{3}}$

On rationalising we get,

$x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x = \frac{60\sqrt{3}}{3}$

$\Rightarrow x = 20\sqrt{3}$

$\Rightarrow$ x = 20 $\times$ 1.732 = 34.64 m

Thus, the horizontal distance between AB and CD is 34.64 m.

40.

Class	**Frequency f_i**	**Mid- value x_i**	$u_i = \left(\frac{x_i - A}{h}\right)$	**$f_i u_i$**	**Cumulative frequency**
100 - 150	6	125	-2	-12	6
150 -					

200	7	175	-1	-7	13
200 - 250	12	225	0	0	25
250 - 300	3	275	1	3	28
300 - 350	2	325	2	4	30
	$\sum f_i = 30$			$\sum f_i u_i = -12$	

Let assumed mean = 225 and h = 50

i. Mean = $A + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$

$= 225 + 50\left(\frac{-12}{30}\right)$

$= 225 - 20 = 205$

ii. $\frac{N}{2} = \frac{30}{2}$ = 15

Cumulative frequency just after 15 is 25.Corresponding Class Interval is 200 - 250

$\therefore$ Median class is 200 - 250.

$l = 200$, f = 12, $\frac{N}{2}$ = 15, h = 50, c.f. = 13

$\therefore$ Median $= l + h\left(\frac{\frac{N}{2} - c.f.}{f}\right)$

$= 200 + 50\left(\frac{15-13}{12}\right)$

$= 200 + \frac{50\times2}{12}$

$= 200 + \frac{25}{3}$

$= 200 + 8.33 = 208.33$