CBSE Class 10th Mathematics
Standard Sample Paper - 10

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. Any ____________ is of the form 4q + 1 or 4q + 3 for some integer ‘q’.

   a. positive even integer

   b. positive odd integer

   c. prime number

   d. composite number

2. All non-terminating and non-recurring decimal numbers are

   a. rational numbers

   b. irrational numbers

c. integers

d. natural numbers

3. An ‘ogive’ is used to determine

a. mode

b. None of these

c. median

d. mean

4. A train travels 360km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey, then the actual speed of the train is

a. 48 km/hr

b. 36 km /hr

c. 40 km/hr

d. 45 km/hr

5. The angle of elevation and the angle of depression from an object on the ground to an object in the air are related as

a. greater than

b. equal

c. less than

d. None of these

6. If $sin\ \theta\ = \frac{1}{2}$ and $cos\phi = \ \frac{1}{2}$, then the value of $(\theta\ + \phi)$ is

a. 0°

b. 30°

c. 90°

d. 60°

7. $\frac{1-\tan^2 45^{\circ}}{1+\tan^2 45^{\circ}} =$

a. $sin 45^{\circ}$

b. 0

c. $cos 45^{\circ}$

d. $tan 45^{\circ}$

8. One number is chosen randomly from the integers 1 to 50. The probability that it is divisible by 4 or 6 is

a. $\frac{2}{25}$

b. $\frac{8}{25}$

c. $\frac{4}{25}$

d. $\frac{6}{25}$

9. The distance between $(at^2, 2at)$ and $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ is

a. $a\left(t^2 + \frac{1}{t^2}\right)$ *units*

b. $a\left(t - \frac{1}{t}\right)^2$ *units*

c. $a\left(t + \frac{1}{t}\right)^2$ *units*

d. $\left(t + \frac{1}{t}\right)^2$ *units*

10. If the point M(– 1, 2) divides the line segment PQ in the ratio 3 : 4, where the co – ordinates of P are (2, 5), then the co – ordinates of Q are

a. (– 5, 2)

b. (– 5, – 2)

c. (5, – 2)

d. (5, 2)

11. Fill in the blanks:

Total outer surface area of Right circular hollow cylinder = 2πrh + ________ sq units.

12. Fill in the blanks:

The number of zeroes of the cubic polynomial $x^3 - 3x^2 - x + 3$ is ________.

OR

Fill in the blanks:

If α, β and γ are zeroes of a cubic polynomial p(x) = $ax^3 + bx^2 + cx + d$, then $\alpha\beta\gamma$ = ________.

13. Fill in the blanks:

The probability of an event is greater than or equal to ________ and less than or equal to ________.

14. Fill in the blanks:

210 is the ________ term of the AP: 21, 42, 63, 84, ...,.

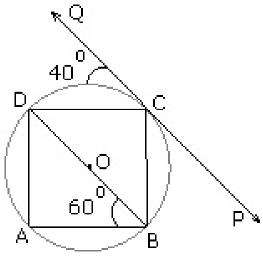
15. Fill in the blanks:

If a point P lies on the circle, then ________ tangents can be drawn.

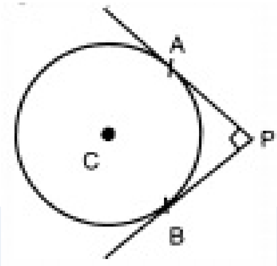
16. If $\frac{p}{q}$ is a rational number (q $\neq$0), what is condition of q so that the decimal expansion of $\frac{p}{q}$ is terminating?

17. In the given figure, ABCD is a cyclic quadrilateral and PQ is a tangent to the circle at C.

If BD is a diameter, $\angle OCQ = 40^o$ and $\angle ABD = 60^o$, find $\angle BCP$



18. In fig., PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then find the length of each tangent.



19. If the first and last terms of an A.P. are 1 and 11 respectively and the sum of its terms is 36, then find the number of terms in the A.P.

OR

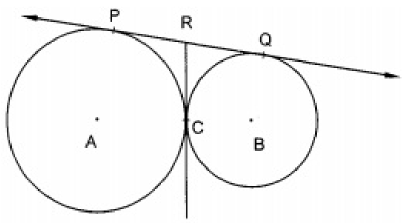
If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a.

20. Determine the nature of the roots of quadratic equation:
$3x^2 - 2\sqrt{6}x + 2 = 0$

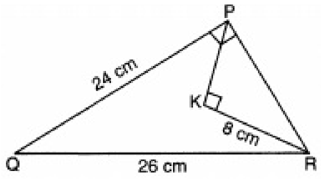
Section B

21. A Box contains cards numbered 3,5,7,9,..,35,37. A card is drawn at random from the box. Find the probability that the number on the card is a prime number.

22. In figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

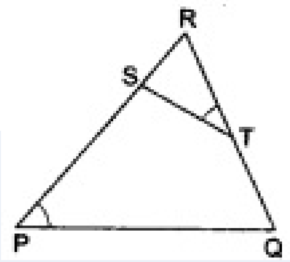


23. In the given triangle PQR, $\angle$QPR = 90^o, PQ = 24 cm and QR = 26 cm and in $\triangle$PKR, $\angle$PKR = 90^o and KR = 8 cm, find PK.

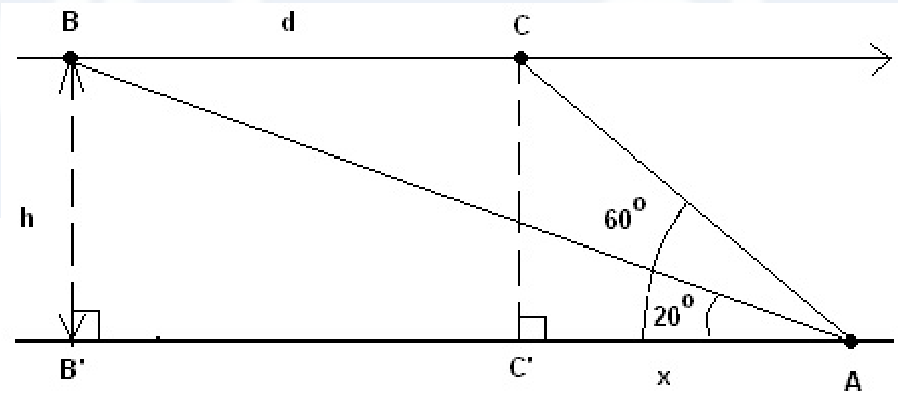


OR

In the given figure, S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.



24. An airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is 20^o and at 10:01 am, it is 60^o.



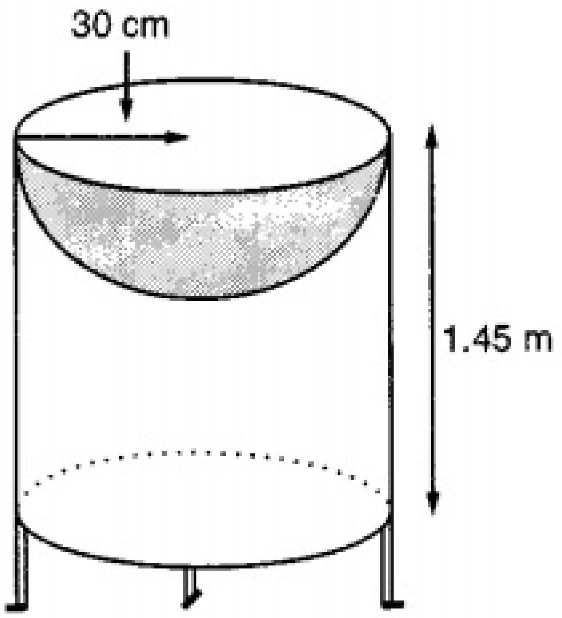
i. What is the distance 'd' is covered by airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour?
ii. What is the altitude 'h' of the airplane? (round answer to 2 decimal places).

25. Find x in terms of a, b and c:
$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$

OR

The length of a rectangle is twice its breadth and its area is 288 cm^2. Find the dimensions of the rectangle.

26. Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm.



By using the above-given information, find the following:

i. The curved surface area of the hemisphere.
ii. The total surface area of the bird-bath. (Take π = 22/7)

Section C

27. Prove that (3 - $\sqrt{5}$) is an irrational number.

OR

Prove that $\sqrt{5}$ is irrational .

28. Prove that the points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) are the vertices of a square.

29. A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the

digits are reversed. Find the number.

OR

The monthly incomes of A and B are in the ratio 8 : 7 and their expenditures are in the ratio 19:16. If each saves Rs 5000 per month, find the monthly income of each.

30. If α, β are the zeros of the polynomial $2x^2 - 4x + 5$. find the value of (i) $\alpha^2 + \beta^2$ (ii) $(\alpha - \beta)^2$.

31. Find the sum of all natural numbers less than 200 which are divisible by 5.

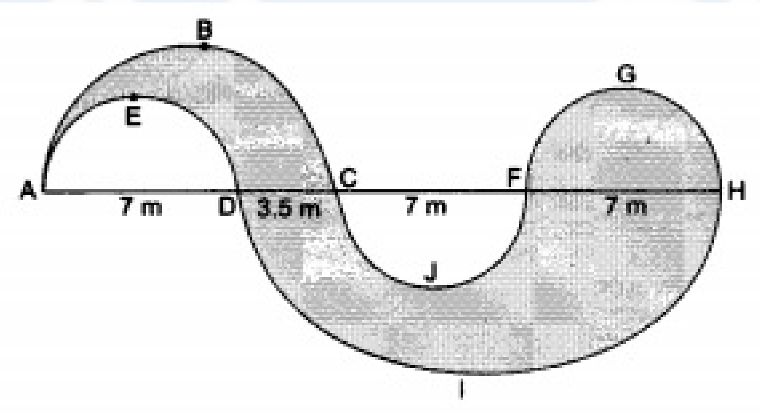
32. Prove: $\frac{\tan A+\sec A-1}{\tan A-\sec A+1} = \frac{1+sinA}{cosA}$.

OR

Prove the identity:

$(\sin^8\theta - \cos^8\theta) = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

33. Find the area of the shaded region of the figure given below.



34. A die is thrown twice. What is the probability that

i. 5 will not come up either time?
ii. 5 will come up at least once?

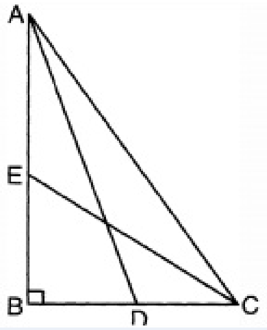
Section D

35. Draw a quadrilateral ABCD with AB = 3 cm, AD = 2.7 cm, DB = 3.6 cm, $\angle$B =110^{o} and BC = 4.2 cm. Construct a quadrilateral A'BC'D similar to the quadrilateral ABCD so that the diagonal D'B may be 4.8.

OR

Construct a triangle ABC in which BC = 6 cm, $\angle$BAC = 60^{0} and median through A is 4.5 cm. Construct a $\triangle$A'BC' similar to $\triangle$ABC with BC' = 8 cm. Write steps of construction.

36. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE.



37. Solve for x and y: $\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}, \frac{5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$

OR

The perimeter of a rectangle is 52 cm, where length is 6 cm more than the width of the rectangle. Form the pair of linear equations for the above situation and find the dimensions of the rectangle graphically.

38. A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm, the radius of the hemisphere is 60 cm and height of the cone is 120 cm, assuming that the hemisphere and the cone have common base.

OR

How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.

39. The elevation of a tower at a station A due north of is α and at a station B due west of A is β. prove that the height of the tower is $\frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha-\sin^2\beta}}$.

40. The frequency distribution of marks obtained by 53 students out of 100 in a certain examination is given below:

Marks	Number of students
0 - 10	5
10 - 20	3
20 - 30	4
30 - 40	3
40 - 50	3
50 - 60	4
60 - 70	7
70 - 80	9
80 - 90	7
90 - 100	8

i. Draw the cumulative frequency curve or an ogive (of the less than type).
ii. Draw the cumulative frequency curve or an ogive (of the more than type).

Solution

Section A

1. (b) positive odd integer
   Explanation:

   Let a be a given positive odd integer.

   Applying Euclid's Division Lemma to a and $b = 4$,

   We have, $a = 4q + r$ where $0 \leqslant r < 4$

   $\Rightarrow r$ = 0, 1, 2, 3

   $\Rightarrow a = 4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$

   But $a = 4q$ and $4q + 2 = 2(2q + 1)$ are clearly even.

   Also $a = 4q, 4q + 1, 4q + 2, 4q + 3$ are consecutive integers,

   therefore any positive odd integer is of the form $4q + 1$ and $4q + 3$

   where q is some integer.

2. (b) irrational numbers
   Explanation:

   All non-terminating and non-recurring decimal numbers are irrational numbers. A number is rational if and only if its decimal representation is repeating or terminating.

3. (c) median
   Explanation:

   An ogive is used to determine how many data values lie above or below a particular value in a data set. In other words it is used to determine the Median of a grouped data

4. (c) 40 km/hr

   Explanation:

   Let the actual speed of the train be x km/hr
   Time taken to cover 360 km at this speed = $\frac{360}{x}$ hrs.
   Time taken to cover 360 km at the increased speed = $\frac{360}{x+5}$ hrs.
   According to condition, $\frac{360}{x} - \frac{360}{x+5} = 1$
   $\Rightarrow 360\left[\frac{1}{x} - \frac{1}{x+5}\right] = 1$
   $\Rightarrow 360\left[\frac{x+5-x}{x(x+5)}\right] = 1$
   $\Rightarrow 360\left[\frac{5}{x(x+5)}\right] = 1$
   $\Rightarrow x^2 + 5x - 1800 = 0$
   $\Rightarrow x^2 + 45x - 40x - 1800 = 0$
   $\Rightarrow x(x+45) - 40(x+45) = 0$
   $\Rightarrow (x-40)(x+45) = 0$
   $\Rightarrow x - 40 = 0$ and $x + 45 = 0$
   $\Rightarrow x = 40$ km/hr and $x = -45$ km/hr [But $x = -45$ is not possible]
   Therefore, the actual speed of train is 40 km/hr.

5. (b) equal

   Explanation:

   The angle of elevation and the angle of depression from an object on the ground to an object in the air are related as equal if the height of objects are the same.

6. (c) 90°

   Explanation:

   Given: $\sin\theta = \frac{1}{2}$
   $\Rightarrow \sin\theta = \sin 30^\circ$
   $\Rightarrow \theta = 30^\circ$
   And $\cos\phi = \frac{1}{2}$
   $\Rightarrow \cos\phi = \cos 60^\circ$

$\Rightarrow \theta = 60^\circ$

$\theta + \phi = 30^\circ + 60^\circ = 90^\circ$

7. (b) 0

   Explanation:

   Given: $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

   = $\frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1}$ = 0/2

   = 0

8. (b) $\frac{8}{25}$

   Explanation:

   numbers divisible by 4 = 4 8 12 16 20 24 28 32 36 40 44 48

   numbers divisible by 6 = 6 12 18 24 30 36 42 48

   Number of possible outcomes = {4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 48} = 16

   Number of Total outcomes = 50

   ∴ Required Probability = $\frac{16}{50} = \frac{8}{25}$

9. (c) $a\left(t + \frac{1}{t}\right)^2$ $units$

   Explanation:

   The distance between $(at^2, 2at)$ and $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

   = $\sqrt{\left(\frac{a}{t^2} - at^2\right)^2 + \left(\frac{-2a}{t} - 2at\right)^2}$

   = $a\sqrt{\frac{1}{t^4} + t^4 - 2 + \frac{4}{t^2} + 4t^2 + 8}$

   = $a\sqrt{\frac{1}{t^4} + t^4 + \frac{4}{t^2} + 4t^2 + 6}$

   = $a\sqrt{\frac{1}{t^4} + t^4 + 4 + 2 + \frac{4}{t^2} + 4t^2}$

   = $a\sqrt{\left(t^2 + \frac{1}{t^2} + 2\right)^2}$

   = $a\left(t^2 + \frac{1}{t^2} + 2\right)$

= $a\left(t+\frac{1}{t}\right)^2$ units

10. (b) (– 5, – 2)

    Explanation:

    Let the coordinates of Q be (x, y) .
    $\therefore -1 = \frac{3\times x+4\times 2}{3+4} \Rightarrow$
    $-7 = 3x + 8 \Rightarrow 3x = -15 \Rightarrow$
    $x = -5$

    And $2 = \frac{3\times y+4\times 5}{3+4} \Rightarrow 14 = 3y + 20 \Rightarrow$
    $3y = -6 \Rightarrow y = -2$
    Therefore, the required coordinates are $(-5, -2)$.

11. $2\pi(R^2 - r^2)$

12. 3 OR $\frac{-d}{a}$

13. 0, 1

14. 10^{th}

15. one and only one

16. The rational number of the form , $\frac{p}{q}$ will have a terminating decimal expansion,

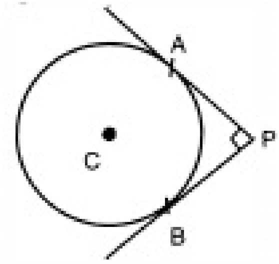
    If q is power of 10
    or q is power of 2×5

    or q is of form $2^n \times 5^m$
    Any rational number $\frac{p}{q}$ will have terminating decimal expansion,if q is of the form $2^n \times 5^m$ where n and m are positive integers.

17. $\because$ BD is a diameter

    $\therefore \angle BCD = 90^o$ [Angle in the semi-circle]

    $\therefore \angle BCP = 180^o - 90^o - 40^o = 50^o$

18. 

Construction: Join AC and BC

Now, $AC \perp AP \; and \; CB \perp BP$

$\angle APB = 90^0$

Therefore, CAPB will be a square

CA = AP = PB = BC = 4 cm

$\therefore$ Length of tangent = 4 cm.

19. We are given that

a = 1

$a_n = 11$

$s_n = 36$

36 = $\frac{n}{2}$ {(a + a_n)} { using formula for sum to n terms}

36 = $\frac{n}{2}$ [12]

72 = 12 n

$n = \frac{72}{12}$

n = 6

OR

Given $\frac{4}{5}$, a, 2 are in AP

$\Rightarrow$ common difference d = a - $\frac{4}{5}$ = 2 - a

$\Rightarrow$ 5a - 4 = 10 - 5a

$\Rightarrow$ 10a = 14

$\Rightarrow$ a = $\frac{7}{5}$

20. We have the equation

$3x^2 - 2\sqrt{6}x + 2 = 0$

by comparing with $ax^2 + bx + c = 0$ we get

Here, a = 3, b = $-2\sqrt{6}$ and c = 2

For knowing the nature of roots, we have to find discriminant

$\therefore$ D = b^2 - 4ac

$= (-2\sqrt{6})^2 - 4 \times 3 \times 2$

= 24 - 24

= 0

Since, D = 0, therefore, roots of the given equation are real and equal.

Section B

21. Given numbers 3, 5, 7, 9 ......, 35, 37 form an AP with a = 3 and d = 2.

Let T_n = 37. Then,

3 + (n – 1)2 = 37

⇒ 3 + 2n – 2 = 37

⇒ 2n = 36

⇒ n = 18

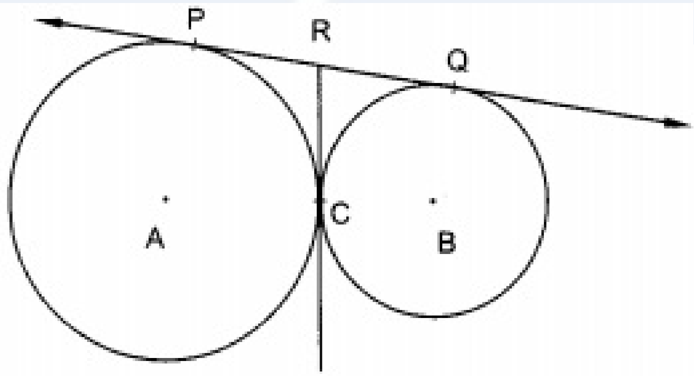
Thus, total number of outcomes = 18.

Let E be the event of getting a prime number.

Out of these numbers, the prime numbers are 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37.

The number of favorable outcomes = 11.

Therefore, P(getting a prime number) = P(E) = $\frac{\text{Number of outcomes favorable to } E}{\text{Number of all possible outcomes}} = \frac{11}{18}$

22. 

We know that the tangents drawn from an external point to a circle are equal.

$\therefore$ RP = RC and RC = RQ

$\Rightarrow$ RP = RQ

$\Rightarrow$ R is the mid-point of PQ.

23. According to the question,

(Given) $\angle$QPR =90^0

By Pythagoras theorm,

$QR^2 = QP^2 + PR^2$

$\therefore$ PR = $\sqrt{26^2 - 24^2}$

$= \sqrt{100}$ = 10 cm

$\angle$PKR = 90^o

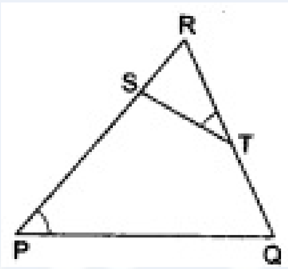
By Pythagoras theorm,

PK = $\sqrt{PR^2 - KR^2} = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$

= $\sqrt{36}$ = 6 cm

OR

According to questions it is given that S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$



To Prove $\triangle RPQ \sim \triangle RTS$

Proof In $\triangle RPQ$ and $\triangle RTS$, we have

$\angle P = \angle RTS$ (given)

$\angle R = \angle R$ (common)

$\therefore \quad \triangle RPQ \sim \triangle RTS$ [by AA-similarity].

24. i. Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

Given: Speed = 600 miles/hour

Thus, distance d = 600 $\times$ $\frac{1}{60}$ = 10 miles

ii. Now, tan 20^o = $\frac{BB'}{B'A}$ = $\frac{h}{10+x}$ ....eq(1)

And tan 60^o = $\frac{CC'}{C'A} = \frac{BB'}{C'A}$ = $\frac{h}{x}$

x = $\frac{h}{tan 60^o}$ = $\frac{h}{\sqrt{3}}$

Putting the value of x in eq(1), we get,

tan 20^o = $\frac{h}{10+\frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

$0.364(10\sqrt{3} + h) = \sqrt{3}\ h$

$6.3 + 0.364\ h = 1.732\ h$

$1.368\ h = 6.3$

$h = 4.6$

Thus, the altitude 'h' of the airplane is 4.6 miles.

25. $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

$\Rightarrow \frac{a(x-b)+b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$

$\Rightarrow [(a + b)x - 2ab]\ (x - c) = 2c\ [x^2 - (a + b)x + ab]$

$\Rightarrow (a + b)x^2 - 2abx - c(a + b)x + 2abc = 2cx^2 - 2c(a + b)x + 2abc$

$\Rightarrow (a + b - 2c)x^2 = (2ab + ac + bc - 2ca - 2bc)x$

$\Rightarrow (a + b - 2c)x = (2ab - ca - bc)$

$\Rightarrow x = \frac{2ab-ca-bc}{a+b-2c}$

OR

Let the breadth be x cm

Then, the length is 2x cm

$\therefore$ Area = length $\times$ breadth = 288 cm^2

$\Rightarrow 2x(x) = 288$

$\Rightarrow 2x^2 = 288$

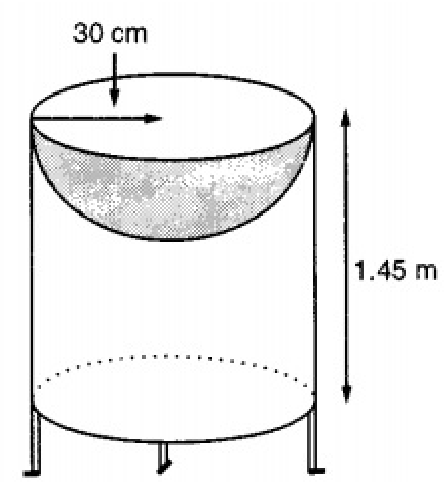
$\Rightarrow x^2 = 144$

$\Rightarrow x = \sqrt{144} \Rightarrow x = \pm 12$

$\Rightarrow x = 12$ [$\because$breadth can't be negative]

Therefore, breadth = 12 cm

and length = 2(12) = 24 cm

26. 

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

i. Curved surface area of the hemisphere = $2\pi r^2$

= $2 \times 3.14 \times 30^2$ = 0.56 m^2

ii. Let S be the total surface area of the bird-bath.

S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow \quad S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000 \text{ cm}^2 = 3.3 \text{ m}^2$

Section C

27. Let $3 - \sqrt{5} = \frac{p}{q}$

$\therefore 3 - \sqrt{5} = \frac{P}{q}($ where p and q are integers, co-prime and $q \neq 0)$

$\Rightarrow \quad 3 - \frac{p}{q} = \sqrt{5}$

$\Rightarrow \quad \frac{3q-p}{q} = \sqrt{5}$

$(3q - p)$ and q are integers, so $\left(\frac{3q-p}{q}\right)$ is a rational

number, but $\sqrt{5}$ is an irrational number. This contradiction arises because of our wrong assumption.

So $(3 - \sqrt{5}$ is an irrational number

OR

Let take √5 as a rational number

If a and b are two co-prime number and b is not equal to 0.

We can write √5 = a/b

Multiply by b both side we get

b√5 = a

To remove root, Squaring on both sides, we get

$5b^2 = a^2$ ....(1)

Therefore, 5 divides a^2 and according to a theorem of rational number, for any prime number p which is divided 'a^2' then it will divide 'a' also.

That means 5 will divide 'a'. So we can write

a = 5c

and putting the value of a in equation (1) we get

$5b^2 = (5c)^2$

$5b^2 = 25c^2$

Divide by 25 we get

$b^{2/5} = c^2$

again using the same theorem we get that b will divide by 5

and we have already get that a is divided by 5

but a and b are co-prime number. so it is contradicting.

Hence √5 is an irrational number

28. Let A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) be the given points. One way of showing that ABCD is a square is to use the property that all its sides should be equal both its diagonals should also be equal. Now,

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$
$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$
$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$
$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$
$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$
$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

29. Let us suppose that the digit at unit place be x

Suppose the digit at tens place be y.
Thus, the number is 10y + x.

According to question it is given that the number is 4 times the sum of the two digits.
Therefore, we have
$10y + x = 4(x + y)$
$\Rightarrow 10y + x = 4x + 4y$
$\Rightarrow 4x + 4y - 10y - x = 0$
$\Rightarrow 3x - 6y = 0$
$\Rightarrow 3(x - 2y) = 0$
$\Rightarrow x - 2y = 0$
After interchanging the digits, the number becomes 10x + y.

Again according to question If 18 is added to the number, the digits are reversed.
Thus, we have
$(10y + x) + 18 = 10x + y$
$\Rightarrow 10x + y - 10y - x = 18$
$\Rightarrow 9x - 9y = 18$
$\Rightarrow 9(x - y) = 18$
$\Rightarrow x - y = \frac{18}{9}$
$\Rightarrow x - y = 2$
Therefore, we have the following systems of equations
$x - 2y = 0$ ....(1)

$x - y = 2$...............(2)
Here x and y are unknowns. Now let us solve the above systems of equations for x and y.
Subtracting the equation (1) from the (2), we get
$(x - y) - (x - 2y) = 2 - 0$
$\Rightarrow x - y - x + 2y = 2$
$\Rightarrow y = 2$
Now, substitute the value of y in equation (1), we get
$x - 2 \times 2 = 0$
$\Rightarrow x - 4 = 0$
$\Rightarrow x = 4$

Therefore the number is $10 \times 2 + 4 = 24$

Thus the number is 24

OR

Let the monthly incomes of A and B be Rs.8x and Rs. 7x respectively, and let their expenditures be Rs 19y and Rs 16 y respectively. Then, A's monthly savings = $Rs(8x - 19y)$.

And, B's monthly savings = $Rs(7x - 16y)$.

But, the monthly saving of each is Rs 5000.

$\therefore 8x - 19y = 5000$ ... (i)

and $7x - 16y = 5000$. ... (ii)

Multiplying (ii) by 19, (i) by 16 and subtracting the results, we get

$(19 \times 7 - 16 \times 8)x = (19 \times 5000 - 16 \times 5000)$

$\Rightarrow (133 - 128)x = 5000 \times (19 - 16)$

$\Rightarrow \quad 5x = 15000 \Rightarrow x = 3000$

$\therefore$ A's monthly income = Rs. (8x) = Rs.(8×3000) = Rs .24000.

And, B's monthly income = Rs. (7x) = Rs. (7×3000) = Rs. 21000.

30. $P(x) = 2x^2 - 4x + 5$

Here, a = 2 , b = – 4, c = 5

Let zeroes be α, β

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{2} = 2$

Product of zeroes $\alpha \times \beta = \frac{c}{a} = \frac{5}{2}$

i. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(\frac{5}{2})$

$= 4 - 5$

$\Rightarrow \alpha^2 + \beta^2 = -1$

ii. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (2)^2 - 4(\frac{5}{2})$

$= 4 - 2(5)$

$= 4 - 10$

= - 6

$(\alpha - \beta)^2 = -6$

31. All natural numbers less than 200 which are divisible by 5 are

5, 10, 15, 20, ..., 195

Here, $a_1 = 5$

$a_2 = 10$

$a_3 = 15$

$a_4 = 20$

: :

$\therefore a_2 - a_1 = 10 - 5 = 5$

$a_3 - a_2 = 15 - 10 = 5$

$a_4 - a_3 = 20 - 15 = 5$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ...$ (= 5 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 5.

Here, a = 5

d = 5

l = 195

Let the numbers of terms be n. Then,

l = a + (n - 1)d

$\Rightarrow$ 195 = 5 + (n - 1)d

$\Rightarrow$ (n - 1)5 = 190

$\Rightarrow$ n - 1 = 38

$\Rightarrow$ n = 39

$\therefore S_n = \frac{n}{2}(a+l)$

$= \left(\frac{39}{2}\right)(5+195)$

$= \left(\frac{39}{2}\right)(200)$

= (39) (100)

= 3900

32. $\frac{\tan A+\sec A-1}{\tan A-\sec A+1} = \frac{\tan A+\sec A-(\sec^2 A-\tan^2 A)}{\tan A-\sec A+1}$ $[\because 1 + \tan^2 A = \sec^2 A \Rightarrow \sec^2 A - \tan^2 A = 1]$

$= \frac{\tan A+\sec A-\{(\sec A-\tan A)(\sec A+\tan A)\}}{\tan A-\sec A+1}$

Now, take (tanA + secA) as a common term, we get

$= \frac{(\tan A+\sec A)(1-\sec A+\tan A)}{\tan A-\sec A+1}$

= tanA + secA

$= \frac{\sin A}{\cos A} + \frac{1}{\cos A}$

$= \frac{1+\sin A}{\cos A}$

$\therefore$ Hence proved

OR

We have,

LHS = $\sin^8\theta - \cos^8\theta = (\sin^4\theta)^2 - (\cos^4\theta)^2 = (\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$

$\Rightarrow$ LHS = $(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta)$

$\Rightarrow$ LHS = $(\sin^2\theta-\cos^2\theta)\{(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta\}$

$\Rightarrow$ LHS = $(\sin^2\theta - \cos^2\theta)\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\}$

$\Rightarrow$ LHS = $(\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$ = RHS

33. Area of the shaded region

= ar(ABCDEA) + ar(FGHF) + ar(HIDCJFH)

= ar(semicircle ABC) - ar(semicircle AED) + ar(semicircle FGH) + ar(semicircle HID) - ar(semidrcle FJC)

$= \left[\left\{\frac{1}{2}\pi \times \frac{21}{4} \times \frac{21}{4}\right\} - \left\{\frac{1}{2}\pi \times \frac{7}{2} \times \frac{7}{2}\right\} + \left\{\frac{1}{2}\pi \times \frac{7}{2} \times \frac{7}{2}\right\}\right.$

$\left. + \left\{\frac{1}{2}\pi \times \frac{35}{4} \times \frac{35}{4}\right\} - \left\{\frac{1}{2}\pi \times \frac{7}{2} \times \frac{7}{2}\right\}\right] \mathrm{m}^2$

$= \left\{\left(\frac{22}{7} \times \frac{441}{32}\right) + \left(\frac{1}{2} \times \frac{22}{7} \times \frac{35}{4} \times \frac{35}{4}\right) - \left(\frac{22}{7} \times \frac{49}{8}\right)\right\} \mathbf{m}^2$

$= \left\{\left(\frac{11\times63}{16}\right) + \left(\frac{55\times35}{16}\right) - \frac{77}{4}\right\} \mathrm{m}^2 = \left(\frac{693}{16} + \frac{1925}{16} - \frac{77}{4}\right) \mathrm{m}^2$

$= \left(\frac{693+1925-308}{16}\right) m^2$

$= \frac{1155}{8} \mathrm{m}^2$

= 144.38 m^2

Hence, the required area is 144.38 m^2

34. The possible outcomes are:

(1, 1), (1, 2), (1, 3), (1, 4), (1,5),(1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2,5),(2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3,5),(3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4,5),(4, 6),
(5, 1), (5, 2), (5, 3), (5, 4),(5,5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4),(6,5), (6, 6)
∴ Number of all possible outcomes = 6 $\times$ 6 = 36

$$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$$

i. Let E be the event that 5 will not come up either time. Then, the favourable outcomes are :
(1, 1), (1, 2), (1, 3), (1, 4), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 6)
∴ Number of favourable outcomes = 25
∴ P (E) = P (5 will not come up either time) = $\frac{25}{36}$

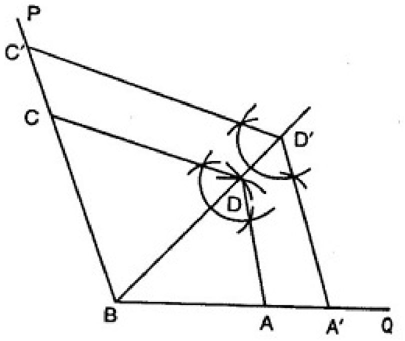
ii. Let E be event that 5 will come up at least once. Then, the favourable outcomes are:
(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 6)
∴ Number of favourable outcomes = 11
∴ P(E) = P(5 will come up at least once)= $\frac{11}{36}$

Section D

35. Steps of construction

i. Draw AB = 3 cm
ii. Make $\angle PBQ = 110^\circ$
iii. From BP, cut BC = 4.2 cm and from BQ, cut BA = 3 cm.
iv. Taking B and A as a centre, draw arcs of radii 3.6 cm and 2.7 cm respectively, let the arcs intersect in D. Join AD, DC. ABCD is the quadrilateral with the given data.
v. Now produce BD and cut BD' = 4.8 cm.

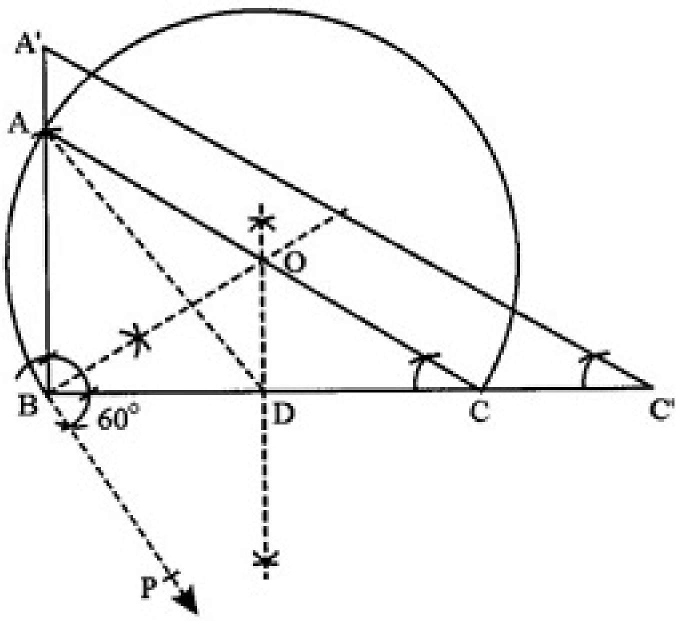
vi. Through D', draw lines parallel to DA and DC meeting BQ and BC' in A' and C' respectively.
vii. Join D'A' and D'C'.
ABC'S' is the required quadrilateral.



OR

Steps of construction:

i. Draw a line segment BC = 6 cm.
ii. Draw a ray BP making an angle of 60° with BC.
iii. Through B, draw $OB \perp BP$.
iv. Draw the right bisector of BC which intersects BC at D and OB at O.
v. With O as centre and OB as radius draw a circle.
vi. With D as centre and 4.5 cm as radius, draw an arc which intersects the circle at A.
vii. Join AB and AC, $\triangle ABC$ is the required triangle.
viii. Produce BC to C' such that BC' = 8 cm.
ix. Through C', draw C'A' || CA which meets BA produced at A'.
x. $\triangle A'BC'$ is the required triangle which is similar to $\triangle ABC$.



36. ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively.

AC = 5 cm

AD = $\frac{3\sqrt{5}}{2}$ cm

In $\triangle ABC, \triangle B = 90^\circ$, AD and CE are two medians.

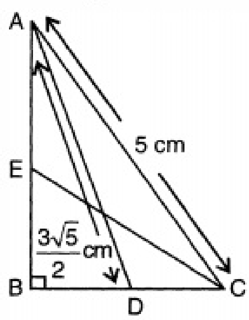
$\therefore AC^2 = AB^2 + BC^2 = (5)^2 = 25$.....(i)

In $\triangle ABD$, Using pythagoras theorem

$AD^2 = AB^2 + BD^2$

$\Rightarrow \left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$

$\Rightarrow \frac{45}{4} = AB^2 + \frac{BC^2}{4}$ ...(ii)



Similarly In $\triangle EBC$,

$CE^2 = BC^2 + \frac{AB^2}{4}$ ...(iii)

Subtracting equation (ii) from equation (i),

$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$

$\Rightarrow BC^2 = \frac{55}{3}$ ...(iv)

From equation (ii),

$AB^2 + \frac{55}{12} = \frac{45}{4}$

$\Rightarrow AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$

From equation (iii),

$CE^2 = \frac{55}{3} + \frac{20}{3\times4}$

$= \frac{240}{12} = 20$

$CE = \sqrt{20} = 2\sqrt{5}\text{cm}$

$\therefore CE = 2\sqrt{5}\ cm$

37. The given equations are

$\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = -\frac{3}{2}$.....(1)

and $\frac{5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$....(2)

Putting $\frac{1}{x+2y}$ =u and $\frac{1}{3x-2y}$ = v in equation (1) & equation (2) so that we may get two linear equations in the variables u & v as following:-

$\frac{1}{2}u + \frac{5}{3}v = -\frac{3}{2}$.....................(1)

$\frac{5}{4}u - \frac{3}{5}v = \frac{61}{60}$.................(2)

Multiplying (1) by 36 and (2) by 100, we get

$18u + 60v = -54$...............(3)

$125u - 60v = \frac{305}{3}$.............(4)

Adding (3) and (4),we get

$143u = \frac{305}{3} - 54 = \frac{305-162}{2} = \frac{143}{3}$

$\therefore \quad u = \frac{1}{3} = \frac{1}{x+2y}$

$\therefore x + 2y = 3$...........(5)

Putting value of u in (3), we get

$1 + 10v = -9$ (after dividing by 3)

$\therefore$ 10v $= -10$ or v $= -1$

$\Rightarrow -1 = \frac{1}{3x-2y}$

$\Rightarrow 3x - 2y = -1$......(6)

Adding (5) and (6), we get

$4x = 2$

$\therefore x = \frac{1}{2}$

Putting value of x in (5),

$\frac{1}{2} + 2y = 3$

or $2y = 3 - \frac{1}{2} = \frac{5}{2}$

$\therefore \quad y = \frac{5}{4}$

The required solution is $\text{x} = \frac{1}{2}, \text{y} = \frac{5}{4}$

OR

Let length of the rectangle = x cm and the width = y cm

According to question:

Perimeter = 52 cm

$\Rightarrow$ 2(x + y) = 52 [Since, perimeter of rectangle = 2(l + b)]

$\Rightarrow$ x + y = 26 ...(i)

Also, given length of the rectangle is 6 cm greater than width of rectangle.

$\Rightarrow$ x = y + 6

$\Rightarrow$ x - y = 6 ...(ii)

$\therefore$ Pair of linear equations are,

x + y = 26 ...(i) and x - y = 6 ...(ii)

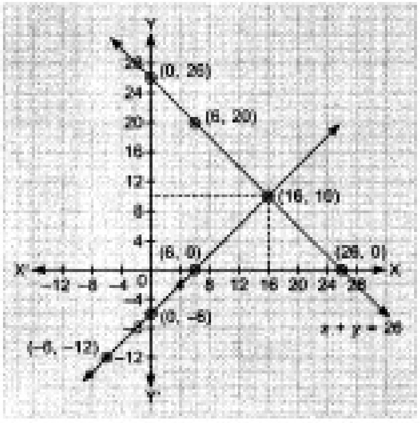
For graphical solution :

Points Coordinate Table for x + y = 26 is

x	0	26	6
y	26	0	20

Points Coordinate Table for x - y = 6 is

x	0	6	-6
y	-6	0	-12



From graph x = 16, y = 10

∴ Length = 16 cm, Breadth = 10 cm

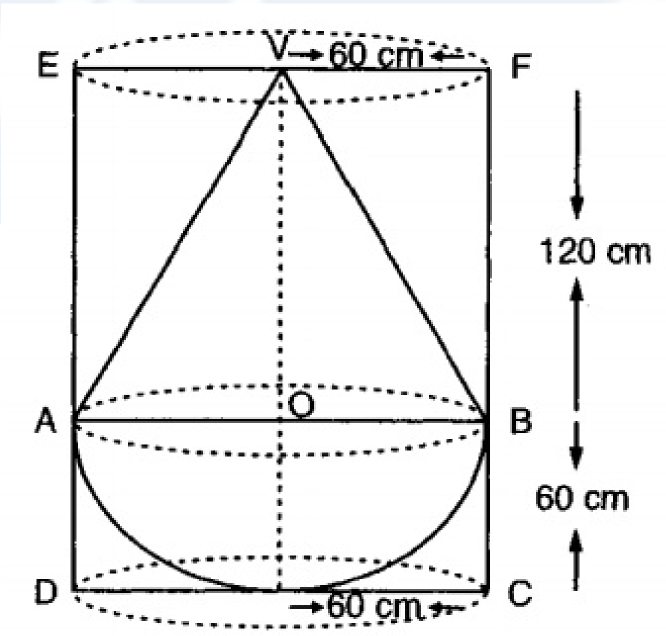
38. We have radius of cylinder = radius of cone = radius of hemisphere = 60 cm

Height of cone = 120 cm

∴ Height of cylindrical vessel = 120 + 60 =180 cm

∴ V = Volume of water that the cylinder contains = $\pi r^2 h = \left\{\pi \times (60)^2 \times 180\right\} \mathrm{cm}^3$

Let V_1 be the volume of the conical part. Then,



$V_1 = \frac{1}{3}\pi r^2 h_1$

$\Rightarrow \quad V_1 = \frac{1}{3} \times \pi \times 60^2 \times 120 \mathrm{cm}^3 = \left\{\pi \times 60^2 \times 40\right\} \mathrm{cm}^3$

For hemispherical part r = Radius = 60 cm

Let V_2 be the volume of the hemisphere. Then,

$V_2 = \left\{\frac{2}{3}\pi \times 60^3\right\} \mathrm{cm}^3$

$\Rightarrow \quad V_2 = \left\{2\pi \times 20 \times 60^2\right\} \mathrm{cm}^3 = \left\{40\pi \cdot 60^2\right\} \mathrm{cm}^3$

Let V_3 the the volume of the water left-out in the cylinder. Then,

$V_3 = V - V_1 - V_2$

$V_3 = \left\{\pi \times 60^2 \times 180 - \pi \times 60^2 \times 40 - 40\pi \times 60^2\right\} \mathrm{cm}^3$

$V_3 = \pi \times 60^2 \times \{180 - 40 - 40\} \mathrm{cm}^3$

$V_3 = \frac{22}{7} \times 3600 \times 100 \mathrm{cm}^3$

$\Rightarrow \quad V_3 = \frac{22 \times 360000}{7} \mathrm{cm}^3 = \frac{22 \times 360000}{7 \times (100)^3} \mathrm{m}^3 = \frac{22 \times 36}{700} \mathrm{m}^3 = 1.1314 \mathrm{m}^3.$

OR

Let the total number of bullets be x.

Edge of the solid cube = 44 cm

The diameter of the bullet = 4 cm

The radius of a spherical bullet = $\frac{4}{2} \mathrm{cm} = 2 \mathrm{cm}$

Now, the volume of a spherical bullet = $\frac{4}{3}\pi \times (2)^3 \mathrm{cm}^3 = \left(\frac{4}{3} \times \frac{22}{7} \times 8\right) \mathrm{cm}^3$

$\therefore$ The volume of x spherical bullets = $\left(\frac{4}{3} \times \frac{22}{7} \times 8 \times x\right) \mathrm{cm}^3$

The volume of the solid cube = $(44)^3$ cm^3

Clearly, Volume of x spherical bullets = Volume of cube

$\Rightarrow \quad \frac{4}{3} \times \frac{22}{7} \times 8 \times x = (44)^3$

$\Rightarrow \quad \frac{4}{3} \times \frac{22}{7} \times 8 \times x = 44 \times 44 \times 44$

$\Rightarrow \quad x = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 8} = 2541$

Hence, the total number of spherical bullets that can be made is 2541.

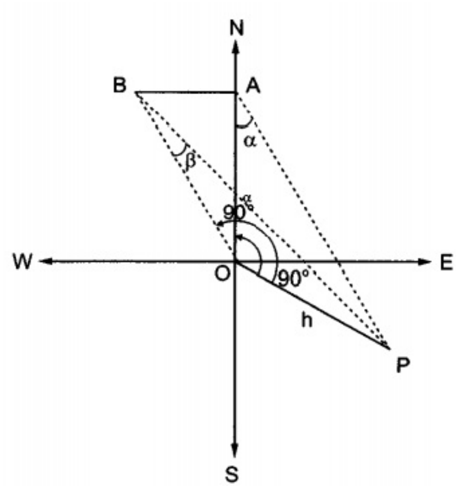
39. Let OP be the tower and let A be a point due north of the tower OP and let B be the point due west of A. Such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let h be the height of the tower.

In right-angled triangle OAP and OBP, we have

$\tan \alpha = \frac{h}{OA}$ and $\tan \beta = \frac{h}{OB}$

$\Rightarrow OA = h \cot \alpha$ and $OB = h \cot \beta$

In $\triangle OAB$, we have



OB ^{2} = OA ^{2} + AB

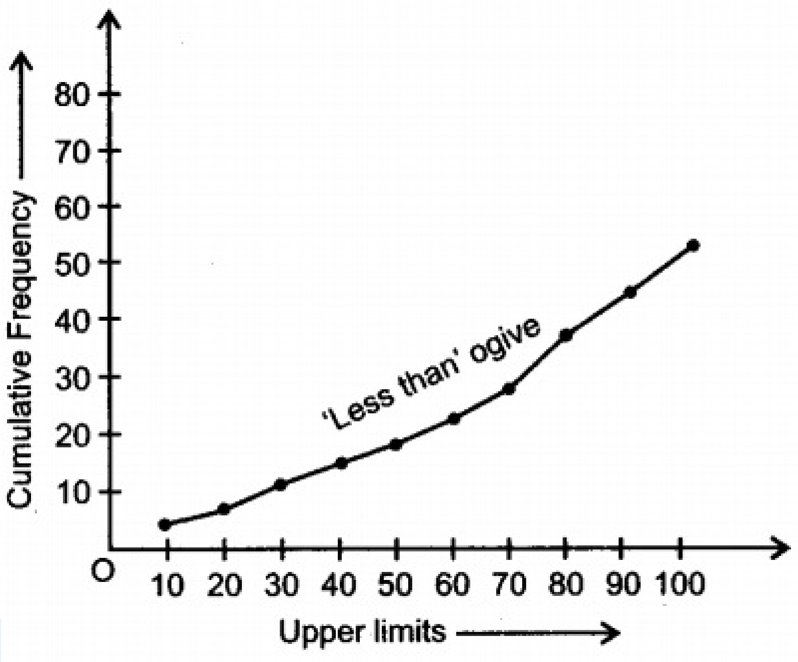
$\Rightarrow$ AB^2 = OB^2 - OA ^{2}

$$\Rightarrow AB^2 = h^2\left[\cot^2\beta - \cot^2\alpha\right]$$
$$\Rightarrow AB^2 = h^2\left[\left(cosec^2\beta - 1\right) - \left(cosec^2\alpha - 1\right)\right]$$
$$\Rightarrow AB^2 = h^2\left(cosec^2\beta - cosec^2\alpha\right)$$
$$\Rightarrow AB^2 = h^2\left(\frac{\sin^2\alpha - \sin^2\beta}{\sin^2\alpha\sin^2\beta}\right)$$
$$\Rightarrow h = \frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}$$

40. i.

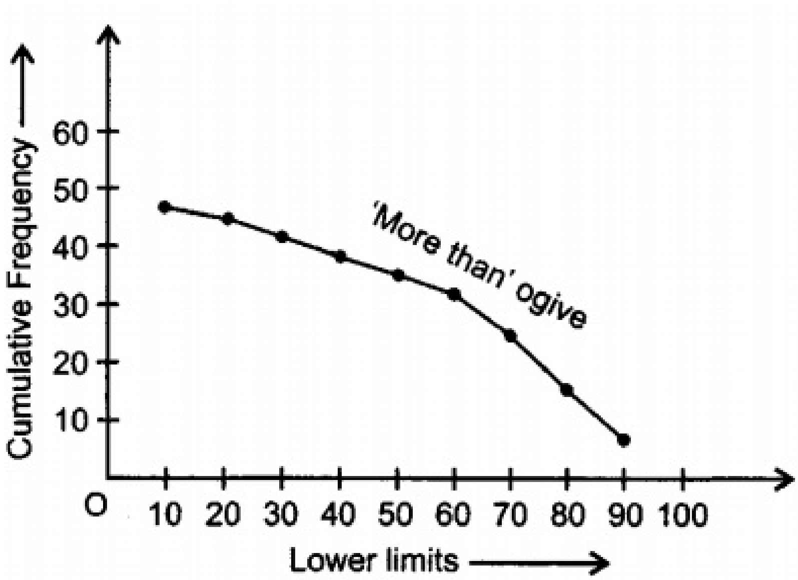
Marks Obtained	Cumulative frequency
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15
Less than 50	15 + 3 = 18
Less than 60	18 + 4 = 22
Less than 70	22 + 7 = 29
Less than 80	29 + 9 = 38
Less than 90	38 + 7 = 45
Less than 100	45 + 8 = 53

Graph:



i.

Marks Obtained more than or equal to	Cumulative frequency
0	53
10	53 - 5 = 48
20	48 - 3 = 45
30	45 - 4 = 41
40	41 - 3 = 38
50	38 - 3 = 35
60	35 - 4 = 31
70	31 - 7 = 24
80	24 - 9 = 15
90	15 - 7 = 8


Cumulative Frequency
60
50
40
30
20
10
O
10 20 30 40 50 60 70 80 90 100
Lower limits
'More than' ogive