**CBSE Class 10th Mathematics**

**Standard Sample Paper- 01**

**Maximum Marks:**

**Time Allowed: 3 hours**

**General Instructions:**

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.



## Section A

1. Every positive odd integer is of the form ________ where ‘q’ is some integer.

   a. 2q + 2

   b. 5q + 1

   c. 3q + 1

   d. 2q + 1

2. By Euclid’ division lemma x = qy + r, x > y the value of q and r for x = 27 and y = 5 are:

   a. q = 5, r = 2

b. cannot be determined

c. q = 6, r = 3

d. q = 5, r = 3

3. For the following distribution

| **Class** | 60 – 70 | 70 – 80 | 80 – 90 | 90 – 100 | 100 – 110 |
|---|---|---|---|---|---|
| **Frequency** | 10 | 15 | 12 | 20 | 9 |

the sum of lower limits of the median class and modal class is

a. 190

b. 20

c. 180

d. 170

4. If the sum of a number and its reciprocal is $2\frac{1}{2}$, then the numbers are

a. $3$ and $\frac{1}{3}$

b. $1$ and $\frac{3}{2}$

c. None of these

d. $2$ and $\frac{1}{2}$

5. If $x = a\ cos\ \theta$ and $y = b\ sin\ \theta$, then the value of $b^2x^2 + a^2y^2$ is

a. a + b

b. $a^2b^2$

c. a – b

d. ab

6. The value of $2\ tan^2 45^\circ + \ cos^2 30^\circ \ - \ sin^2 60^\circ$ is

a. 0

b. 1

c. 2

d. – 2

7. An observer 1.5 m tall is 23.5 m away from a tower 25m high. The angle of elevation of the top of the tower from the eye of the observer is

a. $60^\circ$

b. None of these

c. $30^\circ$

d. $45^\circ$

8. The point ( – 3, 5) lies in the __________ quadrant

a. IV

b. II

c. III

d. I

9. The point where the perpendicular bisector of the line segment joining the points A(2, 5) and B(4, 7) cuts is:

a. (3, 6)

b. (0, 0)

c. (2, 5)

d. (6, 3)

10. Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the dice is 7 is

   a. 7

   b. 2

   c. 0

   d. 1

11. Fill in the blanks:

   The shape of a glass tumbler is usually in the form of ________.

12. Fill in the blanks:

   The degree of polynomial p(x) $= x + \sqrt{2 + 1}$ is ________.

   OR

   Fill in the blanks:

   The number of zeroes of the cubic polynomial $x^3 - 3x^2 - x + 3$ is ________.



13. Fill in the blanks:

   In $\triangle$ABC and $\triangle$DEF, if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{9}$, then the ratio of ar($\triangle$ABC): ar($\triangle$DEF) is ________.

14. Fill in the blanks:

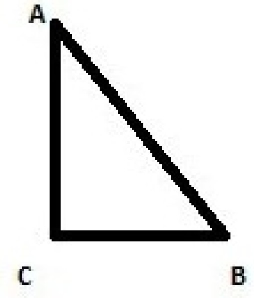
   Four consecutive terms in an AP are denoted by: a - 3d, a - d, a + d, ________.

15. Fill in the blanks:

   The Abscissa is ________ to the right of y-axis and is ________ to the left of y-axis.

16. State whether $\frac{17}{8}$ will have terminating decimal expansion or a non-terminating repeating decimal expansion.

17. ABC is an isosceles right triangle right-angled at C. Prove that $AB^2 = 2AC^2$

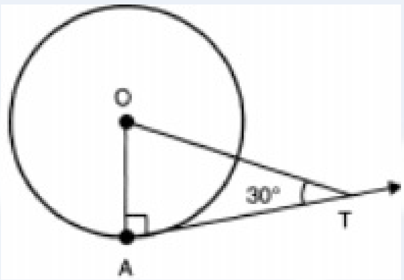



18. Write down the first four terms of the sequences whose general terms are $T_1 = 2$, $T_n = T_{n-1} + 5$, $n \geq 2$.

OR

Find the sum of first five multiples of 2.

19. In given figure, if AT is a tangent to the circle with centre O, such that OT = 4 cm and $\angle OTA = 30^o$, then find the length of AT (in cm).




20. Is it quadratic equation?

$3x^2 - 2\sqrt{x} + 8 = 0$

## Section B

21. A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is(i) red (ii) black or white (iii) not black.

22. Find the values of k for which the given equation has real and equal roots:

$2x^2 - 10x + k = 0$

23. In a $\Delta$ABC, D and E are points on the sides AB and AC respectively such that $DE \| BC$. If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.

OR

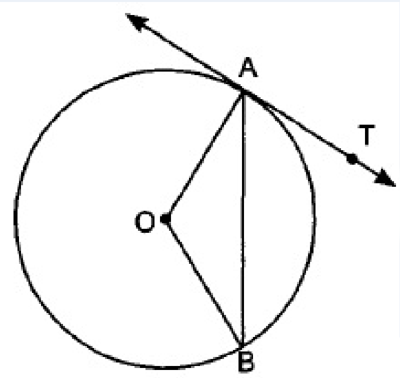
In a $\Delta$ ABC, D and E are points on the sides AB and AC respectively such that $DE \| BC.$ If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.

24. Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. If $\alpha, \beta$ be the elevations of the top of the tower from these stations, prove that its inclination $\theta$ to the horizontal is given by $\cot \theta = \frac{b \cot \alpha - a \cot \beta}{b-a}$

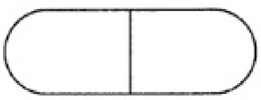
25. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA.

OR

In given figure, O is the centre of the circle, AB is a chord and AT is the tangent at A. If $\angle$AOB = 100° then find $\angle$BAT.




26. Seema a class 10th student went to a chemist shop to purchase some medicine for her mother who was suffering from Dengue. After purchasing the medicine she found that the upcount capsule used to cure platelets has the dimensions as followed:
The shape of the upcount capsule was a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm.



By reading the above-given information, find the following:

i. The surface area of the cylinder.
ii. The surface area of the capsule.

## Section C

27. Show that one and only one out of n, (n + 2) or (n + 4) is divisible by 3, where n EN.

OR

Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

28. Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and $7^{th}$ term is two more than thrice of its 3rd term.

29. Solve the given pair of linear equations by the elimination method and the substitution method: x + y = 5, 2x - 3y = 4.

OR

Solve:
37x + 41y=70
41x + 37y =86

30. If $\alpha$, β are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are ($2\alpha$) and (2β).

31. If three points $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$ lie on the same line, then prove that $\frac{y_2-y_3}{x_2x_3} + \frac{y_3-y_1}{x_3x_1} + \frac{y_1-y_2}{x_1x_2} = 0.$

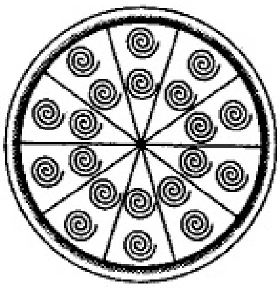
32. Find the value of cosec 30° geometrically.

OR

Prove the identity:
$(\tan A + \operatorname{cosec} B)^2 - (\cot B - \sec A)^2 = 2\tan A \cot B (\operatorname{cosec} A + \sec B)$

33. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:

i. the total length of the silver wire required.
ii. the area of each sector of the brooch.



34. Find the mean of each of the following frequency distributions:

| **Class interval** | 0-8 | 8-16 | 16-24 | 24-32 | 32-40 |
|---|---|---|---|---|---|
| **Frequency** | 5 | 9 | 10 | 8 | 8 |

**Section D**

35. Construct a triangle with sides 5cm, 6cm and 7cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of first triangle.

OR

Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the $1^{st}$ triangle.

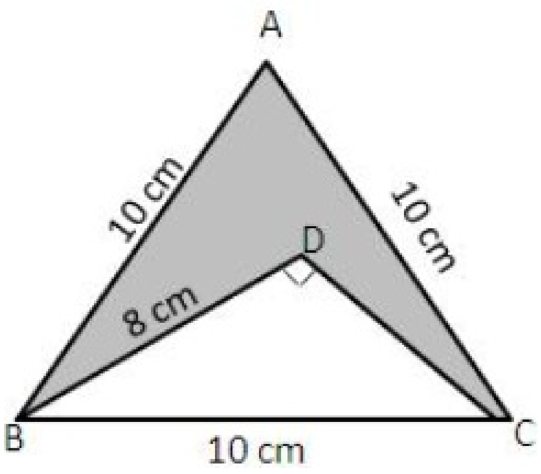
36. In an equilateral $\Delta ABC$, $AD \perp BC$, prove that $AD^2 = 3\ BD^2$

37. Determine whether the following system of linear equations is inconsistent or not: $3x - 5y = 20$ ; $6x - 10y = -40$.

OR

The path of a train A is given by the equation $3x + 4y - 12 = 0$ and the path of another train B is given by the equation $6x + 8y - 48 = 0$. Represent this situation graphically.

38. In the given figure, $\triangle ABC$ is a equilateral triangle the length of whose side is equal to 10 cm, and $\triangle DBC$ is right-angled at D and BD = 8 cm. Find the area of the shaded region. [Take $\sqrt{3}$ = 1.732.]




OR

A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere and total height of the toy, if the height of the cone is 3 times the radius.

39. An aeroplane at an altitude of 1200 metres finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are $60^{o}$ and $30^{o}$ respectively. Find the distance between the two ships.

40. By changing the following frequency distribution to less than type distribution, draw its ogive.

| **Classes** | 0-15 | 15-30 | 30-45 | 45-60 | 60-75 |
|---|---|---|---|---|---|
| **Frequency** | 6 | 8 | 10 | 6 | 4 |

## Solution

### Section A

1. (d) 2q + 1
   Explanation:

   Let $a$ be any positive integer and b = 2
   Then by applying Euclid's Division Lemma,
   we have, a = 2q +r,
   where $0 \leqslant r < 2 \Rightarrow r$ = 0 or 1 $\therefore$ a = 2q or 2q + 1 .
   Therefore, it is clear that a = 2q i.e., a is an even integer.
   Also, 2q and are two 2q + 1 consecutive integers, therefore, 2q + 1 is an odd integer.

2. (a) q = 5, r = 2
   Explanation:

   x = qy + r

   $\Rightarrow 27 = 5 \times 5 + 2$

   $\Rightarrow q = 5, r = 2$

3. (d) 170
   Explanation:

| **Class** | 60 – 70 | 70 – 80 | 80 – 90 | 90 – 100 | 100 – 110 |
|---|---|---|---|---|---|
| **Frequency** | 10 | 15 | 12 | 20 | 9 |
| **Cumulative Frequency** | 10 | 25 | 37 | 57 | 66 |

   Here N = 66
   $\Rightarrow \frac{\text{N}}{2} = 33 \therefore$ The median class is 80 – 90 and Modal class is 90 – 100
   Sum of lower limits of Median class and Modal class = 80 + 90 = 170

4. (d) $2$ and $\frac{1}{2}$

Explanation:

Let the one number be $x$ then its reciprocal will be $\frac{1}{x}$ According to question,
$x + \frac{1}{x} = 2\frac{1}{2}$
$\Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$
$\Rightarrow 2x^2 + 2 = 5x$
$\Rightarrow 2x^2 - 5x + 2 = 0$

using factorisation method
$\Rightarrow 2x^2 - 4x - x + 2 = 0$
$\Rightarrow 2x(x-2) - 1(x-2) = 0$
$\Rightarrow (x-2)(2x-1) = 0$
$\Rightarrow x - 2 = 0$ and $2x - 1 = 0$
$\Rightarrow x = 2$ and $x = \frac{1}{2}$
Therefore, the numbers are 2 and $\frac{1}{2}$.

5. (b) $a^2b^2$

Explanation:

Given: $x = a\cos\theta$ and $y = b\sin\theta$
$\therefore b^2x^2 + a^2y^2$
= $b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2$
= $b^2a^2\cos^2\theta + a^2b^2\sin^2\theta$
$\Rightarrow b^2x^2 + a^2y^2$
= $a^2b^2\left(\cos^2\theta + \sin^2\theta\right)$
= $a^2b^2$
$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

6. (c) 2

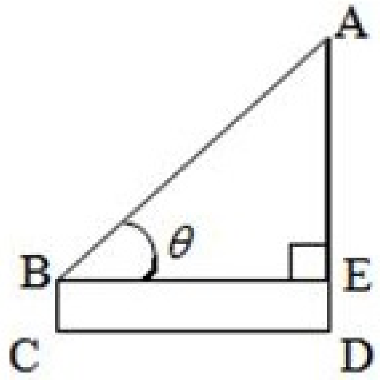
Explanation:

Given: $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

= $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$ = 2

7. (d) $45^\circ$

   Explanation:

   Let $\theta$ be the angle of elevation,
   The height of the tower AD = 25 m
   And CD = 23.5 m
   In triangle ABE,
   $\therefore \tan\theta = \frac{\text{AE}}{\text{BE}} = \frac{\text{AD}-\text{ED}}{\text{CD}}$
   $\Rightarrow \tan\theta = \frac{25-1.5}{23.5} = \frac{23.5}{23.5} = 1$
   $\Rightarrow \tan\theta = \tan 45^\circ\ \theta$
   $\Rightarrow\ \theta = 45^\circ$

8. (b) II
   Explanation:

   Since $x-$coordinate is negative and $y-$coordinate is positive.
   Therefore, the point $(-3, 5)$ lies in II quadrant.

9. (a) (3, 6)
   Explanation:

   Since, the point, where the perpendicular bisector of a line segment joining the points A(2, 5) and B(4, 7) cuts, is the mid-point of that line segment.
   $\therefore$ Coordinates of Mid-point of line segment AB = $\left(\frac{2+4}{2}, \frac{5+7}{2}\right)$ = $(3, 6)$

10. (c) 0

    Explanation:

    Elementary events are

    (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

    (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

    (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

    (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

    (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

    (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

    $\therefore$ Number of Total outcomes = 36

    And Number of possible outcomes (product of numbers appearing on die is 7) = 0

    $\therefore$ Required Probability = $\frac{0}{36} = 0$

11. frustum of a cone

12. 1 OR 3

13. 25:81

14. a + 3d

15. +ve, -ve

16. According to the question, $\frac{17}{8} = \frac{17}{2^3 \times 5^0}$

    So, the denominator 8 of $\frac{17}{8}$ is of the form $2^m \times 5^n$, where m, n are non-negative integers.

    Hence, $\frac{17}{8}$ has terminating decimal expansion. The decimal expansion of $\frac{17}{8}$ terminates after three places of decimals.

17. In right-angled $\triangle$ABC, right angled at C

    $\Rightarrow$ $AB^2 = AC^2 + BC^2$ [By pythagoras theorem]

    $\Rightarrow$ $AC^2 + AC^2 = 2AC^2$ [$\because$ BC = AC(given)]

    $\Rightarrow$ $AB^2 = 2AC^2$

18. $T_1 = 2, T_n = T_{n-1} + 5, n > 2$

$\Rightarrow T_2 = T_{2-1} + 5 = T_1 + 5$=2+5=7

$T_3 = T_{3-1} + 5 = T_2 + 5 = 7 + 5 = 12$ and $T_4 = T_{4-1} + 5 = T_3 + 5 = 12 + 5 = 17$

$\therefore$ 1st four terms are 2, 7,12 and 17.

OR

Here, $a = 2, d = 2, n = 5$

S = $\frac{n}{2}$[2a +(n - 1)d]

$\therefore S_5 = \frac{5}{2}[2 \times 2 + (5 - 1)2]$

= $\frac{5}{2}[4 + 4 \times 2]$

$= \frac{5}{2}[4 + 8]$

$= \frac{5}{2} \times 12$

$= 5 \times 6$

$= 30$

19. In given figure, AT is a tangent to the circle with center O, such that OT = 4 cm and $\angle$OTA = $30^0$, then we have to find the length of AT (in cm).

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$\frac{AT}{OT} = \cos 30^\circ$

$\therefore \text{AT} = \text{OT} \cos 30^\circ$

or, $AT = 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}\text{cm}$

20. $3x^2 - 2\sqrt{x} + 8$ is not of the form $ax^2 + bx + c = 0$.

$\therefore 3x^2 - 2\sqrt{x} + 8$ is not a quadratic equation.

**Section B**

21. $Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

Total number of balls = 5 + 7 + 3 = 15

i. Number of red balls = 7

$\therefore$ P(drawing a red ball) = $\frac{7}{15}$

ii. Number of black or white balls = 5 + 3 = 8

$\therefore$ P(drawing a black or white ball) = $\frac{8}{15}$

iii. Number of balls which are not black = 15 - 5=10

$\therefore$ P(drawing a ball that is not black)= $\frac{10}{15} = \frac{2}{3}$

Hence, the probability of getting a red ball, a black or white ball and a not black ball are $\frac{7}{15}$, $\frac{8}{15}$ and $\frac{2}{3}$ respectively.

22. The given equation is $2x^2 - 10x + k = 0$.

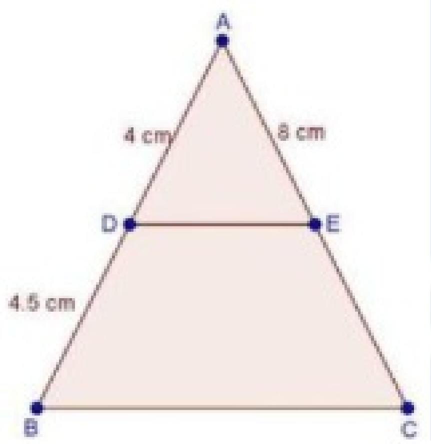
Here, a = 2, b = -10 and c = k $\therefore$ D = $b^2 - 4ac = (-10)^2 - 4 \times 2 \times k$

= 100 - 8k

The given equation will have real and equal roots, if

$D = 0 \Rightarrow 100 - 8k = 0 \Rightarrow k = \frac{100}{8} = \frac{25}{2}$

23. It is given that In a ΔABC D and E are the points on the sides AB and AC respectively such that




DE || BC

It is also given that AD = 4, DB = 4.5, AE = 8

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{4.5} = \frac{8}{EC}$

$\Rightarrow EC = \frac{8 \times 4.5}{4}$

$\Rightarrow$ EC = 9 cm

Now, AC = AE + EC

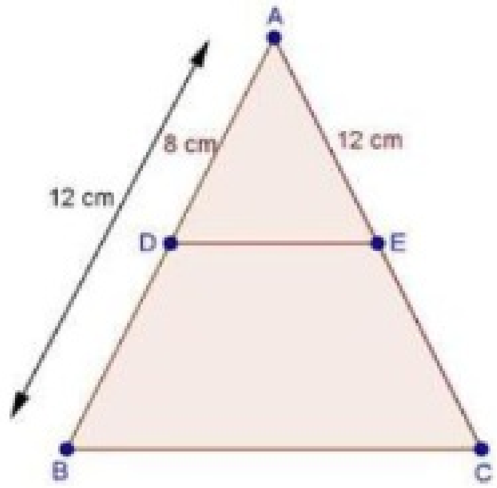
= 8 + 9

= 17 cm

$\Rightarrow$ AC = 17 cm

OR

We have,




AD = 8 cm, AB = 12 cm

$\therefore$ BD = AB - AD

= 12 - 8

$\Rightarrow$ BD = 4 cm

And, DE || BC

Therefore, by basic proportionality theorem, We have,

$\frac{AD}{BD} = \frac{AE}{CE}$

$\Rightarrow \frac{8}{4} = \frac{12}{CE}$

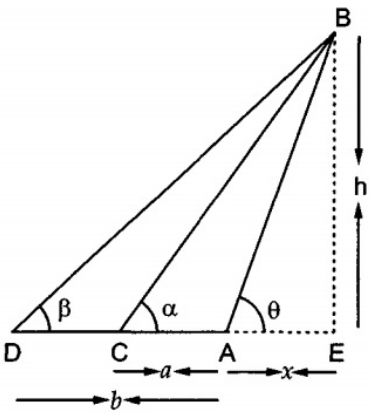
$\Rightarrow CE = \frac{12\times4}{8} = \frac{12}{2}$

$\Rightarrow$ CE = 6 cm

24. Let AB be the leaning tower and let C and D be two given stations at distances a and b respectively from the foot A of the tower.

Let AE = x and BE = h

In $\triangle AEB,$ we have



$\tan\theta = \frac{BE}{AE}$

$\Rightarrow \quad \tan\theta = \frac{h}{x}$

$\Rightarrow \quad x = h\cot\theta$ .......(i)

In $\Delta CEB$, we have

$\tan\alpha = \frac{BE}{CE}$

$\Rightarrow \quad \tan\alpha = \frac{h}{a+x}$

$\Rightarrow \quad a + x = h\cot\alpha$

$\Rightarrow \quad x = h\cot\alpha - a$.........(ii)

In $\Delta DEB$, we have

$\tan\beta = \frac{BE}{DE}$

$\Rightarrow \quad \tan\beta = \frac{h}{b+x}$

$\Rightarrow \quad b + x = h\cot\beta$

$\Rightarrow \quad x = h\cot\beta - b$ ...........(iii)

On equating the values of x obtained from equations (i) and (ii), we have

$h\cot\theta = h\cot\alpha - a$

$\Rightarrow \quad h(\cot\alpha - \cot\theta) = a$

$\Rightarrow \quad h = \frac{a}{\cot\alpha - \cot\theta}$ .........(iv)

On equating the values of x obtained from equations (i) and (iii), we get

$h\cot\theta = h\cot\beta - b$

$\Rightarrow \quad h(\cot\beta - \cot\theta) = b$

$\Rightarrow \quad h = \frac{b}{\cot\beta - \cot\theta}$ .........(v)

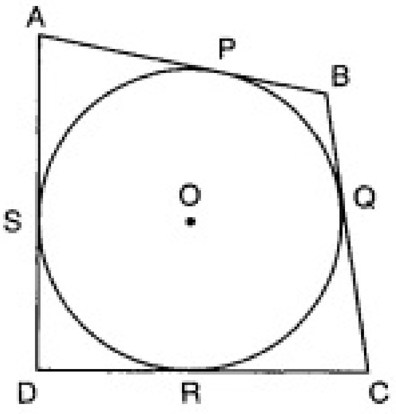
Equating the values of h from equations (iv) and (v), we get

$\frac{a}{\cot\alpha - \cot\theta} = \frac{b}{\cot\beta - \cot\theta}$

$\Rightarrow \quad a(\cot\beta - a\cot\theta) = b(\cot\alpha - \cot\theta)$

$\Rightarrow \quad (b-a)\cot\theta = b\cot\alpha - a\cot\beta$

$\Rightarrow \quad \cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$

25. 


Since tangents drawn from an exterior point to a circle are equal in length.

AP = AS [From A] ...(i)

BP = BQ [FromB] ...(ii)

CR = CQ [From C] ...(iii)

and, DR = DS [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP + BP + CR+DR = AS + BQ + CQ + DS

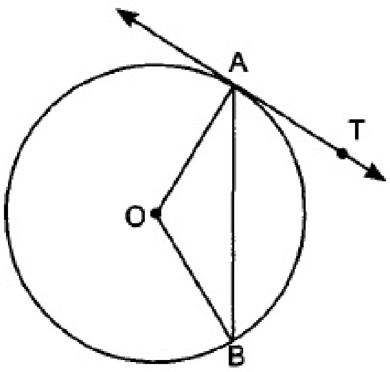
$\Rightarrow$ (AP + BP) + (CR+D R) = (AS + D S)+(BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

Hence, AB + CD = BC + DA

OR

Given, $\angle$AOB =$100^{0}$




Let $\angle$OAB = x = $\angle$OBA

$\angle$AOB + $\angle$OAB + $\angle$OBA = 180°

$\Rightarrow$ 100° + x + x = 180°

$\Rightarrow$ 2x = $180^{0}$ - $100^{0}$

$\Rightarrow$ 2x = 80°

$\Rightarrow$ x = 40°

$\angle$OAB + $\angle$BAT =90°

$40^\circ$ + $\angle$BAT = 90°

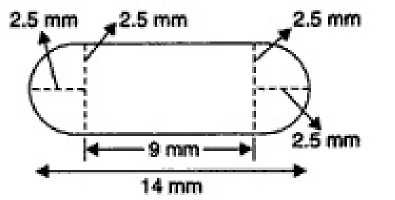
Therefore, $\angle$BAT = 50°

26. Let r = radius, h = cylindrical height

The radius of the hemisphere or cylinder, r $= \frac{5}{2}$mm

Height of cylinder, h = Total height - 2$\times$ radius of hemisphere

$h = 14 - 2 \times 2.5 = 9$ mm




i. Surface area of cylinder $= 2\pi rh$

$= 2\pi\left(\frac{5}{2}\right)(9) = 45\pi \text{ mm}^2$

ii. Surface area of the capsule = curved surface area of cylinder + 2 $\times$ surface area of the hemisphere

$= 2\pi rh + 2(2\pi r^2)$

$= 2\pi\left(\frac{5}{2}\right)(9) + 2\left[2 \cdot \pi \cdot \left(\frac{5}{2}\right)^2\right]$

$= 45\pi + 25\pi$

$= 70\pi = 70 \times \frac{22}{7} = 220 \text{ mm}^2$

**Section C**

27. Let the number be (3q + r)

$n = 3q + r \quad 0 \leq r < 3$

or $3q, 3q+1, 3q+2$

If $n = 3q$ then, numbers are $3q, (3q+1), (3q+2)$

$3q$ is divisible by 3.

If $n = 3q+1$ then, numbers are $(3q+1), (3q+3), (3q+4)$

$(3q+3)$ is divisible by 3.

If $n = 3q+2$ then, numbers are $(3q+2), (3q+4), (3q+6)$

$(3q+6)$ is divisible by 3.

$\therefore$ out of $n, (n+2)$ and $(n+4)$ only one is divisible by 3.

OR

Let a be the positive integer and b = 6.

Then, by Euclid's algorithm, a = 6q + r for some integer $q \geq 0$ and r = 0, 1, 2, 3, 4, 5 because $0 \leq r < 5$.

So, a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5.

$(6q)^2 = 36q^2 = 6(6q^2)$

= 6m, where m is any integer.

$(6q + 1)^2 = 36q^2 + 12q + 1$

$= 6(6q^2 + 2q) + 1$

= 6m + 1, where m is any integer.

$(6q + 2)^2 = 36q^2 + 24q + 4$

$= 6(6q^2 + 4q) + 4$

= 6m + 4, where m is any integer.

$(6q + 3)^2 = 36q^2 + 36q + 9$

$= 6(6q^2 + 6q + 1) + 3$

= 6m + 3, where m is any integer.

$(6q + 4)^2 = 36q^2 + 48q + 16$

$= 6(6q^2 + 7q + 2) + 4$

= 6m + 4, where m is any integer.

$(6q + 5)^2 = 36q^2 + 60q + 25$

$= 6(6q^2 + 10q + 4) + 1$

= 6m + 1, where m is any integer.

Hence, The square of any positive integer is of the form 6m, 6m + 1, 6m + 3, 6m + 4 and cannot be of the form 6m + 2 or 6m + 5 for any integer m.

28. Here, we have the sum of first 20 terms of an A.P., in which 3rd term is 7 and $7^{th}$ term is two more than thrice of its 3rd term.

Let "a" be the first term and" d"be the common difference of the given A.P.

Therefore, $a_3 = 7$ and $a_7 = 3a_3 + 2$ [Given]

$\Rightarrow a + 2d = 7$ and $a + 6d = 3(a + 2d) + 2$

$\Rightarrow$ a + 2d = 7 and a + 6d = 3a + 6d + 2

$\Rightarrow$ a + 2d = 7 and a -3a= 6d-6d+2

$\Longrightarrow$ a+2d=7 and -2a=2

$\Longrightarrow$ a+2d=7 and a=-1

$\Longrightarrow$ -1+2d=7

$\Longrightarrow$ 2d=7+1=8

$\Longrightarrow$ d=4

$\Rightarrow$ a = -1 and d = 4

Putting n = 20, a = - 1 and d = 4 in $S_n = \frac{n}{2}\{2a + (n-1)d\}$,we get

$S_{20} = \frac{20}{2}\{2 \times -1 + (20-1) \times 4\} = \frac{20}{2}(-2+76) = 740$

29. y = 5 .......... (1)

2x - 3y=4 ............. (2)

I. **Elimination method:**

Multiplying equation (1) by 2, we get equation (3)

2x +2y =10 ............. (3)

2x−3y =4 ........... (2)

Subtracting equation (2) from (3), we get

5y =6 ⇒ y = $\frac{6}{5}$

Putting value of y in (1), we get

x + $\frac{6}{5}$ =5

⇒ x =5− $\frac{6}{5}$ = $\frac{19}{5}$

Therefore, x = $\frac{19}{5}$ and y = $\frac{6}{5}$

II. **Substitution method:**

x +y =5 .......................... (1)

2x−3y =4 ......................... (2)

From equation (1), we get,

x =5−y

Putting this in equation (2), we get

2(5−y )−3y =4

$\Rightarrow 10-2y-3y = 4$

$\Rightarrow 5y = 6$

$\Rightarrow y = \frac{6}{5}$

Putting value of y in (1), we get

$x = 5 - \frac{6}{5} = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

OR

We have,

$37x + 41y = 70$ ..............(i)

$41x + 37y = 86$ ..............(ii)

Adding equation (i) and (ii), we get

$37x + 41y + 41x + 37 = 70 + 86$

$78x + 78y = 156$

$\Rightarrow x + y = 2$ ..................(iii)

Subtracting equation (i) from equation (ii), we get

$(37x + 41y) - (41x + 37) = 70 - 86$

$-4x + 4y = -16$

$4x - 4y = 16$

$\Rightarrow x - y = 4$ ...................(iv)

Adding equation (iii) and (iv), we get

$x + y + x - y = 2 + 4$

$2x = 6$

$\Rightarrow x = 3$

Subtracting equation (iii) and (iv), we get

$x + y - x + y = 2 - 4$

$2y = -2$

$y = -1$

Hence, $x = 3$ and $y = -1$ is the solution of the given system of equations.

30. Polynomial $p(x) = x^2 - 2x - 15$

$a = 1$, $b = -2$ and $c = -15$

Let $\alpha, \beta$ be the zeroes of p(x)

$\therefore$ Sum of zeroes $\alpha + \beta = -\frac{b}{a}$
$\Rightarrow \alpha + \beta = -\frac{(-2)}{1} = 2$
Product of zeroes $\alpha \times \beta = \frac{c}{a}$
$\Rightarrow \alpha \times \beta = \frac{-15}{1} = -15$
Given zeroes are $2\alpha$, $2\beta$
$\therefore$ Sum of zeroes = $2\alpha + 2\beta = 2(\alpha + \beta) = 2 \times 2 = 4\ [\because \alpha + \beta = 2]$
Product of zeroes = $2\alpha \times 2\beta = 4\alpha\beta = 4(-15) = -60[\because \alpha\beta = -15]$

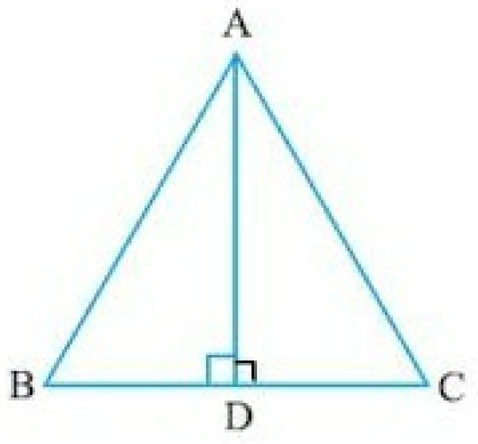
Required polynomial = $x^2$ – (sum of zeroes) x + product of zeroes

= $x^2 - 4x - 60$

31. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the given points.
So, Area of $\Delta ABC$
$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$
Since, it is given that the given points lie on the same line [ I.e. they are colinear]
$\therefore$ Area of $\Delta$ABC = 0
$\Rightarrow \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$
$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

Dividing both sides of the equation by $x_1x_2x_3$, we get
$\frac{x_1(y_2-y_3)}{x_1x_2x_3} + \frac{x_2(y_3-y_1)}{x_1x_2x_3} + \frac{x_3(y_1-y_2)}{x_1x_2x_3} = \frac{0}{x_1x_2x_3}$
$\Rightarrow \frac{y_2-y_3}{x_2x_3} + \frac{y_3-y_1}{x_3x_1} + \frac{y_1-y_2}{x_1x_2} = 0$
Hence proved.

32. Let us take an equilateral $\Delta$ ABC.




Each angle in an equilateral triangle is 60°

Therefore, $\angle A = \angle B = \angle C = 60^\circ$

Let the perpendicular AD be drawn from A to side BC.

Now in $\triangle$ABD and $\triangle$ACD;

$\angle ADB = \angle ADC$ (as AD is perpendicular to BC)

AB = AC (Since, ΔABC is an equilateral triangle)

AD = AD (Common side)

Hence, by RHS criterion of congruency of triangles-

$\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$

$\therefore$ BD = DC (CPCT)

$\angle BAD = \angle CAD = 30^\circ$

Let the length of AB be 2a. therefore AB = BC = CD = 2a

BD = half of BC = a

Therefore, in ΔBAD

sin 30° $= \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

$\therefore$ Cosec 30° = 2

OR

We have,

LHS = (tanA + cosec B)$^2$ - (cotB - sec A)$^2$

$\Rightarrow$ LHS = (tan$^2$A + cosec$^2$B + 2tanA cosecB ) - (cot$^2$B + sec$^2$A - 2cotB secA)

$\Rightarrow$ LHS = $(\tan^2A - \sec^2A)+(\text{cosec}^2B - \cot^2B)+2\tan A \text{ cosec}B + 2\cot B \sec A$

But, $\text{Sec}^2A - \tan^2A = 1$ & $\text{cosec}^2A - \cot^2 A = 1$

$\therefore$ LHS = -1 + 1 + 2 tanA cosecB + 2cotB secA

$\Rightarrow$ LHS = 2 (tanA cosecB + cotB secA)

$\Rightarrow$ LHS = 2 tanA cotB$\left(\frac{cosec\ B}{\cot B} + \frac{\sec A}{\tan A}\right)$ [Dividing and multiplying by tanA cotB]

$\Rightarrow$ LHS = 2tan A cotB$\left\{\frac{\frac{1}{\sin B}}{\frac{\cos B}{\sin B}} + \frac{\frac{1}{\cos A}}{\frac{\sin A}{\cos A}}\right\}$ [Since, CosecA.SinA =1 , SecA.cosA =1, (sinA/cosA)= tanA & (cosA/SinA) =cotA ]

$\Rightarrow$ LHS = 2 tanA cotB$\left(\frac{1}{\cos B} + \frac{1}{\sin A}\right)$ = 2tanA cotB ( secB + cosecA ) = RHS. Hence, proved.

33. i. $\because$ Diameter = 35 mm

$\therefore$ Radius = $\frac{35}{2}$mm

$\therefore$ Circum ference = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2}$ = 110mm ..... (1)

Length of 5 diameters

$= 35 \times 5$ = 175 mm ...... ( 2 )

$\therefore$ The total length of the silver wire required

= 110 + 175 = 285 mm

ii. $r = \frac{35}{2}mm, \theta = \frac{360^\circ}{10} = 36^\circ$

$\therefore$ The area of each sector of the brooch

$= \frac{\theta}{360} \times \pi r^2$

$= \frac{36}{360} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{385}{4}\text{mm}^2$

34.

| **Class interval** | **Mid value $x_i$** | **$d_i = x_i - 20$** | **$u_i = (x_i-20)/8$** | **Frequency $f_i$** | **$f_iu_i$** |
|---|---|---|---|---|---|
| 0-8 | 4 | -16 | -2 | 5 | -10 |
| 8-16 | 12 | -8 | -1 | 9 | -9 |
| 16-24 | 20 | 0 | 0 | 10 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 24-32 | 28 | 8 | 1 | 8 | 8 |
| 32-40 | 36 | 16 | 2 | 8 | 16 |
| | | | | **N = 40** | **Sum = 5** |

Let the assumed mean (A) = 20

A = 20, h = 8

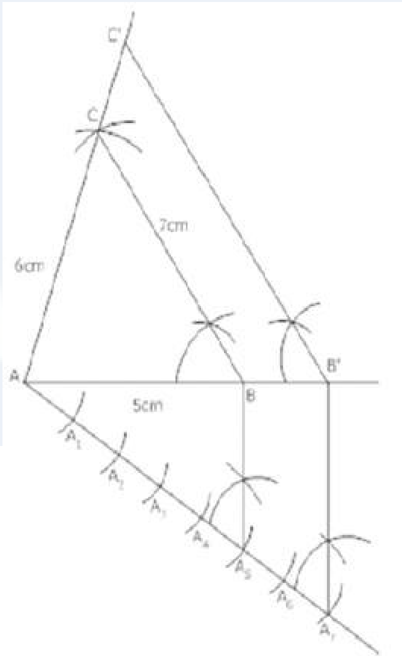
Mean =$A + h\frac{sum}{N}$

= $20 + 8\left(\frac{5}{40}\right)$

=20+1

=21

**Section D**

35. 


Steps of construction:

i. Draw a line segment AB = 5 cm.
ii. With A as centre and radius 6 cm draw an arc.
iii. Again B as centre and radius 7 cm draw another arc cutting the previous arc at C. Join AC and BC, then ABC is required triangle.
iv. Draw any ray AX making an acute angle.
v. Locate 7 points $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$ and $A_7$ on AX so that $AA_1 = A_1A_2 = A_2A_3 =$

$A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

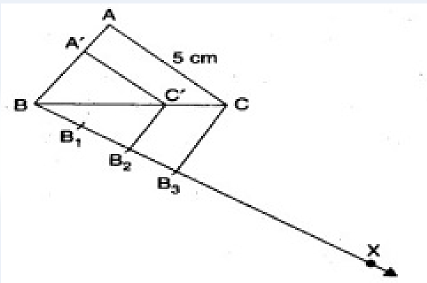
vi. Join $A_5$ to B and draw a line through $A_7$ parallel to $A_5B$ intersecting the extended line segment AB at B'.

vii. Draw B'C' || BC, then $\triangle AB'C$ is the required triangle

OR

Required:
To construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle.
Steps of construction:

i. Draw a triangle ABC of sides 4 cm, 5 cm and 6 cm
ii. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
iii. Locate 3 points $B_1$, $B_2$ and $B_3$ on BX such that $BB_1 = B_1B_2 = B_2B_3$.




iv. Join $B_3C$ and draw a line through $B_2$ parallel to $B_3$ C intersecting BC at C'.
v. Draw a line through C' parallel to the line CA to intersect BA at A'. Then, $\triangle A'BC'$ is the required triangle.

Justification:
$\therefore \quad B_3C \| B_2C'$ [By construction]
$\therefore \frac{\mathrm{BB_2}}{\mathrm{B_2B_3}} = \frac{\mathrm{BC'}}{\mathrm{C'C}}$ [By the basic proportionality theorem]
But $\frac{BB_2}{B_2B_3} = \frac{2}{1}$ [By construction]
$\therefore \quad \frac{BC'}{C'C} = \frac{2}{1}$
$\therefore \quad \frac{C'C}{BC'} = \frac{1}{2}$
$\Rightarrow \quad \frac{\mathrm{C'C}}{\mathrm{BC'}} + 1 = \frac{1}{2} + 1$
$\Rightarrow \quad \frac{\mathrm{C'C+BC'}}{\mathrm{BC'}} = \frac{1+2}{2}$

$\Rightarrow \quad \frac{BC}{BC'} = \frac{3}{2}$

$\Rightarrow \quad \frac{BC'}{BC} = \frac{2}{3}$ .............(1)

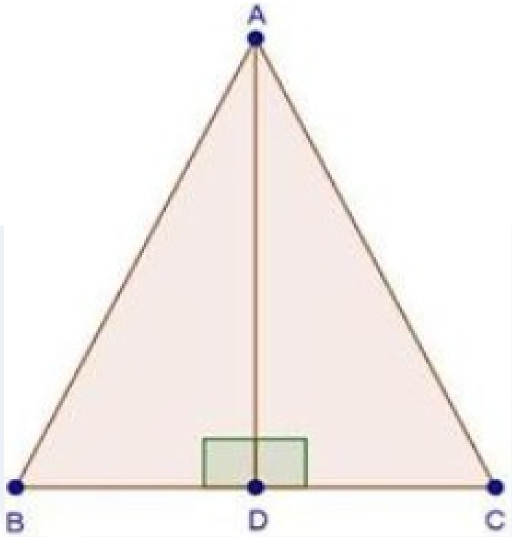
$\therefore$ CA | C'A' By construction]

$\therefore \quad \triangle BC'A' \sim \triangle BCA$ [AA similarity cirterion]

$\frac{A'^{B}B}{AB} - \frac{AC'}{AC} - \frac{BC}{BC}\left(-\frac{2}{3}\right)$

[By the basic proportionality theorem]

36. We have,




$\Delta$ABC is an equilateral $\Delta$ and AD$\perp$BC

In $\Delta$ADB and $\Delta$ADC

$\angle ADB = \angle ADC$ [Each 90°]

AB = AC [Given]

AD = AD [Common]

Then, $\Delta ADB \cong \Delta ADC$ [By RHS condition]

$\therefore BD = CD = \frac{BC}{2}$ ....(i) [ by c.p.c.t ]

In $\Delta$ABD, by pythagoras theorem

$AB^2 = AD^2 + BD^2$

$\Rightarrow BC^2 = AD^2 + BD^2$ [AB = BC given]

$\Rightarrow [2BD]^2 = AD^2 + BD^2$ [From (i)]

$\Rightarrow 4BD^2 - BD^2 = AD^2$

$\Rightarrow 3BD^2 = AD^2$

37. The given system of linear equation is

3x - 5y =20...........(1)

6x - 10y = -40..........(2)

We can write these equations as

3x - 5y - 20 = 0..........(3)

6x - 10y = -40................(4)

Here, $a_1$ = 3, $b_1$ = -5, $c_1$= -20;

$a_2$ = 6, $b_2$= -10, $c_2$= 40

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines representing the given pair of linear equations are parallel.

Therefore, equation(1) and (2) have a common solution, i.e, the given pair of linear equations is inconsistent.

OR

The paths of two trains are given by the following pair of linear equations.

$3x + 4y - 12 = 0$ ..... (i)

$6x + 8y - 48 = 0$..... (ii)

In order to represent the following sets of lines graphically we need two points for a single equation

We have,

$3x + 4y - 12 = 0$

Putting $y = 0$

$3x + 4(0) = 12$

⇒ $3x = 12$

⇒ $x = 4$

Hence the coordinate is (4,0)

Putting $x = 0$

$3(0) + 4y = 12$

⇒ $4y = 12$

⇒ $y = 3$

Hence the coordinate is (0,3)

We have,

6x + 8y - 48 = 0

Putting x = 0

$6(0) + 8y = 48$

$\Rightarrow\ 8y = 48$

$\Rightarrow\ y = 6$

Hence the coordinate is (0, 6)

$6x + 8y - 48 = 0$

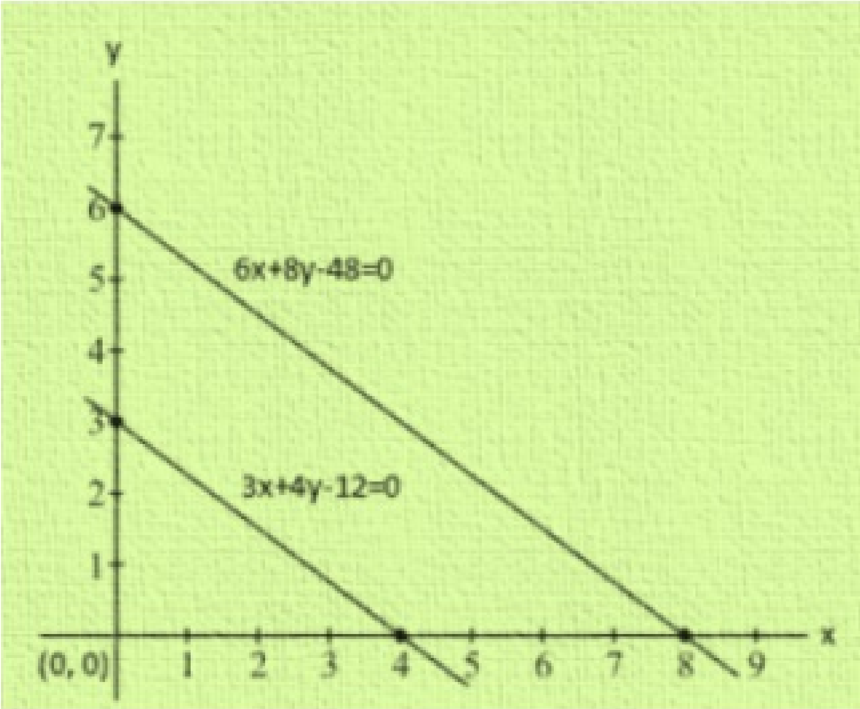
Putting y = 0

$6x + 8(0) = 48$

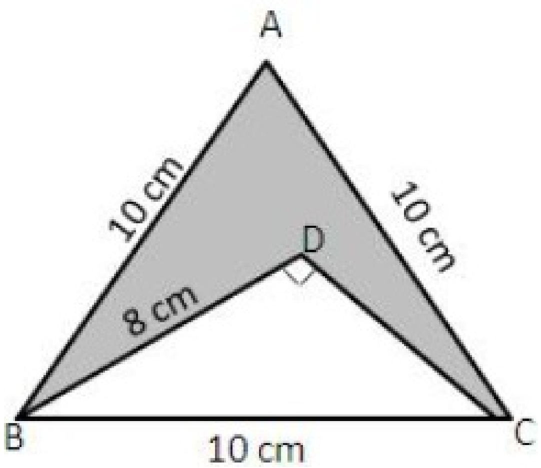
$\Rightarrow\ 6x = 48$

$\Rightarrow\ x = 8$

Hence the coordinate is (8,0)




38. Given, $\triangle ABC$ is a equilateral triangle, The length of side is equal to 10 cm.
    Also, $\triangle DBC$ is right-angled at D and BD = 8 cm.




Area of shaded region = Area of $\triangle$ABC - Area of $\triangle$DBC

First we find area of $\triangle$ABC

$\therefore$ Area = $\frac{\sqrt{3}}{4}a^2$

= $\left(\frac{\sqrt{3}}{4} \times 10 \times 10\right)$

= 43.30 $cm^2$

Second we find area of $\triangle$DBC which is right angled.

DC = $\sqrt{BC^2 - DB^2}$

= $\sqrt{10^2 - 8^2}$

= $\sqrt{100 - 64} = \sqrt{36}$ cm

= 6 cm

Area of $\triangle$DBC = $\frac{1}{2} \times$ Base $\times$ Height

Area of $\triangle$DBC = $\frac{1}{2} \times DB \times DC$

= $\left(\frac{1}{2} \times 8 \times 6\right)$ $cm^2$

= 24 $cm^2$

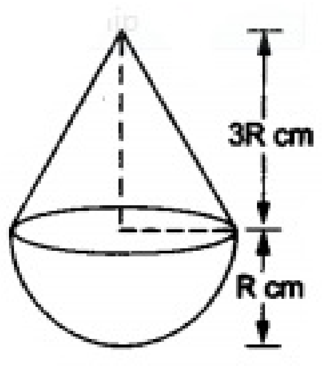
Area of shaded region = Area of $\triangle$ABC - Area of $\triangle$DBC

= (43.30 - 24) $cm^2$

= 19.30 $cm^2$

Therefore, Area of shaded region = 19.3 $cm^2$

OR




Radius of the cylinder, r = 6cm and

height of the cylinder, h = 15cm.

$\therefore$ volume of the cylinder = $\pi r^2 h$

= $(\pi \times 6 \times 6 \times 15)\mathrm{cm}^3$

=$(540\pi)\mathrm{cm}^3$

Volume of 12 toys = $(540\pi)\mathrm{cm}^3$

$\therefore \quad$ volume of 1 toy $= \left(\frac{540\pi}{12}\right)\mathrm{cm}^3$ = $(45\pi)\mathrm{cm}^3$.

Let the radius of each of the hemisphere and cone be R cm.

Then, height of the cone, H = (3R) cm.

Volume of 1 toy = volume of the hemisphere + volume of the cone

$= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 H$

$= \left(\frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 \times 3R\right)\mathrm{cm}^3$

$= \left(\frac{5\pi R^3}{3}\right)\mathrm{cm}^3$

$\therefore \quad \frac{5\pi R^3}{3} = 45\pi \Rightarrow R^3$

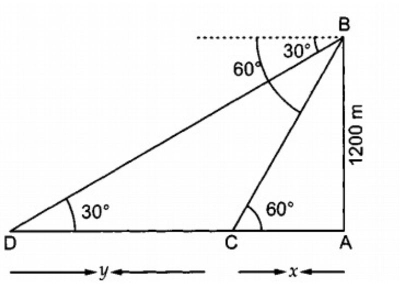
$= \left(45 \times \frac{3}{5}\right) = 27 = 3^3 \Rightarrow R = 3.$

Total height of the toy = (R + 3R)cm = 4R cm

= $(4\times3)$ cm = 12cm.

39. Let the aeroplane be at B and let the two ships be at C and D such that their angles of depression from B are $30^o$ and $60^o$ respectively.

We have, AB = 1200 metres. Let AC = x and CD = y.




In $\triangle CAB$, we have

$\tan 60^\circ = \frac{AB}{CA}$

$\Rightarrow \quad \sqrt{3} = \frac{1200}{x}$

$\Rightarrow \quad x = \frac{1200}{\sqrt{3}} = 400\sqrt{3}$.......(1)

In $\triangle BAD$, we have

$\tan 30^\circ = \frac{AB}{AD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1200}{x+y}$

$\Rightarrow \quad x + y = 1200\sqrt{3}$

$\Rightarrow \quad y = 1200\sqrt{3} - x$

$\Rightarrow \quad y = 1200\sqrt{3} - 400\sqrt{3}$ (using 1)

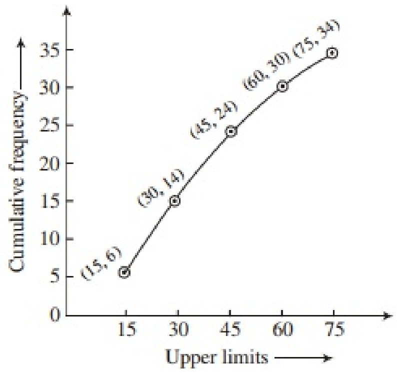
$= 800\sqrt{3} = 800 \times 1.732 = 1385.6$

Hence, the distance between the two ships is 1385.6 metres.

40.

| Classes | Frequency | Classes | Cumulative frequency |
|---|---|---|---|
| 0-15 | 6 | Less than 15 | 6 |
| 15-30 | 8 | Less than 30 | 14 |
| 30-45 | 10 | Less than 45 | 24 |
| 45-60 | 6 | Less than 60 | 30 |
| 60-75 | 4 | Less than 75 | 34 |

We can plot the the less than ogive curve as follows by plotting the points (15,6), (30,14), (45,24), (60,30), (75,24).


Cumulative frequency
Upper limits
(15, 6)
(30, 14)
(45, 24)
(60, 30)
(75, 34)
0
5
10
15
20
25
30
35
15
30
45
60
75