Physics Delhi (Set 3)

General Instructions:

Read the following instructions very carefully and strictly follow them:
(i) This question paper comprises **four** sections – A, B, C and D.
(ii) There are **37** questions in the question paper. **All** questions are compulsory.
(iii) Section A : Q. no. **1** to **20** are very short-answer type questions carrying **1** mark each.
(iv) Section B : Q. no. **21** to **27** are short-answer type questions carrying 2 marks each.
(v) Section C : Q. no. **28** to **34** are long-answer type questions carrying 3 marks each.
(vi) Section D : Q. no. **35** to **37** are also long answer type questions carrying **5** marks each.
(vii) There is no overall choice in the question paper. However, an internal choice has been provided in **two** questions of **one** mark, **two** questions of **two** marks, **one** question of **three** marks and all the **three** questions of **five** marks. You have to attempt **only one** of the choices in such questions.
(viii) However, separate instructions are given with each section and question, wherever necessary.
(ix) Use of calculators and log tables is not permitted.
(x) You may use the following values of physical constants wherever necessary.

$c = 3 \times 10^8$ m/s
$h = 6.63 \times 10^{-34}$ Js
$e = 1.6 \times 10^{-19}$ C
$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
$\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2}
$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}
Mass of electron (m_e) = 9.1×10^{-31} kg
Mass of neutron = 1.675×10^{-27} kg
Mass of proton = 1.673×10^{-27} kg
Avogadro's number = 6.023×10^{23} per gram mole
Boltzmann constant = 1.38×10^{-23} JK^{-1}

Question 1

A biconcave lens of power *P* vertically splits into two identical plano concave parts. The power of each part will be
(a) 2*P*
(b) P/2
(c) *P*
(d) P/√2

SOLUTION:
Let the focal of the biconcave lens be *f* and that of the plano-concave lens be *f*'.

Let the radius of curvature for the biconcave lens be *R*.
Applying Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{(-R)} - \frac{1}{R}\right) = -\frac{2(\mu-1)}{R}$$

For the plano-concave lens, the focal length can be calculated as:

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{(-R)}\right) = -\frac{(\mu-1)}{R}$$

Thus, *f*' = 2*f*

Thus, power of the plano-concave lens will be P'=P/2
Hence, the correct answer is option (b)

Question 2

The power factor of a series LCR circuit at resonance will be
(a) 1
(b) 0
(e) 1/2
(d) √12

SOLUTION: At resonance condition, the alternating current and voltage are in same phase, thus the phase difference between them *Φ* = 0, and the power factor, cos *Φ* = 1. Hence the correct answer is option (a).

Question 3

If photons of frequency *v* are incident on the surfaces of metals. A & B of threshold frequencies v/2 and v/3 respectively, the ratio of the maximum kinetic energy of electrons emitted from A to that from B is
(a) 2 : 3
(b) 3 : 4
(c) 1 : 3
(d) √3 : √2

SOLUTION: According to Einstein's photoelectric equation,

$hv = hv_0 + Kmax$

Where,
v = frequency of the incident light
v0 = threshold frequency of the metal
KEmax = maximum kinetic energy of the emitted photoelectrons

$$K.E_{max} = h\nu - h\nu_0$$
$$K.E_{max,\,A} = h\nu - \frac{h\nu}{2} = \frac{h\nu}{2}$$
$$K.E_{max,\,B} = h\nu - \frac{h\nu}{3} = \frac{2h\nu}{3}$$
$$\frac{K.E_{max,\,A}}{K.E_{max,\,B}} = \frac{\frac{h\nu}{2}}{\frac{2h\nu}{3}} = \frac{3}{4} = 3 : 4$$

Hence, the correct answer is option (b).

Question 4

The electric flux through a closed Gaussian surface depends upon
(a) Net charge enclosed and permittivity of the medium
(b) Net, charge enclosed, permittivity of the medium and the size of the Gaussian surface
(c) Net charge enclosed only
(d) Permittivity of the medium only

Solution:
The electric flux through a closed Gaussian surface is given by:

$$\oint \overrightarrow{E}.\mathrm{d}\,\overrightarrow{s} = \frac{q}{\epsilon}$$

Where, *q* is the net charge enclosed by the Gaussian and∈∈is the permittivity of the medium.

Hence, the correct answer is option (a).

Question 5

A charge particle after being accelerated through a potential difference '*V*' enters in a uniform magnetic field and moves in a circle of radius *r*. If *V* is doubled, the radius of the circle will become
(a) 2*r*
(b) √2r

(c) 4*r*
(d) r/√2

SOLUTION: The relation between the accelerating potential and the accelerating voltage is given as:

$$r = \frac{\sqrt{2mqV}}{qB}$$

As the potential is doubled the radius of curvature becomes 2–√2 times.

Hence, the correct answer is option (b).

Question 6

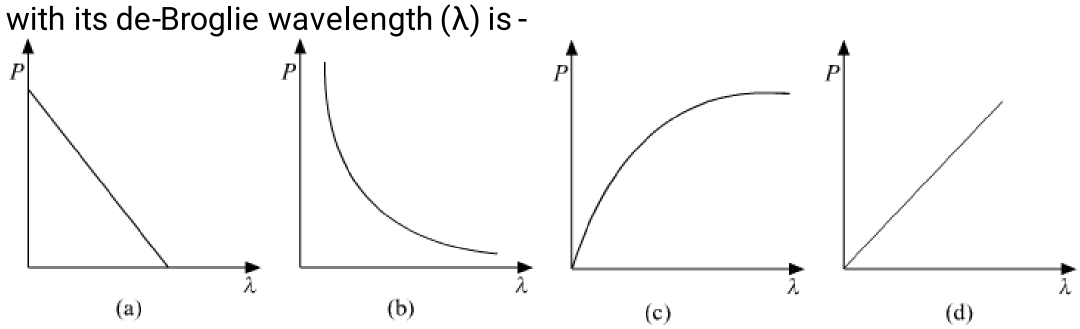
The wavelength and intensity of light emitted by a LED depend upon
(a) forward bias and energy gap of the semiconductor
(b) energy gap of the semiconductor and reverse bias
(c) energy gap only
(d) forward bias only

SOLUTION: The wavelength and intensity of light emitted by an LED depends on both energy gap and bias of the diode. Only when the diode is forward biased, it emits photons. The wavelength of the emitted light depends on the energy gap of the semiconductor.

Hence, the correct answer is option (a).

Question 7

The graph showing the correct variation of linear momentum (*p*) of a charge particle with its de-Broglie wavelength (λ) is -



Solution:

The relation between de-Broglie wavelength and the momentum is:

$$\lambda = \frac{h}{p}$$
$$\Rightarrow p = \frac{h}{\lambda}$$

By the above relation, we can conclude that the graph between the momentum and the de-Broglie wavelength is a rectangular hyperbola.

Hence, the correct answer is option (b).

Question 8

The selectivity of a series LCR a.c. circuit is large, when
(a) *L* is large and *R* is large
(b) *L* is small and *R* is small
(c) *L* is large and *R* is small
(d) *L* = *R*

Solution: Selectivity of a circuit depends on the quality of resonance. The quality factor is given by:

$$Q = \frac{\omega_0 L}{R}$$

High value of quality factor make sure that the resonance curve is sharp. Sharper the resonance curve is more selective is the LCR circuit. Thus, the selectivity of the LCR circuit is large when *L* is large and *R* is small.

Hence, the correct answer is option (c).

Question 9

Photo diodes are used to detect
(a) radio waves
(b) gamma rays
(c) IR rays
(d) optical signals

Solution: Photodiodes are used to detect the visible light, out of the given options optical signals is the most appropriate.

Hence, the correct answer is option (d).

Question 10

The relationship between Brewester angle 'θ' and the speed of light 'v' in the denser medium is –
(a) $v \tan \theta = c$
(b) $c \tan \theta = v$
(c) $v \sin \theta = c$
(d) $c \sin \theta = v$

SOLUTION: Let the absolute refractive index of the given medium be μ and the speed of light in vacuum be c.

From Brewster's law:

$\tan \theta = \mu$

The refractive index can also be written as:
$\mu = \frac{c}{v}$
$\Rightarrow \tan\theta = \frac{c}{v}$
$\Rightarrow c = v\tan\theta$

Hence, the correct answer is option (a).

Question 11

The ability of a junction diode to _________ an alternating voltage, is based on the fact that it allows current to pass only when it is forward biased.

Solution:

Rectify

Question 12

A point charge is placed at the centre of a hollow conducting sphere of internal radius 'r' and outer radius '$2r$'. The ratio of the surface charge density of the inner surface to that of the outer surface will be________.

Solution:

Let the point charge be q.
by gauss's law the charge on the inner surface will be $-q$
Surface charge density of the inner surface $\sigma_i = -\frac{q}{4\pi r^2}$
by charge conservation on the hollow sphere the outer surface will have charge q
Surface charge density of the inner surface $\sigma_o = \frac{q}{4\pi(2r)^2} = \frac{q}{16\pi r^2}$

$$\text{ratio} = \frac{\sigma_i}{\sigma_o} = \frac{\frac{-q}{4\pi r^2}}{\frac{q}{16\pi r^2}} = -\frac{4}{1}$$

Question 13

The __________, a property of materials C, Si and Ge depends upon the energy gap between their conduction and valence bands.

Solution: Conductivity

Question 14

A copper wire of non-uniform area of cross-section is connected to a d.c. battery. The physical quantity, which remains constant along the wire is __________.

Solution: Electric current.

Question 15

The physical quantity having SI unit $NC^{-1}m$ is __________.

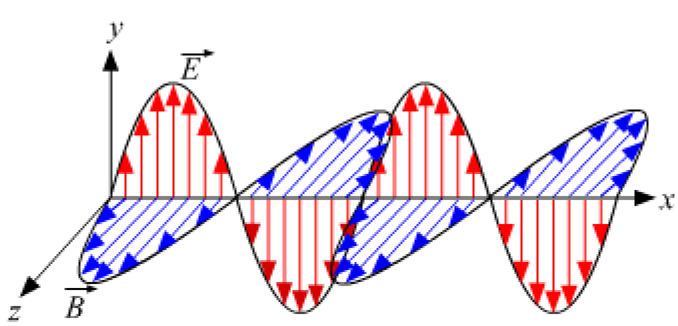
Solution:

Electric Potential.

Question 16

Depict the fields diagram of an electromagnetic wave propagating along positive X-axis with its electric field along Y-axis.

Solution:



Question 17

Write the conditions on path difference under which (i) constructive (ii) destructive interference occur in Young's double slit experiment.

Solution:

I_1 = intensity of light from slit 1

I_2 = intensity of light from slit 2

phase difference between 2 light waves $= \theta = \frac{2\pi\Delta x}{\lambda}$, where Δx = path difference

resultant intensity I is given by,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$

for constructive interference I should by maximum $\Rightarrow \cos\theta = 1$

$\theta = 2n\pi$, n = Integer

$2n\pi = \frac{2\pi\Delta x}{\lambda}$

$\Delta x = n\lambda$

for distructive interference I should by minimum $\Rightarrow \cos\theta = -1$

$\theta = \left(2n+1\right)\pi$, n = Integer

$2n\pi = \frac{(2n+1)\pi\Delta x}{\lambda}$

$\Delta x = \frac{(2n+1)\lambda}{2}$

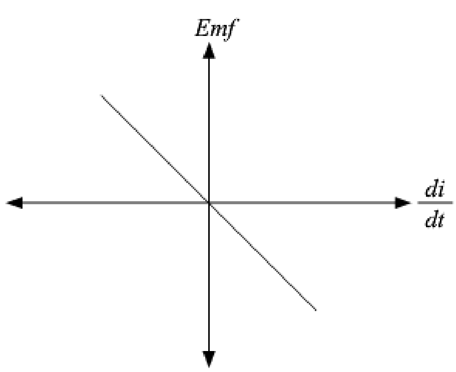
Question 18

Plot a graph showing variation of induced e.m.f. with the rate of change of current flowing through a coil.

OR

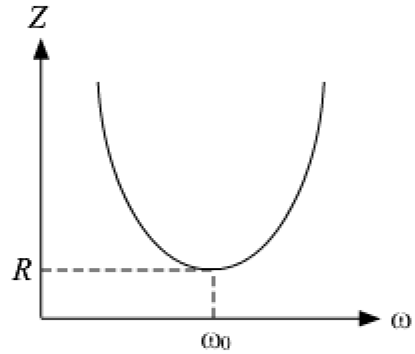
A series combination of an inductor (L), capacitor (C) and a resistor (R) is connected across an ac source of emf of peak value E_0, and angular frequency (ω). Plot a graph to show variation of impedance of the circuit with angular frequency (ω).

Solution:



OR

The graph showing the variation of impedance (Z) of the circuit with angular frequency (ω) is as shown below:



Here, ω_0 represents the resonance frequency for the LCR circuit and R is the resistance of the circuit.

Question 19

Define the term 'current sensitivity' of a moving coil galvanometer.

Solution:

Current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit current flows through it.

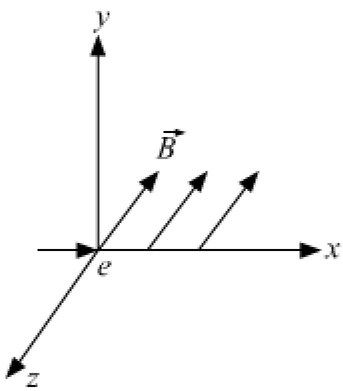
Mathematically it can be given by:

$$I_S = \frac{NBA}{k}$$

Where k is the couple per unit twist.

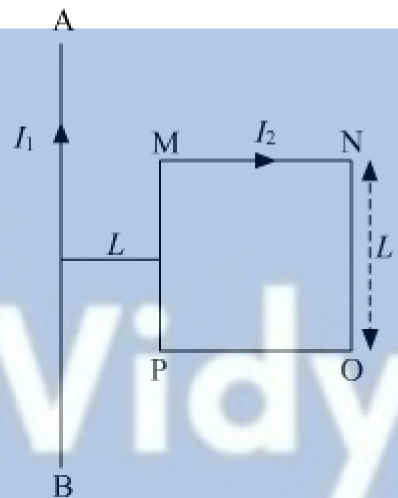
Question 20

An electron moves along $+x$ direction. It enters into a region of uniform magnetic field. $\vec{B}$ directed along $-z$ direction as show in fig. Draw the shape of trajectory followed by the electron after entering the field.



OR

A square shaped current carrying loop MNOP is placed near a straight long current carrying wire AB as shown in the fig. The wire and the loop lie in the same plane. If the loop experiences a net force *F* towards the wire, find the magnitude of the force on the side 'NO' of the loop.

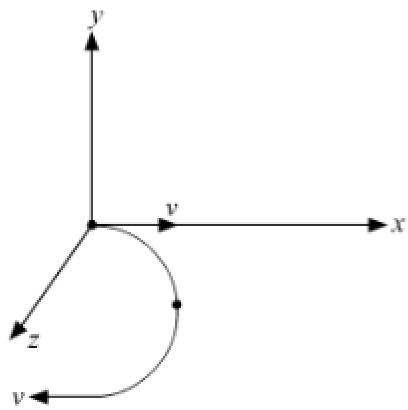


Solution:

Force on the electron is given by

$$\vec{F} = -q\left(\vec{v} \times \vec{B}\right)$$

So, the electron will follow a semi circular path in the magnetic field.



The force acting on the section MN and force on section PO will cancel as the wires are located at equal distance from the infinite wire but have current flowing in opposite directions.

The force acting on the whole loop,

$$F = \frac{\mu_o I_1 I_2 L}{2\pi L} - \frac{\mu_o I_1 I_2 L}{2\pi(2L)} = \frac{\mu_o I_1 I_2 L}{4\pi L}$$

Towards the wire.
The force acting on the side 'NO' is given by

$$F_{NO} = \frac{\mu_0\ I_1\ I_2}{2\pi(2L)} L = \frac{\mu_0\ I_1\ I_2}{4\pi} = F$$

Away from the wire.

Question 21

Write shortcomings of Rutherford atomic model. Explain how these were overcome by the postulates of Bohr's atomic model.

Solution:

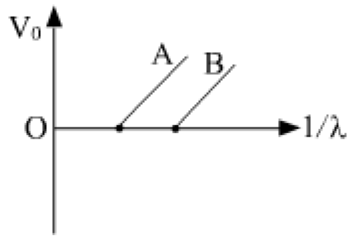
As per Rutherford's model, electrons revolve around the nucleus in a circular path. But particles that are in motion on a circular path would undergo acceleration, and acceleration causes radiation of energy by charged particles.

Eventually, electrons should lose energy and fall into the nucleus. And this points to the instability of atom. But this is not possible because atoms are stable. Hence, Rutherford failed to give an explanation on account of this.

To resolve this problem, Bohr modified the Rutherford model by proposing that the electrons move in orbits of fixed size and energy. The energy of an electron depends on the size of the orbit and is lower for smaller orbits. Radiation can occur only when the electron jumps from one orbit to another.

Question 22

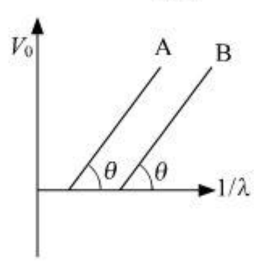
Figure shows the stopping potential (V_0) for the photo electron versus (1λ)1λ graph, for two metals A and B, λ being the wavelength of incident light.



(a) How is the value of Planck's constant determined from the graph?
(b) If the distance between the light source and the surface of metal A is increased, how will the stopping potential for the electrons emitted from it be effected? Justify your answer.

SOLUTION:

(a)
We know that
$eV_\circ = \frac{hc}{\lambda} - \phi$
or $V_\circ = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e} \quad (1)$
where $e =$ charge on electron,
$h =$ Planck's constant
$\phi =$ Work function of metal surface.
Equation (1) is the equation of a straight line as shown in the figure given below.

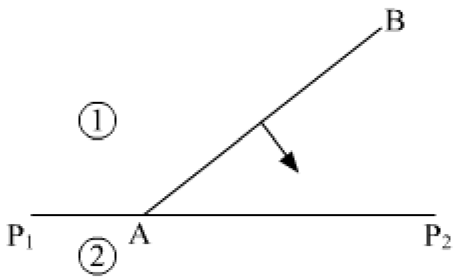


here slope of the line $= \tan\theta = \frac{hc}{e}$
or $h = \frac{e\tan\theta}{c} \quad \ldots(2)$
Planck's constant can easily be determined by substituting the values of the slope of the graph, speed of light and the electronic charge in equation (2).

(b) Stopping potential only depends on frequency of incident light. If the distance is increased, intensity will decrease but the stopping potential will not change.

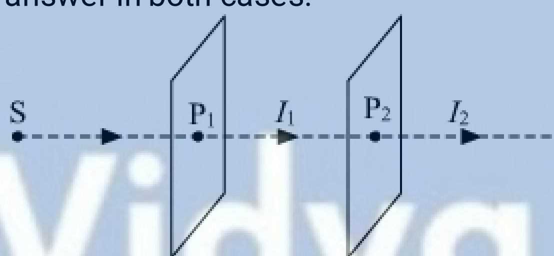
Question 23

Define the term 'wave front of light'. A plane wave front AB propagating from denser medium (1) into a rarer medium (2) is incident of the surface P_1P_2 separating the two media as shown in fig.
Using Huygen's principle, draw the secondary wavelets and obtain the refracted wave front in the diagram.

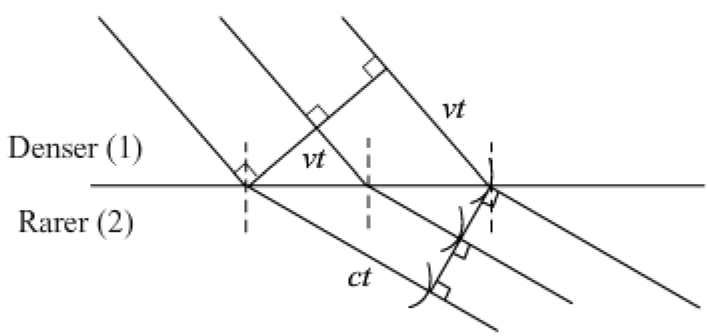


OR

Light from a sodium lamp (S) passes through two polaroid sheets P_1 and P_2 as shown in fig. What will be the effect on the intensity of the light transmitted (i) by P_1 and (ii) by P_2 on the rotating polaroid P_1 about the direction of propagation of light? Justify your answer in both cases.



SOLUTION: It is defined as the locus of all the particles of a medium vibrating in the same phase at a given instant of time.



(i) There is no change in I_1 on rotation of P_1, because the intensity of light does not change irrespective of the orientation of pass–axis of the polaroid.

(ii) I_1 = intensity of polarised light from P_1

I_2 = intensity of polarised light from P_2

θ = Angle between pass axis of polaroid P_1 and P_2

by Malus 's Law , $I_2 = I_1 \cos^2 \theta$

Thus I_2 changes when P_1 is rotated as θ changes

$I_2 = 0$, when $\theta = 90^o$

$I_2 = I_1$, when $\theta = 0^o$

Question 24

Calculate for how many years will the fusion of 2.0 kg deuterium keep 800 W electric lamp glowing. Take the fusion reaction as

$${}_1^2\mathrm{H} + {}_1^2\mathrm{H} \rightarrow {}_2^3\mathrm{He} + {}_0^1\mathrm{n} + 3.27 \text{ Mev}$$

Solution:

2 deuterium nuclei fuse to produce $3.27 \text{ MeV} = 3.27 \times 1.602 \times 10^{-13} \text{ J} = 5.238 \times 10^{-13} \text{ J}$

Energy produced per nuclei = $\frac{5.238 \times 10^{-13} \text{ J}}{2} = 2.61 \times 10^{-13} \text{ J}$

2 kg deuterium atoms will have $= \frac{6.022 \times 10^{23}}{2} \times 2000 = 6.022 \times 10^{26}$ deuterium atoms

6.022×10^{26} deuterium atoms will produce$= 2.61 \times 10^{-13} \times 6.022 \times 10^{26} \text{ J} = 15.77 \times 10^{13} \text{ J}$

$\text{Power} = \frac{E}{t}$

$800 = \frac{15.77 \times 10^{13}}{t}$

$t = \frac{15.77 \times 10^{13}}{800} = 1.971 \times 10^{11}$ seconds

$t = \frac{1.971 \times 10^{11}}{365 \times 24 \times 60 \times 60} = 6.24 \times 10^3$ years

Question 25

In a single slit diffraction experiment, the width of the slit is increased. How will the (i) size and (ii) intensity of central bright band be affected? Justify your answer.

Solution: The size of the central maximum is given by 2λ/a where *a* is the slit width.
It is clear from the above expression if a is increased, the size of the central maximum will decrease.

However, the intensity changes because of two factors.

1. Increasing the width of the slit, causes more light energy to fall on the screen as compared to that with the original width.

2. The light energy is now squeezed into a smaller area on the screen because the size of the central maximum is reduced. The two factors make the intensity increase manyfold.

Question 26

Obtain the expression for the energy stored in a capacitor connected across a dc battery. Hence define energy density of the capacitor.

OR

Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium.

SOLUTION: Energy Stored in a Charged Capacitor

The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.

Let us assume that initially both the plates are uncharged. Now, we have to repeatedly remove small positive charges from one plate and transfer them to the other plate.
Let

q →Total quantity of charge transferred
V →Potential difference between the two plates

Then,

$q = CV$

Now, when an additional small charge dq is transferred from the negative plate to the positive plate, the small work done is given by,

$$dW = Vdq = \frac{q}{C}dq$$

The total work done in transferring charge Q is given by,

$$W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\boxed{W = \frac{Q^2}{2C}}$$

This work done is stored as electrostatic potential energy U in the capacitor.

$$U = \frac{Q^2}{2C}$$

Hence energy stored in the capacitor $= \frac{1}{2}\frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\varepsilon_0 A}$

The surface charge density σ is related to the electric field E between the plates, $E = \frac{\sigma}{\varepsilon_0}$

So, energy stored in the capacitor $= \frac{1}{2}\varepsilon_0 E^2 \times Ad$

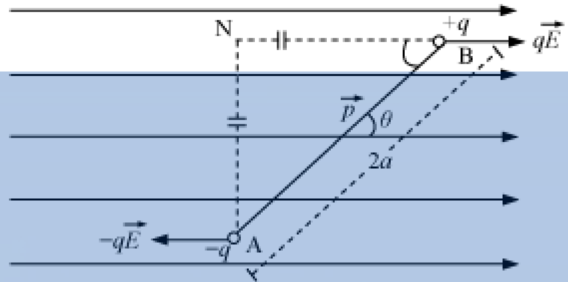
Here, Ad is volume between the plates of capacitor.

We define energy density as energy stored per unit volume of space.

Energy density of electric field $= U = \frac{1}{2}\varepsilon_0 E^2$

OR

Dipole in a Uniform External Field



Consider an electric dipole consisting of charges −*q* and +*q* and of length 2*a* placed in a uniform electric field $\vec{E}$ making an angle *θ* with the electric field.

Force on charge −*q* at $A = -q\vec{E}$ (opposite to $\vec{E}$)

Force on charge +*q* at $B = q\vec{E}$ (along $\vec{E}$)

The Electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole.

τ = Force × Perpendicular distance between the two forces

$\tau = qE\,(AN) = qE\,(2a \sin\theta)$

$\tau = q(2a)\,E \sin\theta$

$\tau = pE \sin\theta$

$\therefore \vec{\tau} = \vec{p} \times \vec{E}$

In a uniform electric field, the net force on dipole will always be zero but torque is zero for θ = 0° and θ = 180°

Now Potential Energy of a dipole in a uniform external electric field is given by the expression $P.E = -\vec{p}\,.\,\vec{E}$

1. For θ = 0°, U = -pE (minimum), the equilibrium will be stable and if the dipole is slightly displaced, it performs oscillations.

2. For θ=180°, U = +pE (maximum), it will be an unstable equilibrium.

Question 27

Gamma rays and radio waves travel with the same velocity in free space. Distinguish between them in terms of their origin and the main application.

Solution:

Gamma rays are produced from radioactive decay of the nucleus while radio waves are produced from rapid acceleration and decelerations of electrons in aerials.

Gamma rays are used as catalyst in the manufacturing of some chemicals. They are also used in treatment of cancer.

Radio waves are used in radio and television communication and broadcasting.

Question 28

(a) Define the term 'half-life' of a radioactive substance

(b) The half life of $^{238}_{92}U$ undergoing alpha decay is 4.5×10^9 years. Calculate the activity of 5 g sample of $^{238}_{92}U$.

Solution:

(a) Half life

The Half-life of a radioactive substance is a characteristic constant. It measures the time it takes for a given amount of the substance to become reduced by half as a consequence of decay

(b) We have,

$$t_{1/2} = 4.5 \times 10^9 \text{ years} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 3600}$$

Number of nuclei present in the sample, $N_0 = \frac{5 \times 6.022 \times 10^{23}}{238}$

$$\text{Activity} = A = \frac{dN}{dt} = \lambda N_o = \left[\frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 3600}\right] \times \left[\frac{5 \times 6.022 \times 10^{23}}{238}\right]$$

$$\simeq 0.61 \times 10^5 \text{ disintegrations /s}$$

Question 29

Explain the formation of potential barrier and depletion region in a p-n junction diode. What is effect of applying forward bias on the width of depletion region?

OR

What is photo diode? Briefly explain its working and draw its V-I characteristics.

Solution:

When a p-n junction is formed, the majority carriers at both sides near the junction diffuse to the opposite side. That is, electrons from the n-side diffuse towards the p-side and holes from the p-side diffuse towards the n-side. This diffusion leaves behind ionized donors on the n-side and ionized acceptors on the p-side.

The ionized donors and acceptors are immobile, as they are bonded to the surrounding atoms of the crystal lattice. This layer of immobile ions is known as **depletion region or space charge region.** That is, there are no free electrons or holes in this region. The thickness of the depletion region is of the order of one-tenth of a micrometer.

Due to this space charge region, an electric field directed from positive charge towards negative charge develops. This field gives rise to the flow of minority carriers. That is, holes in the n-side are attracted towards the p-side and electrons in the p-side are attracted towards the N-side. The motion of charge carriers due to the electric field is known as **drift**.

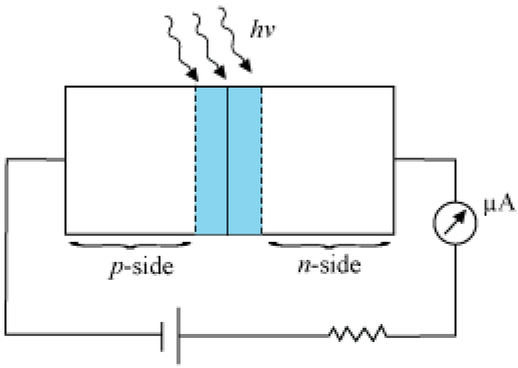
The diffusion and drift continues until there is no net current. The loss of electrons from the n-side and the gain of electrons by the p-side cause a potential difference across the junction. This potential prevents further movement of charge carriers and is called as barrier potential.

If we connect the p-n junction as forward bias then the depletion layer will be reduced whereas when we connect the p-n junction in reverse bias the width of the depletion layer increases.

OR

Photodiode

A Photodiode is a special purpose p-n junction diode fabricated with a transparent window to allow light to fall on the diode. It is operated under reverse bias.

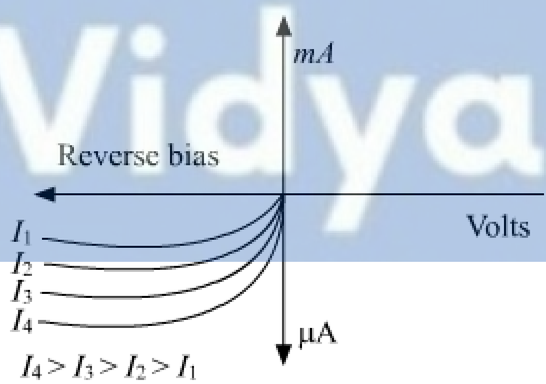


When visible light of energy ($hv > E_g$) enters its depletion region, the electron-hole pairs are generated. These charge carriers are separated by the junction's electric field and are made to flow across the junction and cause reverse saturation current.

The value of reverse saturation current depends on the intensity of incident radiation and is independent of reverse bias.

The photodiode is operated in reverse bias condition because in the reverse bias condition the change in reverse saturation current is directly proportional to the change in incident light intensity.

Thus, the photodiode can be used to detect the optical signals. It cannot be done if the photodiode is forward-biased.



Question 30

Calculate the de-Broglie wavelength associated with the electron in the 2nd excited state of hydrogen atom. The ground state energy of the hydrogen atom is 13.6 eV.

Solution:

the de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$, where p is the momentum of an electron.

Kinetic energy (KE) and momentum (p) are related by , $\mathrm{KE} = \frac{p^2}{2m}$ (m = mass)

$\Rightarrow p = \sqrt{2m(\mathrm{KE})}$

$\Rightarrow \lambda = \frac{h}{\sqrt{2m(\mathrm{KE})}}$

According to Bohr's model, $\text{Kinetic Energy of } e^- = |\text{Total Energy of } e^-| = \left|-\frac{13.6 \times Z^2}{n^2}\right| eV$

for Hydrogen (Z = 1) and second excited state implies n = 3

$$\mathrm{KE} = \frac{13.6 \times 1^2}{3^2} = 1.51 \text{ eV}$$
$$= 1.51 \times 1.6 \times 10^{-19} \text{ J}$$
$$= 2.41 \times 10^{-19} \text{ J}$$

putting the values in formula for wavelength we get,

$$\lambda = \frac{h}{\sqrt{2m(\mathrm{KE})}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2.41 \times 10^{-19}}} = 1.001 \times 10^{-9} \text{ m} \simeq 10 \overset{0}{\text{A}}$$

Question 31

(a) Differentiate between electrical resistance and resistivity of a conductor.
(b) Two metallic rods, each of length L, area of cross A_1 and A_2, having resistivities ρ_1 and ρ_2 are connected in parallel across a d.c. battery. Obtain the expression for the effective resistivity of this combination.

SOLUTION:

Resistance
-Resistance is the property of a conductor to resist the flow of charges through it.
-Its SI unit is ohm.

Resistivity
- Resistivity is the electrical resistance per unit length and per unit of cross-sectional area.
-Its SI unit is ohm metre.

We have,

Area cross section of the 1st wire: A_1
Area cross section of the 1st wire: A_2

Length of each conductor : L
Let, R_1 be the resistance of the 1st conductor.
and R_2 be the resistance of the 2nd conductor.

Now,

$$R_1 = \frac{\rho_1 L}{A_1}$$
$$R_2 = \frac{\rho_2 L}{A_2}$$

In the parallel combination :

$$R_{eq} = \frac{R_1 . R_2}{R_1 + R_2}$$
$$\frac{\rho_{eq} L}{A_1 + A_2} = \frac{\rho_1 \rho_2 L^2 A_1 A_2}{A_1 A_2 L(\rho_1 A_2 + \rho_2 A_1)}$$
$$\rho_{eq} = \frac{\rho_1 \rho_2 (A_1 + A_2)}{(\rho_1 A_2 + \rho_2 A_1)}$$

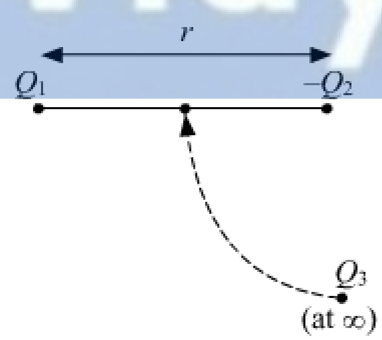
Question 32

(a) Two point charges + Q_1 and $-Q_2$ are placed r distance apart. Obtain the expression for the amount of work done to place a third charge Q_3 at the midpoint of the line joining the two charges.

(b) At what distance from charge + Q_1 on the line joining the two charges (in terms of Q_1, Q_2 and r) will this work done be zero.

Solution:

According to the problem,



(a) Work done:

$$W = \frac{KQ_1Q_3}{\frac{r}{2}} - \frac{KQ_3Q_2}{\frac{r}{2}}$$

$$W = \frac{2KQ_1Q_3}{r} - \frac{2KQ_3Q_2}{r}$$

$$W = \frac{2KQ_3(Q_1 - Q_2)}{r} = \frac{2Q_3(Q_1 - Q_2)}{4\pi\varepsilon_0 r}$$

(b) For work done to be zero

Work done will be zero for the point where the electric potential is zero. Considering a point P to be lying in between the two charges, where electric potential is zero.

Let the distance of the point P from the charge $+Q_1$ be x.

$$V = \frac{KQ_1}{x} - \frac{KQ_2}{(r - x)} = 0$$

$$\frac{Q_1}{x} - \frac{Q_2}{(r - x)} = 0$$

$$\left(rQ_1 - xQ_1 - xQ_2\right) = 0$$

$$\Rightarrow x = \frac{rQ_1}{(Q_1 + Q_2)}$$

Question 33

An optical instrument uses an objective lens of power 100 D and an eyepiece of power 40 D. The final image is formed at infinity when the tube length of the instrument is kept at 20 cm.
(a) Identify the optical instrument.
(b) Calculate the angular magnification produced by the instrument.

Solution:

(a)

$$\text{Power of lens} = \frac{1}{\text{focal length}}$$

$$\text{So, focal length of objective lens} = \frac{1}{100}\ \text{m} = 1\ \text{cm}$$

$$\text{Focal length of eye piece} = \frac{1}{40}\ \text{m} = 2.5\ \text{cm}$$

Since the objective has a smaller focal length than the eyepiece, the instrument is a compound microscope.

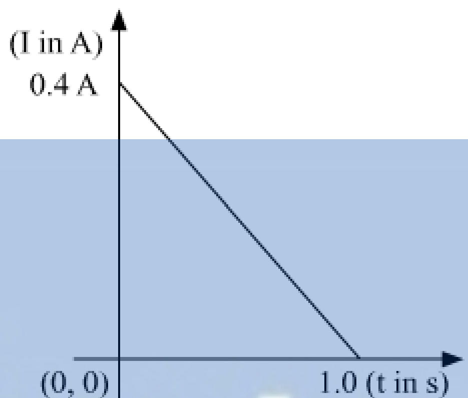
(b)
Magnification produced when the image is formed at infinity is given by,

$$m = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_e}\right) \qquad \text{(where } L \text{ is the tube length)}$$

$$\text{So, } m = \frac{20 \times 25}{1 \times 2.5} = 200$$

Question 34

When a conducting loop of resistance 10 Ω and area 10 cm^2 is removed from an external magnetic field acting normally, the variation of induced current in the loop with time is shown in the figure.



Find the
(i) total charge passed through the loop.
(ii) change in magnetic flux through the loop.
(iii) magnitude of the magnetic field applied.

SOLUTION:

$$I = \frac{dq}{dt} \Rightarrow \int dq = \int \text{Idt}$$

Hence area under the *I-t* curve gives charge flown.

Area of the *I-t* curve (as given in the question) = $= \frac{1}{2} \times 1 \times 0.4 = 0.2$

Total charge passed through the loop, ΔQ = 0.2 C

Resistance of the loop, R = 2 Ω
Let the change in magnetic flux be Δφ

We know that the charge flown throw the loop and the total change in the flux are related as:

$$\Delta Q = \frac{\Delta\varphi}{R}$$
$$\Delta\varphi = \Delta Q \times R = 0.2 \times 10 = 2 \text{ Wb}$$

Total change in magnetic flux through the loop = 2 Wb

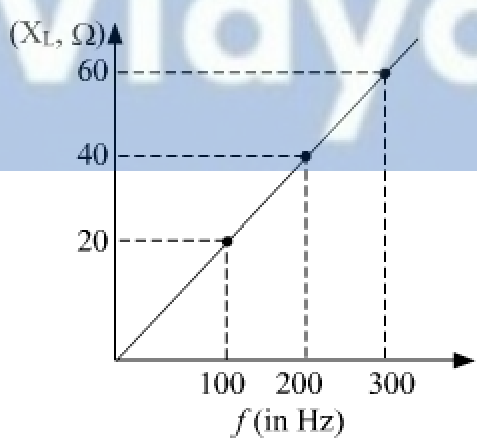
Let the magnetic field applied to the loop be *B*.
total change in area, Δ*A* = 10 cm^2 = 0.001 m^2

$$\Delta\varphi = B\ (\Delta A)$$
$$2 = B\ (0.001)$$
$$B = 2000 \text{ T}$$

The magnitude of the field applied = 2000 T

Question 35

(a) Show that an ideal inductor does not dissipate power in an ac circuit.
(b) The variation of inductive reactance (X_L) of an inductor with the frequency (f) of the ac source of 100 V and variable frequency is shown in the fig.



(i) Calculate the self-inductance of the inductor.
(ii) When this inductor is used in series with a capacitor of unknown value and resistor of 10 Ω at 300 s^{-1}, maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

OR

(a) A conductor of length '*l*' is rotated about one of its ends at a constant angular speed 'ω' in a plane perpendicular to a uniform magnetic field B. Plot graphs to show variations of the emf induced across the ends of the conductor with (i) angular speed ω and (ii) length of the conductor *l*.

(b) Two concentric circular loops of radius 1 cm and 20 cm are placed coaxially.
(i) Find mutual inductance of the arrangement.
(ii) If the current passed through the outer loop is changed at a rate of 5 A/ms, find the emf induced in the inner loop. Assume the magnetic field on the inner loop to be uniform.

SOLUTION:

(a)

Power = $VI\cos\phi$

For pure inductive circuit, the phase difference between current and voltage is $\frac{\pi}{2}$.

$$\therefore \phi = \frac{\pi}{2}, \cos\phi = 0$$

Therefore, zero power is dissipated. This current is sometimes referred to as watt-less current.

(b)

(i)

We know that $X_L = \omega L$ and $\omega = 2\pi f$ where f is frequency in Hz.

$$\text{So, } L = \frac{X_L}{2\pi f} = \frac{20}{2\pi(100)} = \frac{40}{2\pi(200)} = \frac{60}{2\pi(300)} = 31.84 \times 10^{-3} \approx 32 \text{ mH}$$

(ii)

we know that power dissipation is maximum when $X_L = X_C$

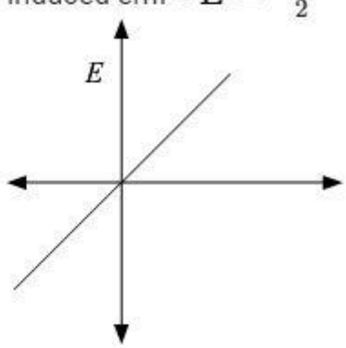
$$\text{or } \omega L = \frac{1}{\omega C} \text{ or } C = \frac{1}{\omega^2 L}$$

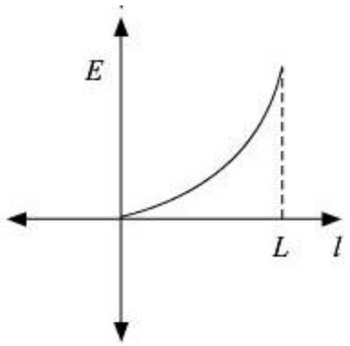
$$\Rightarrow C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\times3.14\times3.14\times300\times300\times32\times10^{-3}} = 8.8\ \mu F$$

OR

(a)

Induced emf = $E = \frac{B\omega l^2}{2}$





(b)
We know $\varphi = \mathrm{MI}$

And magnetic field at the center of the bigger loop $\overrightarrow{\mathrm{B}} = \frac{\mu_0 \mathrm{I}}{2\mathrm{R}} = \frac{4\pi\times10^{-7}\mathrm{I}}{2\times20\times10^{-2}} = \pi \times 10^{-6}\mathrm{I}$

Flux through the smaller loop

$$\varphi = \mathrm{BA_s} = \frac{4\pi\times10^{-5}\mathrm{I}}{40} \times \pi(0.01)^2 = \pi^2 \times 10^{-10} \times \mathrm{I}$$

$$\mathrm{M} = \frac{\varphi}{\mathrm{I}} = \pi^2 \times 10^{-10} = 9.86 \times 10^{-10}\,\mathrm{H}$$

Now emf induced

$$\mathrm{e} = -\frac{\mathrm{d}\varphi}{\mathrm{dt}} = -9.86 \times 10^{-10} \times \frac{\mathrm{dI}}{\mathrm{dt}}$$

$$\mathrm{e} = -9.86 \times 10^{-10} \times 5 = -4.93 \times 10^{-9}\,\mathrm{V}$$

Question 36

(a) Write two important characteristics of equipotential surfaces.
(b) A thin circular ring of radius *r* is charged uniformly so that its linear charge density becomes λ. Derive an expression for the electric field at a point P at a distance *x* from it along the axis of the ring. Hence, prove that at large distances ($x >> r$), the ring behaves as a point charge.

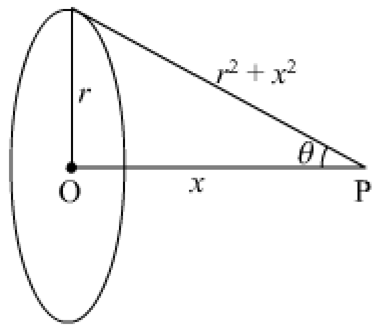
OR

(a) State Gauss's law on electrostatics and drive an expression for the electric field due to a long straight thin uniformly charged wire (linear charge density λ) at a point lying at a distance r from the wire.
(b) The magnitude of electric field (in NC^{-1}) in a region varies with the distance r(in m) as
E = 10r + 5
By how much does the electric potential increase in moving from point at r = 1 1 m to a point at r = 10 m.

SOLUTION: (a) Two important characteristics of equipotential surfaces are:

• Potential remains at all the points on equipotential surface.
• No work is required to move a charge on an equipotential surface.

(b) Let consider a thin circular ring of radius r with charge density as λλ



We need to find the electric field due to this charged ring at a point on the axis of the ring at a distance x from its centre.

Let us consider a small charge element (dxdx) on the ring having small charge dqdq

dq=λdx

the electric field due to this charge element at the point P is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2+x^2)}$$
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(r^2+x^2)}$$

Electric field at the point P will have two components one in the vertical direction and the other one in the horizontal direction.

dE cosθ along the horizontal direction.

dE sinθ along the vertical direction.

The vertical components will cancel out the effect of each other due to the presence of the diametrically opposite element.

So the horizontal component of the electric field will survive at the point P.

From the figure we have the value of $\cos\theta = \frac{x}{\sqrt{r^2+x^2}}$

Now the integration of the horizontal component dEcosθ will be carried out.

$$dE\cos\theta = \frac{\lambda x dx}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}$$

Since the value of $dq = \lambda dx$

$$dE\cos\theta = \frac{x dq}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}$$

Now integrating the above equation and taking x and r quantities as constants we get

$$E_x = \int dE\cos\theta = \int \frac{x dq}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}$$

$$E_x = \int \frac{x dq}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}} = \frac{xQ}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}$$

where Q is the total charge on the ring.

Here Ex is the value of the total electric field at the point P
Special case:

when x >> r, the denominator of the above equation gets modified in the following way:

$$r^2 + x^2 \approx x^2$$

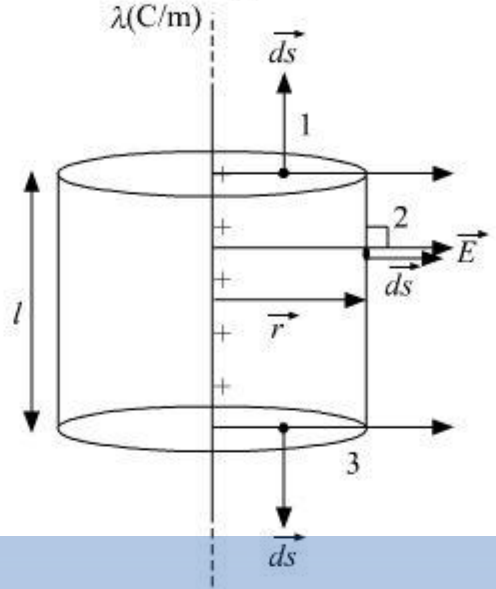
$$E_x = \frac{xQ}{4\pi\varepsilon_0 (x^2)^{3/2}} = \frac{xQ}{4\pi\varepsilon_0 x^3} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

So at large distances (x >> r), the ring behaves as a point charge.

OR

a) Gauss' Law states that the net electric flux through any closed surface is equal to 1/ε0 times the net electric charge within that closed surface.

$$\oint \overrightarrow{E}.d\overrightarrow{s} = \frac{q_{enclosed}}{\varepsilon_o}$$



In the diagram we have taken a cylindrical gaussian surface of radius = *r* and length = *l*.

The net charge enclosed inside the gaussian surface qenclosed = λl

By symmetry we can say that the Electric field will be in radially outward direction.

According to gauss' law,

$$\oint \overrightarrow{E}.d\overrightarrow{s} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\int_1 \overrightarrow{E}.d\overrightarrow{s} + \int_2 \overrightarrow{E}.d\overrightarrow{s} + \int_3 \overrightarrow{E}.d\overrightarrow{s} = \frac{\lambda l}{\varepsilon_o}$$

$\int_1 \overrightarrow{E}.d\overrightarrow{s}$ & $\int_3 \overrightarrow{E}.d\overrightarrow{s}$ are zero , Since $\overrightarrow{E}$ is perpendicular to $d\overrightarrow{s}$

$$\int_2 \overrightarrow{E}.d\overrightarrow{s} = \frac{\lambda l}{\varepsilon_o}$$

at 2, $\overrightarrow{E}$ and $d\overrightarrow{s}$ are in the same direction, we can write

$$E.2\pi rl = \frac{\lambda l}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

b) point A be given at *r* = 1 m, point B be given at *r* = 10 m
V_A = potential at A

V_B = potential at B

The relation between the electric field and potential potential difference is given by the relation,

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{r}$$

$$V_B - V_A = -\int_1^{10}\left(10r + 5\right).dr$$

$$= -\left[\frac{10r^2}{2} + 5r\right]_1^{10}$$

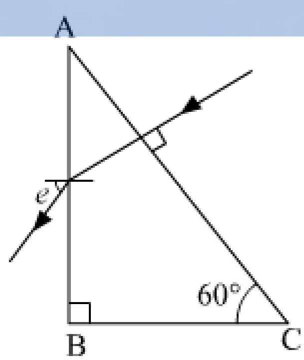
$$= -\left[\left(\frac{10(10)^2}{2} + 5\times 10\right) - \left(\frac{10(1)^2}{2} + 5\times 1\right)\right]$$

$$= -\left[550 - 10\right]$$

$$= -540\ V$$

Question 37

(a) Define the term 'focal length of a mirror'. With the help of a ray diagram, obtain the relation between its focal length and radius of curvature.
(b) Calculate the angle of emergence (e) of the ray of light incident normally on the face AC of a glass prism ABC of refractive index √3. How will the angle of emergence change qualitatively, if the ray of light emerges from the prism into a liquid of refractive index 1.3 instead of air?



OR

(a) Define the term 'resolving power of a telescope'. How will the resolving power be effected with the increase in
(i) Wavelength of light used.
(ii) Diameter of the objective lens.

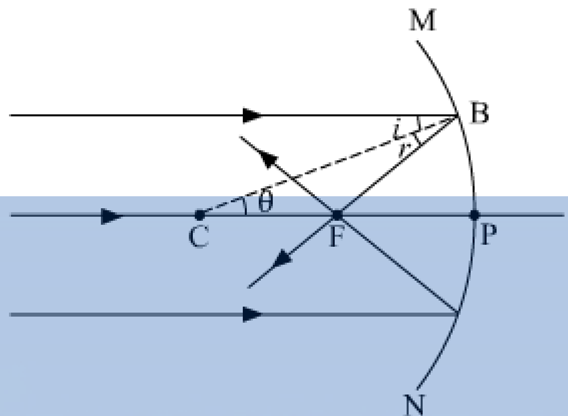
Justify your answers.

(b) A screen is placed 80 cm from an object. The image of the object on the screen is formed by a convex lens placed between them at two different locations separated by a distance 20 cm. determine the focal length of the lens.

SOLUTION:

(a) The distance between the centre of a lens or curved mirror and its focus.

The relationship between the focal length f and the radius of curvature $R = 2f$.



Consider a ray of light AB, parallel to the principal axis and incident on a spherical mirror at point B. The normal to the surface at point B is CB and CP = CB = R is the radius of curvature. The ray AB, after reflection from a mirror, will pass through F (concave mirror) or will appear to diverge from F (convex mirror) and obeys the law of reflection i.e. i = r.

From the geometry of the figure,
$\angle BCP = \theta = i$
In D CBF, $\theta = r$

$\therefore BF = FC$ (because i = r)

If the aperture of the mirror is small, B lies close to P, and therefore BF = PF
Or FC = FP = PF
Or PC = PF + FC = PF + PF
Or R = 2 PF = 2f
Or $f = R/2$

Similar relation holds for convex mirror also. In deriving this relation, we have assumed that the aperture of the mirror is small.

(b)

Snell's law says $\mu_1 \text{Sin(i)} = \mu_2 \text{Sin(r)}$

$\mu_{\text{Prism}} = \sqrt{3}$

$\mu_{\text{Prism}} \sin(30^\circ) = \sin(e)$

$\sqrt{3} \times \frac{1}{2} = \sin(e)$

$e = 60^\circ$

Now when the external medium is changed to liquid of $\mu_{\text{L}} = 1.3$ then,

$\mu_{\text{prism}} \text{Sin}\left(30\right) = \mu_{\text{L}} \text{Sin}\left(\text{e}\right)$

$\sqrt{3} \text{Sin}\left(30^\circ\right) = 1.3 \text{Sin}\left(\text{e}\right)$

$\text{e} = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2\times1.3}\right) = 41.83^\circ$

Hence the angle of emergence reduces to 41.83° from 60°.

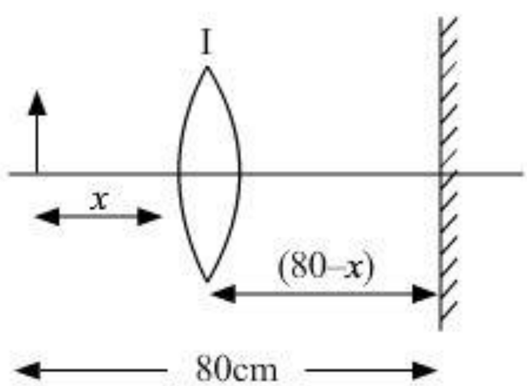
(a) The resolving power of an astronomical telescope is defined as the reciprocal of the smallest angular separation between two point objects whose images can just be resolved by the telescope.

$R.P = \frac{1.22\lambda}{D}$

With the increase in wavelength of light, the resolving power increases whereas with the increase in diameter of the lens, the resolving power decreases.

(b) We have ,

case 1)



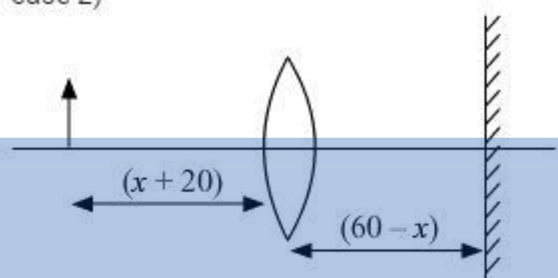
let object distance, $u = x$
image distance, $v = 80 - x$
focal length = f
According to the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{80-x} + \frac{1}{x} \qquad \ldots(1)$$

Similarly,

case 2)



$u = x + 20$
$v = 60 - x$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{60-x} + \frac{1}{20+x} \qquad \ldots(2)$$

On comparing equations 1 and 2 , we get:

$$\frac{1}{f} = \frac{1}{80-x} + \frac{1}{x} = \frac{1}{60-x} + \frac{1}{20+x}$$

$$\frac{80}{x(80-x)} = \frac{80}{(60-x)(20+x)}$$

$$80x - x^2 = 1200 + 40x - x^2$$

$$40x = 1200$$

$$x = 30 \text{ cm}$$

Putting the value of x in equation (1)

$$\frac{1}{f}=\frac{1}{80-30}+\frac{1}{30}$$
$$\frac{1}{f}=\frac{1}{50}+\frac{1}{30}$$
$$\frac{1}{f}=\frac{8}{150}$$
$$f=18.75\ \text{cm}$$