SET-1

Series BVM/3

कोड नं.
Code No. **65/3/1**

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 100*

Time allowed : 3 hours — *Maximum Marks : 100*

सामान्य निर्देश :

(i) ***सभी*** *प्रश्न अनिवार्य हैं ।*

(ii) इस प्रश्न-पत्र में ***29*** *प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द । खण्ड अ में* ***4*** *प्रश्न हैं जिनमें से प्रत्येक* ***एक अंक*** *का है । खण्ड ब में* ***8*** *प्रश्न हैं जिनमें से प्रत्येक* ***दो अंक*** *का है । खण्ड स में* ***11*** *प्रश्न हैं जिनमें से प्रत्येक* ***चार अंक*** *का है । खण्ड द में* ***6*** *प्रश्न हैं जिनमें से प्रत्येक* ***छः अंक*** *का है ।*

(iii) खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं ।

(iv) पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं । फिर भी खण्ड अ के 1 प्रश्न में, खण्ड ब के 3 प्रश्नों में, खण्ड स के 3 प्रश्नों में तथा खण्ड द के 3 प्रश्नों में आन्तरिक विकल्प है । ऐसे सभी प्रश्नों में से आपको एक ही विकल्प हल करना है ।

(v) कैल्कुलेटर के प्रयोग की अनुमति ***नहीं*** *है । यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं ।*

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) The question paper consists of ***29*** *questions divided into four sections : A, B, C and D. Section A comprises of* ***4*** *questions of* ***one mark*** *each, Section B comprises of* ***8*** *questions of* ***two marks*** *each, Section C comprises of* ***11*** *questions of* ***four marks*** *each and Section D comprises of* ***6*** *questions of* ***six marks*** *each.*

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is ***not*** *permitted. You may ask for logarithmic tables, if required.*

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है ।

Questions number 1 to 4 carry 1 mark each.

1. यदि वर्ग आव्यूह A की कोटि 3 और $|A| = 4$ है, तो $|-2A|$ का मान लिखिए ।

If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

2. यदि $y = \sin^{-1} x + \cos^{-1} x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$.

3. अवकल समीकरण

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$$

की कोटि व घात लिखिए ।

Write the order and the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3.$$

4. यदि एक रेखा के दिक्-अनुपात $-18, 12, -4$ हैं, तो इसके दिक्-कोसाइन क्या हैं ?

अथवा

बिन्दु $(-2, 4, -5)$ से गुज़रने वाली उस रेखा का कार्तीय समीकरण ज्ञात कीजिए जो रेखा $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ के समांतर है ।

If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

OR

Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

खण्ड ब

SECTION B

प्रश्न संख्या 5 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।

Questions number 5 to 12 carry 2 marks each.

5. सभी वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ पर परिभाषित संक्रिया $* : a * b = \sqrt{a^2 + b^2}$ है । $\mathbb{R}$ में $*$ के सापेक्ष तत्समक अवयव, यदि इसका अस्तित्व है, ज्ञात कीजिए ।

If $*$ is defined on the set $\mathbb{R}$ of all real numbers by $* : a * b = \sqrt{a^2 + b^2}$, find the identity element, if it exists in $\mathbb{R}$ with respect to $*$.

6. यदि $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ तथा $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ है, तो k, a और b के मान ज्ञात कीजिए ।

If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b.

7. ज्ञात कीजिए :

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}}\, dx, \quad 0 < x < \pi/2$$

Find :

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}} \, dx, \;\; 0 < x < \pi/2$$

8. ज्ञात कीजिए :

$$\int \frac{\sin (x - a)}{\sin (x + a)} \, dx$$

अथवा

ज्ञात कीजिए :

$$\int (\log x)^2 \, dx$$

Find :

$$\int \frac{\sin (x - a)}{\sin (x + a)} \, dx$$

OR

Find :

$$\int (\log x)^2 \, dx$$

9. स्वेच्छ अचरों 'm' तथा 'a' को विलुप्त करते हुए वक्रों के कुल $y^2 = m (a^2 - x^2)$ को निरूपित करने वाला अवकल समीकरण बनाइए ।

Form the differential equation representing the family of curves $y^2 = m (a^2 - x^2)$ by eliminating the arbitrary constants 'm' and 'a'.

10. सदिशों $\vec{a}$ तथा $\vec{b}$, जहाँ $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ तथा $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, दोनों के लम्बवत् एक मात्रक सदिश ज्ञात कीजिए ।

अथवा

दिखाइए कि सदिश $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}$ तथा $\hat{i} - 3\hat{j} + 5\hat{k}$ समतलीय हैं ।

Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

OR

Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

11. एक परिवार की फोटो हेतु माँ, पिता व बेटे को एक लाइन में यादृच्छया खड़ा किया जाता है । यदि दो घटनाएँ A और B निम्न रूप में परिभाषित हों, तो P(B/A) ज्ञात कीजिए :

घटना A : बेटा एक किनारे पर, घटना B : पिता बीच में

Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P(B/A).

12. मान लीजिए X एक यादृच्छिक चर है जिसके संभावित मूल्य x_1, x_2, x_3, x_4 इस प्रकार हैं :

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

X का प्रायिकता बंटन ज्ञात कीजिए ।

अथवा

एक सिक्का 5 बार उछाला जाता है । (i) कम-से-कम 4 चित, और (ii) अधिक-से-अधिक 4 चित प्राप्त करने की प्रायिकता ज्ञात कीजिए ।

Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

Find the probability distribution of X.

OR

A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads, and (ii) at most 4 heads.

खण्ड स

SECTION C

प्रश्न संख्या 13 से 23 तक प्रत्येक प्रश्न के 4 अंक हैं ।

Questions number 13 to 23 carry 4 marks each.

13. दिखाइए कि पूर्णांक समुच्चय Z पर परिभाषित संबंध R = {(a, b) : (a – b), 2 से विभाजित है} एक तुल्यता संबंध है ।

अथवा

यदि $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ है, तो दिखाइए कि सभी $x \neq \frac{2}{3}$ के लिए, $fof(x) = x$ है । f का प्रतिलोम भी ज्ञात कीजिए ।

Show that the relation R on the set Z of all integers, given by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

OR

If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. Also, find the inverse of f.

14. यदि $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$ है, तो x का मान ज्ञात कीजिए और अत: $\sec^{-1}\left(\frac{2}{x}\right)$ का मान ज्ञात कीजिए ।

If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$, find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

15. सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Using properties of determinants, prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

16. यदि $\sin y = x \sin (a + y)$ है, तो सिद्ध कीजिए कि

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

अथवा

यदि $(\sin x)^y = x + y$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $\sin y = x \sin (a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

OR

If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

17. यदि $y = (\sec^{-1} x)^2$, $x > 0$ हो, तो दिखाइए कि

$$x^2 (x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$$

If $y = (\sec^{-1} x)^2$, $x > 0$, show that

$$x^2 (x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$$

18. वक्र $y = \frac{x-7}{(x-2)(x-3)}$ जहाँ x-अक्ष को काटता है, उस बिन्दु से वक्र पर स्पर्श-रेखा व अभिलंब के समीकरण ज्ञात कीजिए ।

Find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

19. ज्ञात कीजिए :

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

Find :

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

20. सिद्ध कीजिए कि

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

अत:

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$ का मूल्यांकन कीजिए ।

Prove that

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$ and hence evaluate

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}.$$

21. अवकल समीकरण $\frac{dy}{dx} = \frac{x+y}{x-y}$ को हल कीजिए ।

अथवा

अवकल समीकरण हल कीजिए :

$$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$$

Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

OR

Solve the differential equation :

$$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$$

22. मान लीजिए $\vec{a}$, $\vec{b}$ और $\vec{c}$ ऐसे तीन सदिश हैं जिनके लिए $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $|\vec{c}| = 3$ हैं । यदि सदिश $\vec{b}$ का सदिश $\vec{a}$ पर प्रक्षेप और सदिश $\vec{c}$ का सदिश $\vec{a}$ पर प्रक्षेप एक-दूसरे के बराबर हैं तथा सदिश $\vec{b}$ और $\vec{c}$ लम्बवत् हों, तो $|3\vec{a} - 2\vec{b} + 2\vec{c}|$ का मान ज्ञात कीजिए ।

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$; and $\vec{b}$, $\vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

23. λ का मान ज्ञात कीजिए जिसके लिए निम्नलिखित रेखाएँ परस्पर लम्बवत् हैं :

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

अत: ज्ञात कीजिए कि क्या ये रेखाएँ एक-दूसरे को काटती हैं या नहीं ।

Find the value of λ for which the following lines are perpendicular to each other :

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

Hence, find whether the lines intersect or not.

खण्ड द

SECTION D

प्रश्न संख्या 24 से 29 तक प्रत्येक प्रश्न के 6 अंक हैं ।

Questions number 24 to 29 carry 6 marks each.

24. यदि $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए ।

अत: निम्न समीकरण निकाय को हल कीजिए :

$x + y + z = 6,$

$y + 3z = 11$

तथा $x - 2y + z = 0$

अथवा

प्रारंभिक रूपांतरणों द्वारा, निम्नलिखित आव्यूह का व्युत्क्रम ज्ञात कीजिए :

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the following system of equations :

$x + y + z = 6,$

$y + 3z = 11$

and $x - 2y + z = 0$

OR

Find the inverse of the following matrix, using elementary transformations :

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

25. दिखाइए कि अधिकतम आयतन के और दिए गए पृष्ठीय क्षेत्रफल के बेलन (जिसका ऊपरी भाग खुला हो) की ऊँचाई, बेलन के आधार की त्रिज्या के बराबर होगी ।

Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

26. समाकलन के प्रयोग से, उस त्रिभुज का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष (–1, 1), (0, 5) तथा (3, 2) हैं ।

अथवा

समाकलन के प्रयोग से वक्रों $(x - 1)^2 + y^2 = 1$ तथा $x^2 + y^2 = 1$ से परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Find the area of the triangle whose vertices are (–1, 1), (0, 5) and (3, 2), using integration.

OR

Find the area of the region bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, using integration.

27. बिन्दुओं (2, 5, – 3), (– 2, – 3, 5) और (5, 3, – 3) से गुज़रने वाले समतल के सदिश व कार्तीय समीकरण ज्ञात कीजिए । यह समतल, एक रेखा, जो बिन्दुओं (3, 1, 5) तथा (– 1, – 3, – 1) से गुज़रती है, को जिस बिन्दु पर काटता है उसे भी ज्ञात कीजिए ।

अथवा

समतलों $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ तथा $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ के प्रतिच्छेदन से होकर जाने वाले उस समतल का समीकरण ज्ञात कीजिए, जो x-अक्ष के समांतर हो । अत: इस समतल की x-अक्ष से दूरी ज्ञात कीजिए ।

Find the vector and cartesian equations of the plane passing through the points (2, 5, – 3), (– 2, – 3, 5) and (5, 3, – 3). Also, find the point of intersection of this plane with the line passing through points (3, 1, 5) and (– 1, – 3, – 1).

OR

Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis.

28. दो डिब्बे I और II दिए गए हैं । डिब्बे I में 3 लाल व 6 काली गेंदें हैं । डिब्बे II में 5 लाल व 'n' काली गेंदें हैं । दोनों डिब्बों I और II में से एक डिब्बे को यादृच्छया चुना जाता है और उसमें से यादृच्छया एक गेंद निकाली जाती है । यदि निकाली गई गेंद लाल है और उसके डिब्बे II से आने की प्रायिकता $\frac{3}{5}$ हो, तो 'n' का मान ज्ञात कीजिए ।

There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.

29. एक कंपनी प्लाइवुड के दो प्रकार के अनूठे स्मृति-चिह्न का निर्माण करती है । A प्रकार के प्रति स्मृति-चिह्न के निर्माण में 5 मिनट काटने और 10 मिनट जोड़ने में लगते हैं । B प्रकार के प्रति स्मृति-चिह्न के लिए 8 मिनट काटने और 8 मिनट जोड़ने में लगते हैं । दिया गया है कि काटने के लिए कुल समय 3 घंटे 20 मिनट तथा जोड़ने के लिए 4 घंटे उपलब्ध हैं । प्रत्येक A प्रकार के स्मृति-चिह्न पर ₹ 50 और प्रत्येक B प्रकार के स्मृति-चिह्न पर ₹ 60 का लाभ होना है । ज्ञात कीजिए कि लाभ के अधिकतमीकरण के लिए प्रत्येक प्रकार के कितने-कितने स्मृति-चिह्नों का कंपनी द्वारा निर्माण होना चाहिए । इस उपर्युक्त समस्या को रैखिक प्रोग्रामन समस्या में परिवर्तित करके आलेख विधि से हल कीजिए तथा अधिकतम लाभ भी ज्ञात कीजिए ।

A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit.

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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/3/1, 65/3/2, 65/3/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/3/1
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \cdot 4 = -32$ $\frac{1}{2}$

2. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

3. order 4, degree 2 $\frac{1}{2}+\frac{1}{2}$

4. $\sqrt{(-18)^2+(12)^2+(-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

6. $kA = k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow 2k = 3a, 3k = 2b \text{ and } -4k = 24$ $\frac{1}{2}$

$\Rightarrow k = -6,\ a = \frac{-12}{3} = -4,\ b = \frac{-18}{2} = -9$ $1\frac{1}{2}$

7. $\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ 1

$= -\log|\sin x + \cos x| + c$ 1

8. $\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int \left[\frac{\sin(x+a)\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x \cdot \cos 2a - \sin 2a \cdot \log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int (\log x)^2 \cdot 1dx = (\log x)^2 \cdot x - \int 2 \cdot \log x \cdot \frac{1}{x} \cdot x\, dx$ 1

$= x \cdot (\log x)^2 - \left\{\log x \cdot 2x - \int \frac{1}{x} \cdot 2x dx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

9. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

10. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j} + \hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3)+2(-6)+3(3)$$
$$= 3-12+9 = 0$$ $1+\frac{1}{2}$

Hence $\vec{a},\vec{b},\vec{c}$ are coplanar

11. A = {(S, F, M), (S, M, F), (M, F, S), (F, M, S)}
B = {(S, F, M), (M, F, S)} 1

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

12. Given $2\,P(X = x_1) = 3P(X=x_2) = P(X=x_3) = 5P(X=x_4)$

Let $P(X=x_3)=k$, then $P(X=x_1)=\frac{k}{2}, P(X=x_2)=\frac{k}{3}$ and $P(X=x_4)=\frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k+\frac{k}{2}+\frac{k}{3}+\frac{k}{5}=1 \Rightarrow k=\frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r\geq 4) = P(4)+P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r\leq 4) = 1-P(5)$

$$= 1-\left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

SECTION C

13. (i) For $a \in Z, (a,a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$$\text{fof}(x) = f\left(\frac{4x+3}{6x-4}\right)$$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow$ $f^{-1} = f$ 1

14. Given $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$

$$\Rightarrow \tan^{-1}x - \left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{\pi}{6}$$ 1

$$\Rightarrow 2\tan^{-1}x = \frac{2\pi}{3} \Rightarrow \tan^{-1}x = \frac{\pi}{3}$$ 1

$$\Rightarrow x = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \qquad \therefore \sec^{-1}\frac{2}{x} = \sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$$ 1+1

15. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ 1

$\therefore \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$ 1

$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$ $\frac{1}{2}$

16. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log\sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1 - y(x+y)\cot x}{(x+y)\log\sin x - 1}$$ $\frac{1}{2}$

17. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2-1}}$$ 1

$$\Rightarrow x\sqrt{x^2-1}\frac{dy}{dx} = 2\sqrt{y}$$ $\frac{1}{2}$

squaring both sides, we get

$$x^2(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$ $\frac{1}{2}$

differentiating w.r.t. x.

$$(x^4 - x^2)2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + (4x^3 - 2x)\left(\frac{dy}{dx}\right)^2 = 4\cdot\frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$ $\frac{1}{2}$

18. Curve $y = \frac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6)\cdot 1 - (x-7)(2x-5)}{(x^2 - 5x + 6)^2}$$ $\frac{1}{2}$

at (7, 0) $\frac{dy}{dx} = \frac{20}{(20)^2} = \frac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\frac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \frac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140$. $\frac{1}{2}$

19. $I = \int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$

Put $\sin^2 x = t \Rightarrow \sin 2x \, dx = dt$ $\frac{1}{2}$

$\therefore I = \int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3} \right) dt$ $1\frac{1}{2}$

$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \frac{1}{2}\log(\sin^2 x + 1) - \frac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

20. RHS $= \int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ $\frac{1}{2}$

Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ ...(i) $\frac{1}{2}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}} dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx = x\Big]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

21. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\,dx = \cot x.dx.$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$ 1

I.F. = $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ 1

$\therefore$ Solution is, $y\cdot(1+x^2) = \int \cot x\,dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore\ \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c} = 0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\,\vec{a}\cdot\vec{b} - 8\,\vec{b}\cdot\vec{c} + 12\,\vec{a}\cdot\vec{c}$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a}-2\vec{b}+2\vec{c}| = \sqrt{61}$ 1

23. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow (5\lambda+2)\cdot 1 + (-5)(2\lambda) + 1(3) = 0$ $\frac{1}{2}$

$-5\lambda + 5 = 0 \Rightarrow \lambda = 1$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7} = \frac{y-2}{-5} - \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $= \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|} = \frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$= \frac{135}{|\vec{b}_1\times\vec{b}_2|} \neq 0$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 1(7) - 1(-3) + 1(-1) = 9$ 1

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ 2

$$\Rightarrow A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\11\\0\end{bmatrix}$

or $AX = B \Rightarrow X = A^{-1}B$ $\frac{1}{2}$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\11\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$ $1\frac{1}{2}$

$\therefore\ x = 1,\ y = 2,\ z = 3$ $\frac{1}{2}$

OR

Let: $\begin{bmatrix}2 & 3 & 1\\2 & 4 & 1\\3 & 7 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$ 1

$$R_1 \leftrightarrow R_3 \begin{bmatrix}3 & 7 & 2\\2 & 4 & 1\\2 & 3 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}A$$

$$\begin{matrix}R_1 \to R_1 - R_3\\R_2 \to R_2 - R_3\end{matrix}\begin{bmatrix}1 & 4 & 1\\0 & 1 & 0\\2 & 3 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\1 & 0 & 0\end{bmatrix}A$$

$$R_3 \to R_3 - 2R_1\begin{bmatrix}1 & 4 & 1\\0 & 1 & 0\\0 & -5 & -1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\3 & 0 & -2\end{bmatrix}A$$

$$R_3 \to R_3 + 5R_2\begin{bmatrix}1 & 4 & 1\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\-2 & 5 & -2\end{bmatrix}A$$ 4

$$\begin{matrix}R_1 \to R_1 - 4R_2\\R_3 \to -R_3\end{matrix}\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & -4 & 1\\-1 & 1 & 0\\2 & -5 & 2\end{bmatrix}A$$

$$R_1 \to R_1 - R_3\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & -1\\-1 & 1 & 0\\2 & -5 & 2\end{bmatrix}A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$ 1

25. Let Given surface area of open cylinder be S.

Then $S = 2\pi rh + \pi r^2$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$$ 1

Volume $V = \pi r^2 h$ $\frac{1}{2}$

$$V = \pi r^2 \left[\frac{S - \pi r^2}{2\pi r}\right] = \frac{1}{2}[Sr - \pi r^3]$$ 1

$$\frac{dV}{dr} = \frac{1}{2}[S - 3\pi r^2]$$ $\frac{1}{2}$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \text{ or } 2\pi rh + \pi r^2 = 3\pi r^2$$ 1

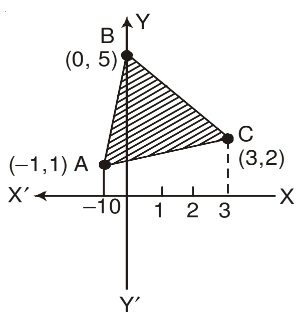
$$\Rightarrow 2\pi rh = 2\pi r^2 \qquad \Rightarrow h = r$$ 1

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$ 1

$\therefore$ For volume to be maximum, height = radius

26. Let the points be A (–1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



Equation of AB : $y = 4x + 5$

BC : $y = 5 - x$

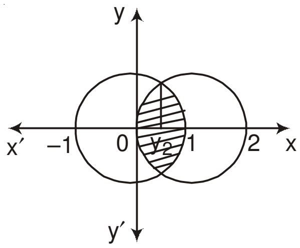
AC : $y = \frac{1}{4}(x + 5)$ $1\frac{1}{2}$

$$\text{Req. Area} = \int_{-1}^{0} (4x+5)\,dx + \int_{0}^{3} (5-x)\,dx - \int_{-1}^{3} \frac{1}{4}(x+5)\,dx$$ $1\frac{1}{2}$

$$\therefore A = \left[\frac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\frac{(5-x)^2}{-2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$= 3+\frac{21}{2}-6=\frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2+y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+2\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]+2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$$ 1

$$\Rightarrow 16(x-2)+24(y-5)+32(z+3)=0$$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$$\frac{x-3}{4}=\frac{y-1}{4}=\frac{z-5}{6} \text{ or } \frac{x-3}{2}=\frac{y-1}{2}=\frac{z-5}{3}$$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$, is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4] = 0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda = 0$ ...(i) 1

Plane (i) is ∥ to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3 = 0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6 = 0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the events be

E_1 : bag I is selected
E_2 : bag II is selected
A : getting a red ball
1

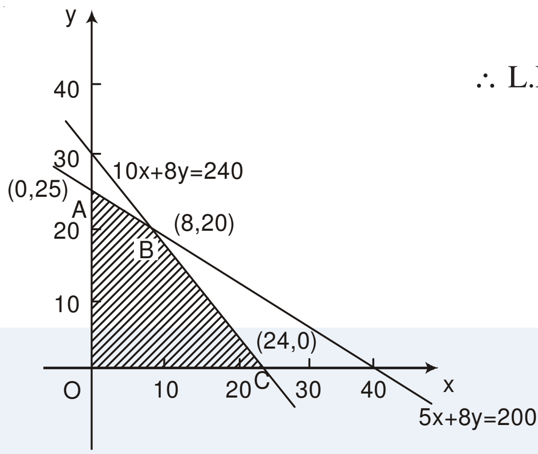
$P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1)=\frac{3}{9}=\frac{1}{3};\ P(A/E_2)=\frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that
$$\left.\begin{aligned} 5x + 8y &\le 200 \\ 10x + 8y &\le 240 \\ x, y &\ge 0 \end{aligned}\right\}$$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. 1

QUESTION PAPER CODE 65/3/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} - \frac{2}{x} \cdot y = 2x \Rightarrow$ I.F. $= e^{-2\log x} = \frac{1}{x^2}$ $\frac{1}{2}+\frac{1}{2}$

2. $y^2 + 2xy\frac{dy}{dx} - 2x = 0 \Rightarrow \frac{dy}{dx} = \frac{2x - y^2}{2xy}$ $\frac{1}{2}+\frac{1}{2}$

3. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \times 4 = -32$ $\frac{1}{2}$

4. $\sqrt{(-18)^2 + (12)^2 + (-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let a = 2, b = 3 $\Rightarrow 2*3 = \frac{2}{4} = \frac{1}{2}$, $3*2 = \frac{3}{3} = 1 \Rightarrow 2*3 \neq 3*2$. 1

$(2*3)*4 = \frac{1}{2}*4 = \frac{\frac{1}{2}}{4+1} = \frac{1}{10}, 2*(3*4) = 2*\frac{3}{5} = \frac{2}{8/5} = \frac{5}{4}$

$\Rightarrow (2*3)*4 \neq 2*(3*4)$ 1

6. $A^2 = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = A$ 1

$\Rightarrow A^3 = A^2 \cdot A = A \cdot A = A^2 = A$ 1

7. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $\quad y\dfrac{dy}{dx} = -mx$ ...(i)

$\quad y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = \dfrac{y}{x}\dfrac{dy}{dx}$ 1

or $\quad xy\dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right)^2 - y\dfrac{dy}{dx} = 0$

8. $\displaystyle\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}}dx = \int \frac{\sin x - \cos x}{\sin x + \cos x}dx$ 1

$= -\log|\sin x + \cos x| + c$ 1

9. $\displaystyle\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$\displaystyle= \int\left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a - \sin 2a\cdot\log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\displaystyle\int (\log x)^2\cdot 1dx = (\log x)^2\cdot x - \int 2\cdot\log x\cdot\frac{1}{x}\cdot x\,dx$ 1

$\displaystyle= x\cdot(\log x)^2 - \left\{\log x\cdot 2x - \int\frac{1}{x}\cdot 2xdx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

10. $A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\}$
$B = \{(S, F, M), (M, F, S)\}$ 1

Total number of possible arrangements = 6

$P(B|A) = \dfrac{P(B\cap A)}{P(A)}$

$= \dfrac{2/6}{4/6} = \dfrac{1}{2}$ 1

11. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \dfrac{k}{2}, P(X = x_2) = \dfrac{k}{3}$ and $P(X = x_4) = \dfrac{k}{5}$ $\frac{1}{2}$

$\therefore\ k + \dfrac{k}{2} + \dfrac{k}{3} + \dfrac{k}{5} = 1 \Rightarrow k = \dfrac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$= {}^5C_4\left(\dfrac{1}{2}\right)^1\left(\dfrac{1}{2}\right)^4 + {}^5C_5\left(\dfrac{1}{2}\right)^5 = 6\left(\dfrac{1}{2}\right)^5 = \dfrac{6}{32}$ or $\dfrac{3}{16}$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$= 1 - \left(\dfrac{1}{2}\right)^5 = \dfrac{31}{32}$ 1

12. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j} + \hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\dfrac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ $\frac{1}{2}$

$$\vec{a} \cdot \vec{b} \times \vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3)$$

$$= 3 - 12 + 9 = 0$$

$1 + \frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

SECTION C

13. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$$fof(x) = f\left(\frac{4x+3}{6x-4}\right)$$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow fof(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow$ $f^{-1} = f$ 1

14. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log \sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1-y(x+y)\cot x}{(x+y)\log \sin x - 1}$$ $\frac{1}{2}$

15. $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\frac{4}{x} = \cos^{-1}\frac{4}{x}$ 1

$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\sqrt{1-\frac{16}{x^2}}\right) \Rightarrow \left(\frac{3}{x}\right)^2 = \frac{x^2-16}{x^2}$ $1\frac{1}{2}$

$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5,\ x = -5$ (rejected) $\therefore\ x = 5$ $\frac{1}{2}+1$

16. $\text{LHS} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix}$ $\left\{\begin{array}{l}\text{Applying} \\ R_1 \to aR_1 \\ R_2 \to bR_2 \\ R_3 \to cR_3\end{array}\right.$ 1

$= \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$ $\{R_1 \to R_1 + R_2 + R_3$ $\frac{1}{2}+1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = 1+a^2+b^2+c^2. \quad \begin{cases} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{cases}$$
$$= \text{RHS}$$ $1\frac{1}{2}$

17. $y = (\cot^{-1} x)^2 \Rightarrow \dfrac{dy}{dx} = 2\cot^{-1} x \cdot \left(\dfrac{-1}{1+x^2}\right)$ 1

$\Rightarrow (1+x^2)\dfrac{dy}{dx} = -2\cot^{-1} x = -2\sqrt{y}$ $\frac{1}{2}$

squaring both sides, we get

$(1+x^2)^2 \cdot \left(\dfrac{dy}{dx}\right)^2 = 4y$ $\frac{1}{2}$

differentiating, w.r.t. x,

$2(1+x^2)2x \cdot \left(\dfrac{dy}{dx}\right)^2 + 2(1+x^2)^2 \cdot \dfrac{dy}{dx} \cdot \dfrac{d^2y}{dx^2} = 4 \cdot \dfrac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow 2x(1+x^2)\dfrac{dy}{dx} + (1+x^2)^2 \dfrac{d^2y}{dx^2} = 2.$ $\frac{1}{2}$

18. $I = \displaystyle\int \frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}\, dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$ $\frac{1}{2}$

$\therefore\ I = \displaystyle\int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3}\right) dt$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log|t+1| - \dfrac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log(\sin^2 x+1) - \dfrac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

19. RHS $= \displaystyle\int_a^b f(a+b-x)\, dx = -\int_b^a f(t)\, dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \displaystyle\int_a^b f(t)\, dt = \int_a^b f(x)\, dx = \text{LHS}$ $\frac{1}{2}$

Let I $= \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$ ...(i) $\frac{1}{2}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\, dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\pi/6}^{\pi/3} 1\cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

20. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2} dv - \frac{1}{2}\int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\, dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{\cot x}{1+x^2}$$ 1

$$\text{I.F.} = e^{\int\frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$$ 1

$\therefore$ Solution is, $y\cdot(1+x^2)=\int \cot x\, dx = \log|\sin x|+c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x|+\frac{c}{1+x^2}$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1};\ \frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$$\Rightarrow (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$$ $\frac{1}{2}$

$$-5\lambda+5=0 \Rightarrow \lambda=1$$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7}=\frac{y-2}{-5}-\frac{z-1}{1};\ \frac{x}{1}=\frac{y+\frac{1}{2}}{2}=\frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$$=\frac{135}{|\vec{b}_1\times\vec{b}_2|}\neq 0$$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore$ $\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c}=0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$ 1

23. Curve $y = \dfrac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$\dfrac{dy}{dx} = \dfrac{(x^2-5x+6)\cdot 1-(x-7)(2x-5)}{(x^2-5x+6)^2}$ $\frac{1}{2}$

at (7, 0) $\dfrac{dy}{dx} = \dfrac{20}{(20)^2} = \dfrac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\dfrac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \dfrac{1}{20}(x-7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x-7)$

or $20x + y = 140$. $\frac{1}{2}$

SECTION D

24. $f(x) = \sin x + \frac{1}{2}\cos 2x \Rightarrow f'(x) = \cos x - \sin 2x$ 1

$f'(x) = 0 \Rightarrow \cos x - 2\sin x \cos x = 0$

$\Rightarrow \cos x\,(1 - 2\sin x) = 0$

$\Rightarrow x = \dfrac{\pi}{2}$ or $x = \dfrac{\pi}{6}$ 1

$x = \dfrac{\pi}{6} \in \left(0, \dfrac{\pi}{2}\right)$ 1

$f''(x) = -\sin x - 2\cos 2x$ 1

$f''(\pi/6) < 0 \Rightarrow \quad x = \dfrac{\pi}{6}$ is a local maxima. 1

Local Max. Value = $f(\pi/6) = \dfrac{3}{4}$ 1

Local extreme values do exist at end points x = 0, x = $\dfrac{\pi}{2}$ but no marks are alotted here for that

25. $A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ 2

$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ 3

or $A \cdot A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow A^2 = A^{-1}$ 1

OR

Given System of equation can be written as

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix}$ or $AX = B$ 1

$|A| = 2(0) + 3\ (-2) + 5\ (1) = -1 \neq 0$ 1

$\therefore\ X = A^{-1} \cdot B$

$(\text{adj. } A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ 2

$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$ x = 2, y = 2, z = 3. $1\frac{1}{2}$

26. Let the events be

$$\left.\begin{array}{l} E_1 : \text{bag I is selected} \\ E_2 : \text{bag II is selected} \\ A : \text{getting a red ball} \end{array}\right\}$$ 1

$P(E_1) = P(E_2) = \dfrac{1}{2}$ $\frac{1}{2}$

$P(A/E_1) = \dfrac{3}{9} = \dfrac{1}{3}$; $P(A/E_2) = \dfrac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow 16(x-2)+24(y-5)+32(z+3)=0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) = 7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6} \text{ or } \frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3}$$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0,$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4] = 0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda = 0$ ...(i) 1

Plane (i) is || to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

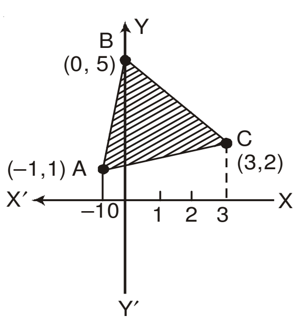
$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3 = 0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6 = 0$

Distance of this plane from x–axis

$= \frac{|-6|}{\sqrt{(-1)^2+(3)^2}} = \frac{6}{\sqrt{10}}$ units 1

28. Let the points be A (–1, 1), B (0, 5) and C (3, 2)



Correct Figure 1

Equation of AB : $y=4x+5$

BC : $y=5-x$

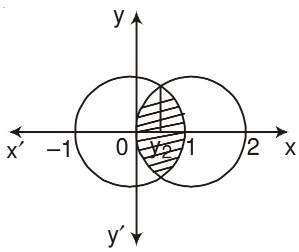
AC : $y=\frac{1}{4}(x+5)$ $1\frac{1}{2}$

Req. Area $= \int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\int_{-1}^{3}\frac{1}{4}(x+5)dx$ $1\frac{1}{2}$

$$\therefore \; A = \left[\frac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\frac{(5-x)^2}{-2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$= 3 + \frac{21}{2} - 6 = \frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2 + y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

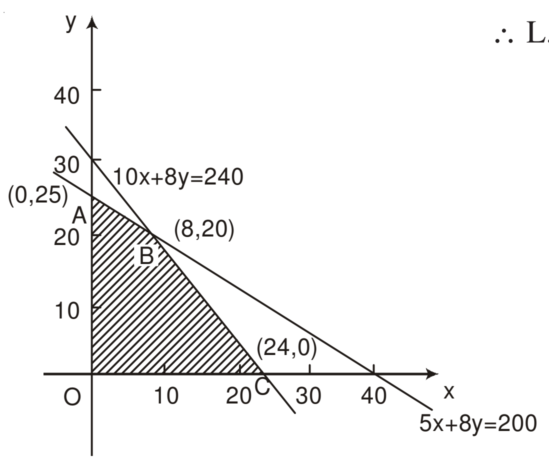
$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] + 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $5x + 8y \le 200$

$10x + 8y \le 240$

$x, y \ge 0$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

QUESTION PAPER CODE 65/3/3
EXPECTED ANSWER/VALUE POINTS
SECTION A

1. DRs are (6, 2, 3) $\therefore$ DC's are $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ $\frac{1}{2}+\frac{1}{2}$

OR

$\frac{x-1}{-3}=\frac{y-7}{p}=\frac{z-3}{2}; \frac{x-1}{-3p}=\frac{y-5}{1}=\frac{z-6}{-5}$ $\frac{1}{2}$

$\Rightarrow 9p + p - 10 = 0 \Rightarrow p = 1$ $\frac{1}{2}$

2. $\frac{dy}{dx}-\frac{2}{x}\cdot y=2x$ $\frac{1}{2}$

$\Rightarrow$ I.F. $= e^{-2\log x}=x^{-2}=\frac{1}{x^2}$ $\frac{1}{2}$

3. $|-2A| = (-2)^3\cdot|A|$ $\frac{1}{2}$

$= -8 \times 4 = -32$ $\frac{1}{2}$

4. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. $B'A'=\begin{bmatrix}4 & 7 & 2\\0 & 1 & 2\\2 & 4 & 6\end{bmatrix}\begin{bmatrix}3 & 1 & 7\\9 & 8 & 5\\0 & -2 & 4\end{bmatrix}$ 1

$=\begin{bmatrix}75 & 56 & 71\\9 & 4 & 13\\42 & 22 & 58\end{bmatrix}$ 1

6. $\int_a^b \frac{\log x}{x}dx=\left[\frac{1}{2}(\log x)^2\right]_a^b$ 1

$=\frac{1}{2}[(\log b)^2-(\log a)^2]$ 1

7. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

8. $\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int \left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a - \sin 2a \cdot \log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int (\log x)^2 \cdot 1 dx = (\log x)^2 \cdot x - \int 2\cdot \log x \cdot \frac{1}{x}\cdot x\, dx$ 1

$= x\cdot(\log x)^2 - \left\{\log x \cdot 2x - \int \frac{1}{x}\cdot 2x dx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

9. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j}+\hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a}\cdot\vec{b}\times\vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c}=\begin{vmatrix}1 & -2 & 3\\ -2 & 3 & -4\\ 1 & -3 & 5\end{vmatrix}=1(3)+2(-6)+3(3)$$
$$=3-12+9=0$$ $1+\frac{1}{2}$

Hence $\vec{a},\vec{b},\vec{c}$ are coplanar

10. A = {(S, F, M), (S, M, F), (M, F, S), (F, M, S)}
B = {(S, F, M), (M, F, S)} 1

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$= \frac{2/6}{4/6}=\frac{1}{2}$$ 1

11. Given $2\,P(X = x_1) = 3P(X=x_2)=P(X=x_3)=5P(X=x_4)$

Let $P(X=x_3)=k$, then $P(X=x_1)=\frac{k}{2}, P(X=x_2)=\frac{k}{3}$ and $P(X=x_4)=\frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k+\frac{k}{2}+\frac{k}{3}+\frac{k}{5}=1\Rightarrow k=\frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r\geq 4)=P(4)+P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5=6\left(\frac{1}{2}\right)^5=\frac{6}{32}\ \text{or}\ \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r\leq 4)=1-P(5)$

$$= 1-\left(\frac{1}{2}\right)^5=\frac{31}{32}$$ 1

12. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

SECTION C

13. $\tan(\sec^{-1}\frac{1}{x}) = \sin(\tan^{-1}2) \Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$ 2

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$ 1

$\Rightarrow 9x^2 = 5 \Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}, \{x > 0\}$ 1

14. $e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1)e^y \cdot \frac{dy}{dx} = 0$ 1

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ 1

$\Rightarrow \frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2}$ 1

$\therefore \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ 1

OR

$y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right) = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$, where $t = 2^x$ 1

$\Rightarrow y = 2\tan^{-1}t$ 1

$\frac{dy}{dt} = \frac{2}{1+t^2}$ and $\frac{dt}{dx} = 2^x \cdot \log 2.$ 1

$\Rightarrow \frac{dy}{dx} = \frac{2}{1+t^2} \cdot 2^x \cdot \log 2 = \frac{2^{x+1} \cdot \log 2}{1+4^x}$ 1

15. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$\Rightarrow f'(x) = 12x^2 - 12x - 72 = 12(x-3)(x+2)$ 1

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ possible intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

$f'(x) < 0$ in $(-2, 3)$

and $f'(x) > 0$ in $(-\infty, -2)$ and $(3, \infty)$

$\Rightarrow$ f(x) is strictly increasing in $(-\infty, -2), (3, \infty)$ or $(-\infty, 2], [3, \infty)$ 1

and strictly decreasing in $(-2, 3)$ or $[-2, 3]$ $\frac{1}{2}$

16. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let $(a, b) \in R$ for $a, b \in Z$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in R \Rightarrow$ R is symmetric ...(ii) 1

For $a, b, c, \in Z$, Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore a - b = 2p, p \in Z$, and $b - c = 2q, q \in Z$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in R$, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$fof(x) = f\left(\frac{4x+3}{6x-4}\right)$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x ⇒ fof = I ⇒ $f^{-1} = f$ 1

17. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ 1

$$\therefore \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$$ 1

$$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$$ $\frac{1}{2}$

18. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2 - 1}}$$ 1

$$\Rightarrow x\sqrt{x^2-1}\frac{dy}{dx} = 2\sqrt{y}$$ $\frac{1}{2}$

squaring both sides, we get

$$x^2(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$ $\frac{1}{2}$

differentiating w.r.t. x.

$$(x^4 - x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3 - 2x)\left(\frac{dy}{dx}\right)^2 = 4 \cdot \frac{dy}{dx}$$ 1

$$\Rightarrow\ x^2(x^2-1)\frac{d^2y}{dx^2}+(2x^3-x)\frac{dy}{dx}-2=0$$ 1

19. RHS $= \int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ $\frac{1}{2}$

Let I $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ...(i) $\frac{1}{2}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

20. $I = \int \frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}\,dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\,dx = dt$ $\frac{1}{2}$

$\therefore\ I = \int \frac{dt}{(t+1)(t+3)} = \int\left(\frac{1/2}{t+1}+\frac{-1/2}{t+3}\right)dt$ $1\frac{1}{2}$

$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \frac{1}{2}\log(\sin^2 x+1) - \frac{1}{2}\log(\sin^2+3) + c.$ $\frac{1}{2}$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow\ (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$ $\frac{1}{2}$

$-5\lambda+5=0 \Rightarrow \lambda=1$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7}=\frac{y-2}{-5}-\frac{z-1}{1};\ \frac{x}{1}=\frac{y+\frac{1}{2}}{2}=\frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$=\frac{135}{|\vec{b}_1\times\vec{b}_2|}\neq 0$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}\ \therefore\ \vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c}=0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$ 1

$=9(1)^2+4(2)^2+4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a}-2\vec{b}+2\vec{c}|=\sqrt{61}$ 1

23. $\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v+x\frac{dv}{dx}=\frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}$

$$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$ $1+\frac{1}{2}$

$$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$$ 1

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\,dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$$ 1

I.F. $= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ 1

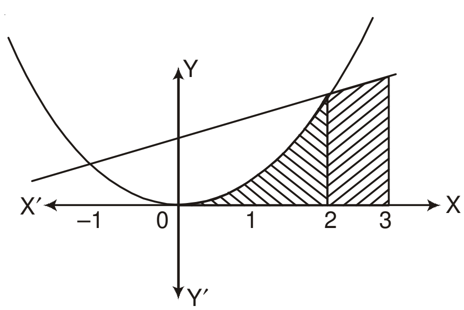
$\therefore$ Solution is, $y\cdot(1+x^2) = \int \cot x\,dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$

SECTION D

24. $\{(x,y): 0 \le y \le x^2, 0 \le y \le x+2, -1 \le x \le 3\}$

Correct Figure 1



$$\text{Area} = \int_{-1}^{2} x^2 dx + \int_{2}^{3} (x+2)dx$$ 2

$$= \left.\frac{x^3}{3}\right]_{-1}^{2} + \left.\frac{(x+2)^2}{2}\right]_{2}^{3}$$ 2

$$= 3 + \frac{9}{2} = \frac{15}{2}$$ 1

OR

$$I = \lim_{h\to 0} h[f(1) + f(1+h) + f(1+2h) + \cdots + f(1+\overline{n-1})n)]$$ 1

where $h = \dfrac{3}{n}$ or $nh = 3$ $\frac{1}{2}$

$$= \lim_{h\to 0} h[(2+e^2) + (2+h+e^{2+2h}) + (2+2h+e^{2+4h}) + \cdots + (2+(n-1)h) + e^{2+2(n-1)h}]$$ 1

$$= \lim_{h\to 0} h\left[2n + h\cdot\frac{n(n-1)}{2}\right] + \lim_{h\to 0} h\cdot e^2[1 + e^{2h} + e^{4h} + \cdots e^{2(n-1)h}]$$ 1

$$= \lim_{h\to 0}\left[2nh + \frac{nh(nh-h)}{2}\right] + \lim_{h\to 0} \not{h}\cdot\frac{e^2}{2\not{h}}\frac{e^{2nh}-1}{\frac{e^{2h}-1}{2h}}$$ $1\frac{1}{2}$

$$= 6 + \frac{9}{2} + \frac{e^2(e^6-1)}{2} = \frac{21}{2} + \frac{e^2(e^6-1)}{2}$$ 1

25. p = (prob. of doublet) = 1/6 ∴ q = 5/6 1

X	0	1	2	3	4
P(X)	$\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$	$4\cdot\left(\frac{5}{6}\right)^3\frac{1}{6} = \frac{500}{1296}$	$6\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2$ $\frac{150}{1296}$	$4\frac{5}{6}\left(\frac{1}{6}\right)^3$ $\frac{20}{1296}$	$\left(\frac{1}{6}\right)^4$ $\frac{1}{1296}$
XP(X)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$
$X^2P(X)$	0	$\frac{500}{1296}$	$\frac{600}{1296}$	$\frac{180}{1296}$	$\frac{16}{1296}$

1

1

1

Mean = $\Sigma XP(X) = \dfrac{864}{1296} = \dfrac{2}{3}$ 1

Variance = $\Sigma X^2\cdot P(X) - [\Sigma X\cdot P(X)]^2 = 1 - \dfrac{4}{9} = \dfrac{5}{9}$ 1

26. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow\ 16(x-2)+24(y-5)+32(z+3)=0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$\frac{x-3}{4}=\frac{y-1}{4}=\frac{z-5}{6}$ or $\frac{x-3}{2}=\frac{y-1}{2}=\frac{z-5}{3}$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+4)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0,$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow\ \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is || to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

27. Let Given surface area of open cylinder be S.

Then S = $2\pi rh + \pi r^2$

$\Rightarrow h = \dfrac{S - \pi r^2}{2\pi r}$ 1

Volume V = $\pi r^2 h$ $\frac{1}{2}$

$V = \pi r^2 \left[\dfrac{S - \pi r^2}{2\pi r}\right] = \dfrac{1}{2}[Sr - \pi r^3]$ 1

$\dfrac{dV}{dr} = \dfrac{1}{2}[S - 3\pi r^2]$ $\frac{1}{2}$

$\dfrac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2$ or $2\pi rh + \pi r^2 = 3\pi r^2$ 1

$\Rightarrow 2\pi rh = 2\pi r^2$ $\quad \Rightarrow h = r$ 1

$\dfrac{d^2V}{dr^2} = -6\pi r < 0$ 1

$\therefore$ For volume to be maximum, height = radius

28. $|A| = 1(7) - 1(-3) + 1(-1) = 9$ 1

$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ 2

$\Rightarrow A^{-1} = \dfrac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

or $AX = B \Rightarrow X = A^{-1}B$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ $1\frac{1}{2}$

$\therefore$ x = 1, y = 2, z = 3 $\frac{1}{2}$

OR

Let: $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$\begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{matrix} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$

$R_3 \to R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A$

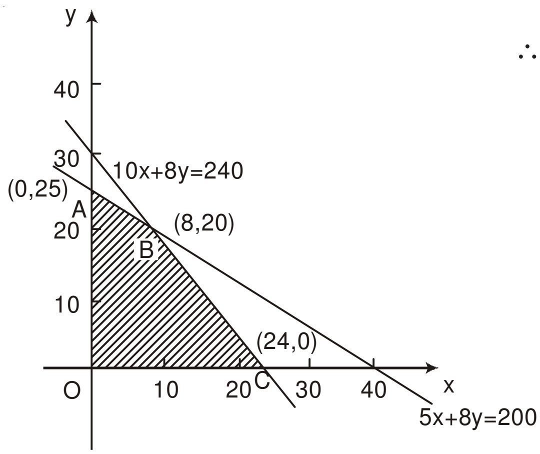
$R_3 \to R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A$ 4

$\begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to -R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$

$R_1 \to R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ — $\frac{1}{2}$

$$\text{such that } \left.\begin{aligned} 5x + 8y &\le 200 \\ 10x + 8y &\le 240 \\ x, y &\ge 0 \end{aligned}\right\}$$ — $2\frac{1}{2}$

Correct Graph — 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. — 1