8 QUADRILATERALS

EXERCISE 8.1

Q.1. *The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.*

Sol. Suppose the measures of four angles are $3x$, $5x$, $9x$ and $13x$.

$\therefore\ 3x + 5x + 9x + 13x = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow\ 30x = 360°$

$\Rightarrow\ x = \frac{360°}{30} = 12°$

$\Rightarrow\ 3x = 3 \times 12° = 36°$

$5x = 5 \times 12° = 60°$

$9x = 9 \times 12° = 108°$

$13x = 13 \times 12° = 156°$

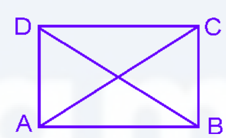
$\therefore$ the angles of the quadrilateral are **36°, 60°, 108° and 156° Ans.**

Q.2. *If the diagonals of a parallelogram are equal, then show that it is a rectangle.*

Sol. **Given :** ABCD is a parallelogram in which AC = BD.

To Prove : ABCD is a rectangle.

Proof : In ΔABC and ΔABD



AB = AB [Common]

BC = AD [Opposite sides of a parallelogram]

AC = BD [Given]

$\therefore\ \Delta ABC \cong \Delta BAD$ [SSS congruence]

$\angle ABC = \angle BAD$ ...(i) [CPCT]

Since, ABCD is a parallelogram, thus,

$\angle ABC + \angle BAD = 180°$ ...(ii) [Consecutive interior angles]

$\angle ABC + \angle ABC = 180°$

$\therefore\ 2\angle ABC = 180°$ [From (i) and (ii)]

$\Rightarrow\ \angle ABC = \angle BAD = 90°$

This shows that ABCD is a parallelogram one of whose angle is 90°.

Hence, ABCD is a rectangle. **Proved.**

Q.3. *Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.*

Sol. **Given :** A quadrilateral ABCD, in which diagonals AC and BD bisect each other at right angles.

To Prove : ABCD is a rhombus.

Proof : In ΔAOB and ΔBOC

$AO = OC$

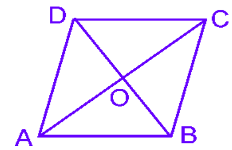
[Diagonals AC and BD bisect each other]

$\angle AOB = \angle COB$ [Each = $90°$]

$BO = BO$ [Common]

$\therefore \Delta AOB \cong \Delta BOC$ [SAS congruence]

$AB = BC$ ...(i) [CPCT]



Since, ABCD is a quadrilateral in which

$AB = BC$ [From (i)]

Hence, ABCD is a rhombus.

[$\because$ if the diagonals of a quadrilateral bisect each other, then it is a parallelogram and opposite sides of a parallelogram are equal] **Proved.**

Q.4. *Show that the diagonals of a square are equal and bisect each other at right angles.*

Sol. **Given :** ABCD is a square in which AC and BD are diagonals.

To Prove : $AC = BD$ and AC bisects BD at right angles, i.e. $AC \perp BD$. $AO = OC$, $OB = OD$

Proof : In ΔABC and ΔBAD,

$AB = AB$ [Common]

$BC = AD$ [Sides of a square]

$\angle ABC = \angle BAD = 90°$ [Angles of a square]

$\therefore \Delta ABC \cong \Delta BAD$ [SAS congruence]

$\Rightarrow AC = BD$ [CPCT]

Now in ΔAOB and ΔCOD,

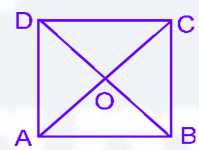
$AB = DC$ [Sides of a square]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate angles]

$\therefore \Delta AOB \cong \Delta COD$ [AAS congruence]

$\angle AO = \angle OC$ [CPCT]



Similarly by taking ΔAOD and ΔBOC, we can show that $OB = OD$.

In ΔABC, $\angle BAC + \angle BCA = 90°$ [$\because \angle B = 90°$]

$\Rightarrow 2\angle BAC = 90°$ [$\angle BAC = \angle BCA$, as $BC = AD$]

$\Rightarrow \angle BCA = 45°$ or $\angle BCO = 45°$

Similarly $\angle CBO = 45°$

In ΔBCO.

$\angle BCO + \angle CBO + \angle BOC = 180°$

$\Rightarrow 90° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 90°$

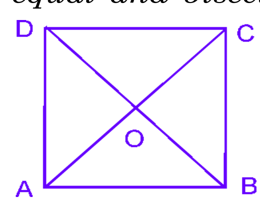
$\Rightarrow BO \perp OC \Rightarrow BO \perp AC$

Hence, $AC = BD$, $AC \perp BD$, $AO = OC$ and $OB = OD$. **Proved.**

Q.5. *Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.*

Sol. **Given :** A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles,

To Prove : ABCD is a square.



Proof : Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

$\Rightarrow$ AB = BC = CD = DA [Sides of a rhombus]

In ΔABC and ΔBAD, we have

AB = AB [Common]

BC = AD [Sides of a rhombus]

AC = BD [Given]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [SSS congruence]

$\therefore$ $\angle ABC = \angle BAD$ [CPCT]

But, $\angle ABC + \angle BAD = 180°$ [Consecutive interior angles]

$\angle ABC = \angle BAD = 90°$

$\angle A = \angle B = \angle C = \angle D = 90°$ [Opposite angles of a $||^{gm}$]

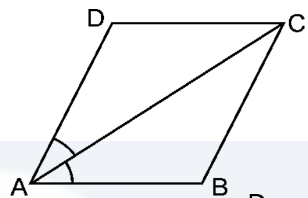
$\Rightarrow$ ABCD is a rhombus whose angles are of 90° each.

Hence, ABCD is a square. **Proved.**

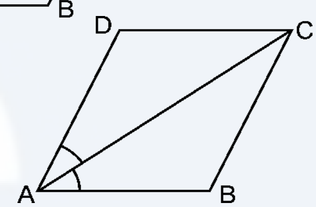
Q.6. *Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig.). Show that*

(i) it bisects ∠C also,

(ii) ABCD is a rhombus.



Given : A parallelogram ABCD, in which diagonal AC bisects $\angle$A, i.e., $\angle DAC = \angle BAC$.



To Prove : (i) Diagonal AC bisects $\angle$C i.e., $\angle DCA = \angle BCA$

(ii) ABCD is a rhomhus.

Proof : (i) $\angle DAC = \angle BCA$ [Alternate angles]

$\angle BAC = \angle DCA$ [Alternate angles]

But, $\angle DAC = \angle BAC$ [Given]

$\therefore$ $\angle BCA = \angle DCA$

Hence, AC bisects $\angle$DCB

Or, AC bisects $\angle$C **Proved.**

(ii) In ΔABC and ΔCDA

AC = AC [Common]

$\angle BAC = \angle DAC$ [Given]

and $\angle BCA = \angle DAC$ [Proved above]

$\therefore$ $\Delta ABC \cong \Delta ADC$ [ASA congruence]

$\therefore$ BC = DC [CPCT]

But AB = DC [Given]

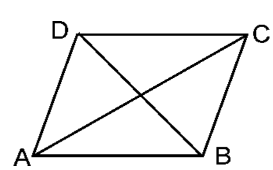
$\therefore$ AB = BC = DC = AD

Hence, ABCD is a rhombus **Proved.**

[$\because$ opposite angles are equal]

Q.7. *ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.*

Sol. Given : ABCD is a rhombus, i.e., AB = BC = CD = DA.



To Prove : $\angle DAC = \angle BAC$,

$\angle BCA = \angle DCA$

$\angle ADB = \angle CDB$, $\angle ABD = \angle CBD$

Proof : In ΔABC and ΔCDA, we have

$AB = AD$ [Sides of a rhombus]

$AC = AC$ [Common]

$BC = CD$ [Sides of a rhombus]

$\Delta ABC \cong \Delta ADC$ [SSS congruence]

So, $\angle DAC = \angle BAC$, $\angle BCA = \angle DCA$ [CPCT]

Similarly, $\angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$.

Hence, diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$. **Proved.**

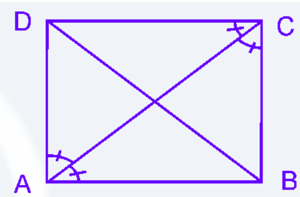
Q.8. *ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that :*

(i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. Given : ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.

To Prove : (i) ABCD is a square.

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.



Proof : (i) In ΔABC and ΔADC, we have

$\angle BAC = \angle DAC$ [Given]

$\angle BCA = \angle DCA$ [Given]

$AC = AC$

$\therefore \Delta ABC \cong \Delta ADC$ [ASA congruence]

$\therefore AB = AD$ and $CB = CD$ [CPCT]

But, in a rectangle opposite sides are equal,

i.e., $AB = DC$ and $BC = AD$

$\therefore AB = BC = CD = DA$

Hence, ABCD is a square **Proved.**

(ii) In ΔABD and ΔCDB, we have

$AD = CD$, $AB = CD$ [Sides of a square]

$BD = BD$ [Common]

$\therefore \Delta ABD \cong \Delta CBD$ [SSS congruence]

So, $\angle ABD = \angle CBD$, $\angle ADB = \angle CDB$ [CPCT]

Hence, diagonal BD bisects $\angle B$ as well as $\angle D$ **Proved.**

Q.9. *In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig.). Show that :*

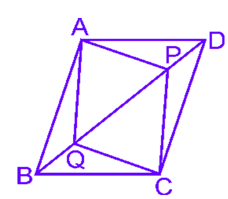
(i) $\Delta APD \cong \Delta CQB$

(ii) $AP = CQ$

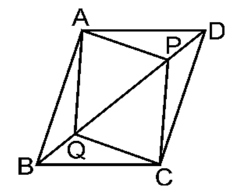
(iii) $\Delta AQB \cong \Delta CPD$

(iv) $AQ = CP$

(v) APCQ is a parallelogram



Sol. **Given :** ABCD is a parallelogram and P and Q are points on diagonal BD such that DP = BQ.



To Prove : (i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

Proof : (i) In ΔAPD and ΔCQB, we have

AD = BC [Opposite sides of a ||gm]

DP = BQ [Given]

$\angle ADP = \angle CBQ$ [Alternate angles]

$\therefore$ $\Delta APD \cong \Delta CQB$ [SAS congruence]

(ii) $\therefore$ AP = CQ [CPCT]

(iii) In ΔAQB and ΔCPD, we have

AB = CD [Opposite sides of a ||gm]

DP = BQ [Given]

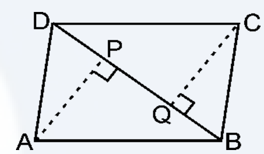
$\angle ABQ = \angle CDP$ [Alternate angles]

$\therefore$ $\Delta AQB \cong \Delta CPD$ [SAS congruence]

(iv) $\therefore$ AQ = CP [CPCT]

(v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram. **Proved.**

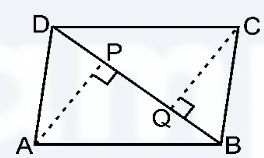
Q.10. *ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that*



(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Sol. **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.



To Prove : (i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

Proof : (i) In ΔAPB and ΔCQD, we have

$\angle ABP = \angle CDQ$ [Alternate angles]

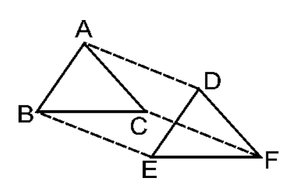
AB = CD [Opposite sides of a parallelogram]

$\angle APB = \angle CQD$ [Each = 90°]

$\therefore$ $\Delta APB \cong \Delta CQD$ [ASA congruence]

(ii) So, AP = CQ [CPCT] **Proved.**

Q.11. *In* ΔABC *and* ΔDEF, *AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.). Show that*



(i) quadrilateral ABED is a parallelogram

(ii) quadrilataeral BEFC is a parallelogram

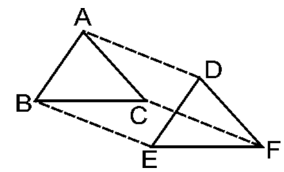
(iii) AD || CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) $\Delta ABC \equiv \Delta DEF$

Sol. **Given :** In DABC and DDEF, AB = DE, AB ||DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F.



To Prove : (i) ABED is a parallelogram

(ii) BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) ACFD is a parallelogram

(v) AC = DF

(vi) $\Delta ABC \cong \Delta DEF$

Proof : (i) In quadrilateral ABED, we have

AB = DE and AB || DE. [Given]

$\Rightarrow$ ABED is a parallelogram.

[One pair of opposite sides is parallel and equal]

(ii) In quadrilateral BEFC, we have

BC = EF and BC || EF [Given]

$\Rightarrow$ BEFC is a parallelogram.

[One pair of opposite sides is parallel and equal]

(iii) BE = CF and BE||BECF [BEFC is parallelogram]

AD = BE and AD||BE [ABED is a parallelogram]

$\Rightarrow$ AD = CF and AD||CF

(iv) ACFD is a parallelogram.

[One pair of opposite sides is parallel and equal]

(v) AC = DF [Opposite sides of parallelogram ACFD]

(vi) In ΔABC and ΔDEF, we have

AB = DE [Given]

BC = EF [Given]

AC = DF [Proved above]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [SSS axiom] **Proved.**

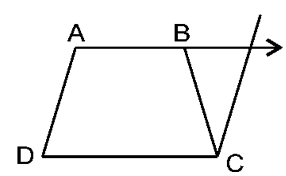
Q.12. *ABCD is a trapezium in which AB || CD and AD = BC (see Fig.). Show that*

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD



Sol. **Given :** In trapezium ABCD, AB || CD and AD = BC.

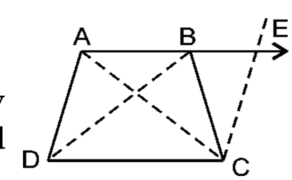
To Prove : (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD

Constructions : Join AC and BD. Extend AB and draw a line through C parallel to DA meeting AB produced at E.



Proof : (i) Since $AB \parallel DC$

$\Rightarrow AE \parallel DC$ ...(i)

and $AD \parallel CE$ ...(ii) [Construction]

$\Rightarrow$ ADCE is a parallelogram [Opposite pairs of sides are parallel

$\angle A + \angle E = 180°$ ...(iii) [Consecutive interior angles]

$\angle B + \angle CBE = 180°$ ...(iv) [Linear pair]

$AD = CE$ ...(v) [Opposite sides of a $\parallel^{gm}$]

$AD = BC$ ...(vi) [Given]

$\Rightarrow BC = CE$ [From (v) and (vi)]

$\Rightarrow \angle E = \angle CBE$ ...(vii) [Angles opposite to equal sides]

$\therefore \angle B + \angle E = 180°$ ...(viii) [From (iv) and (vii)

Now from (iii) and (viii) we have

$\angle A + \angle E = \angle B + \angle E$

$\Rightarrow \angle A = \angle B$ **Proved.**

(ii) $\left.\begin{array}{l}\angle A + \angle D = 180° \\ \angle B + \angle C = 180°\end{array}\right\}$ [Consecutive interior angles]

$\Rightarrow \angle A + \angle D = \angle B + \angle C$ [$\because \angle A = \angle B$]

$\Rightarrow \angle D = \angle C$

Or $\angle C = \angle D$ **Proved.**

(iii) In ΔABC and ΔBAD, we have

$AD = BC$ [Given]

$\angle A = \angle B$ [Proved]

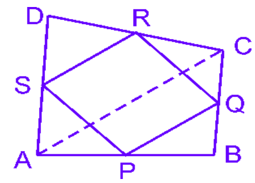
$AB = CD$ [Common]

$\therefore \Delta ABC \cong \Delta BAD$ [ASA congruence]

(iv) diagonal AC = diagonal BD [CPCT] **Proved.**

EXERCISE 8.2

Q.1. *ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. (see Fig.). AC is a diagonal. Show that :*

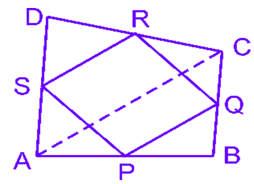


(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Given : ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.



To Prove : (i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram

Proof : (i) In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

∴ PQ || AC and PQ = $\frac{1}{2}$AC ...(1)

[Mid-point theorem]

In ΔADC, R is the mid-point of CD and S is the mid-point of AD

∴ SR || AC and SR = $\frac{1}{2}$AC ...(2)

[Mid-point theorem]

(ii) From (1) and (2), we get

PQ || SR and PQ = SR

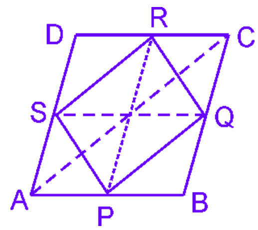
(iii) Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is equal and parallel.

∴ PQRS is a parallelogram. **Proved.**

Q.2. *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

Sol. **Given :** ABCD is a rhombus in which P, Q, R and S are mid points of sides AB, BC, CD and DA respectively :

To Prove : PQRS is a rectangle.



Construction : Join AC, PR and SQ.

Proof : In ΔABC

P is mid point of AB [Given]

Q is mid point of BC [Given]

$\Rightarrow$ PQ || AC and PQ = $\frac{1}{2}$AC ...(i) [Mid point theorem]

Similarly, in ΔDAC,

SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we have PQ||SR and PQ = SR

$\Rightarrow$ PQRS is a parallelogram

[One pair of opposite sides is parallel and equal]

Since ABQS is a parallelogram

$\Rightarrow$ AB = SQ [Opposite sides of a || gm]

Similarly, since PBCR is a parallelogram.

$\Rightarrow$ BC = PR

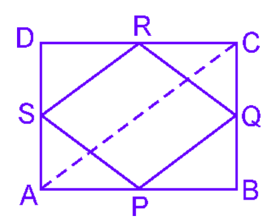
Thus, SQ = PR [AB = BC]

Since SQ and PR are diagonals of parallelogram PQRS, which are equal.

$\Rightarrow$ PQRS is a rectangle. **Proved.**

Q.3. *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilataral PQRS is a rhombus.*

Sol. **Given :** A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively, PQ, QR, RS and SP are joined.



To Prove : PQRS is a rhombus.

Construction : Join AC

Proof : In ΔABC, P and Q are the mid-points of the sides AB and BC.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC ...(i) [Mid point theorem]

Similarly, in ΔADC,

SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we get

PQ || SR and PQ = SR ...(iii)

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From (iii)]

$\therefore$PQRS is a parallelogram.

Now AD = BC ...(iv)

[Opposite sides of a rectangle ABCD]

$\therefore$ $\frac{1}{2}$AD = $\frac{1}{2}$BC

$\Rightarrow$ AS = BQ

In ΔAPS and ΔBPQ

AP = BP [$\because$ P is the mid-point of AB]

AS = BQ [Proved above]

∠PAS = ∠PBQ [Each = 90°]

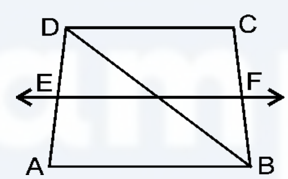
ΔAPS ≅ ΔBPQ [SAS axiom]

$\therefore$ PS = PQ ...(v)

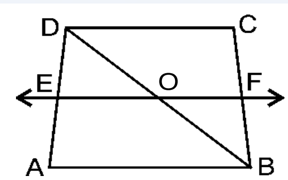
From (iii) and (v), we have

PQRS is a rhombus **Proved.**

Q.4. *ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig.). Show that F is the mid-point of BC.*



Sol. Given : A trapezium ABCD with AB || DC, E is the mid-point of AD and EF || AB.



To Prove : F is the mid-point of BC.

Proof : AB || DC and EF || AB

$\Rightarrow$ AB, EF and DC are parallel.

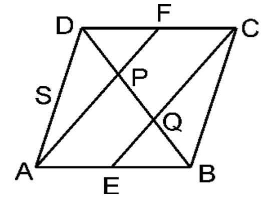
Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.

$\therefore$ Intercepts made by those parallel lines on transversal BC are also equal.

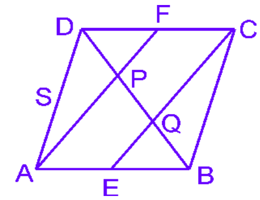
i.e., BF = FC

$\Rightarrow$ F is the mid-point of BC.

Q.5. *In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.). Show that the line segments AF and EC trisect the diagonal BD.*



Sol. **Given :** A parallelogram ABCD, in which E and F are mid-points of sides AB and DC respectively.



To Prove : DP = PQ = QB

Proof : Since E and F are mid-points of AB and DC respectively.

$\Rightarrow$ AE = $\frac{1}{2}$AB and CF = $\frac{1}{2}$DC ...(i)

But, AB = DC and AB || DC ...(ii)

[Opposite sides of a parallelogram]

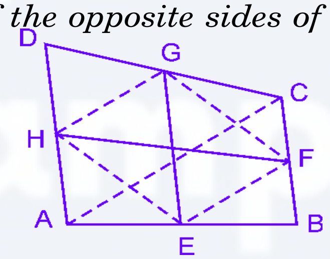
$\therefore$ AE = CF and AE || CF.

$\Rightarrow$ AECF is a parallelogram.

[One pair of opposite sides is parallel and equal]

In ΔBAP,
E is the mid-point of AB
EQ || AP
$\Rightarrow$ Q is mid-point of PB [Converse of mid-point theorem]
$\Rightarrow$ PQ = QB ...(iii)
Similarly, in ΔDQC,
P is the mid-point of DQ
DP = PQ ...(iv)
From (iii) and (iv), we have
DP = PQ = QB
or line segments AF and EC trisect the diagonal BD. **Proved.**

Q.6. *Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.*



Sol. **Given :** ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.

To Prove : EG and FH bisect each other.

Construction : Join EF, FG, GH, HE and AC.

Proof : In ΔABC, E and F are mid-points of AB and BC respectively.

$\therefore$ EF = $\frac{1}{2}$AC and EF || AC ...(i)

In ΔADC, H and G are mid-points of AD and CD respectively.

$\therefore$ HG = $\frac{1}{2}$AC and HG || AC ...(ii)

From (i) and (ii), we get
EF = HG and EF || HG
$\therefore$ EFGH is a parallelogram.

[$\because$ a quadrilateral is a parallelogram if its one pair of opposite sides is equal and parallel]

Now, EG and FH are diagonals of the parallelogram EFGH.

$\therefore$ EG and FH bisect each other.

[Diagonal of a parallelogram bisect each other] **Proved.**

Q.7. *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

(i) D is the mid-point of AC.

(ii) MD ⊥ AC

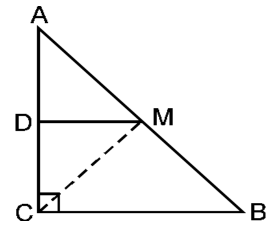
(iii) CM = MA = $\frac{1}{2}$AB

Sol. Given : A triangle ABC, in which ∠C = 90° and M is the mid-point of AB and BC || DM.

To Prove : (i) D is the mid-point of AC [Given]

(ii) DM ⊥ BC

(iii) CM = MA = $\frac{1}{2}$AB



Construction : Join CM.

Proof : (i) In ΔABC,

M is the mid-point of AB. [Given]

BC || DM [Given]

D is the mid-point of AC

[Converse of mid-point theorem] **Proved.**

(ii) ∠ADM = ∠ACB [∵ Coresponding angles]

But ∠ACB = 90° [Given]

∴ ∠ADM = 90°

But ∠ADM + ∠CDM = 180° [Linear pair]

∴ ∠CDM = 90°

Hence, MD ⊥ AC **Proved.**

(iii) AD = DC ...(1) [∵ D is the mid-point of AC]

Now, in ΔADM and ΔCMD, we have

∠ADM = ∠CDM [Each = 90°]

AD = DC [From (1)]

DM = DM [Common]

∴ ΔADM ≅ ΔCMD [SAS congruence]

⇒ CM = MA ...(2) [CPCT]

Since M is mid-point of AB,

∴ MA = $\frac{1}{2}$AB ...(3)

Hence, CM = MA = $\frac{1}{2}$AB **Proved.** [From (2) and (3)]