

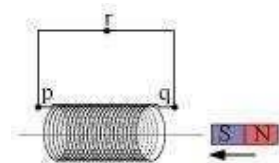
## CHAPTER-6 ELECTROMAGNETIC INDUCTION

### Exercises

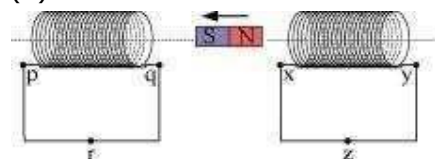
Question 6.1:

Predict the direction of induced current in the situations described by the following Figs. 6.18(a) to (f).

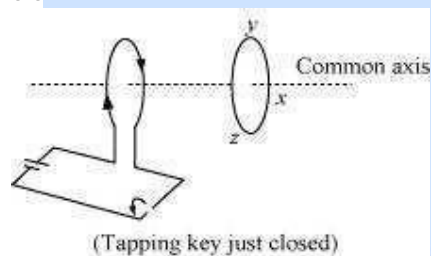
(a)



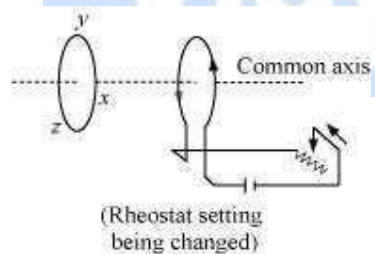
(b)



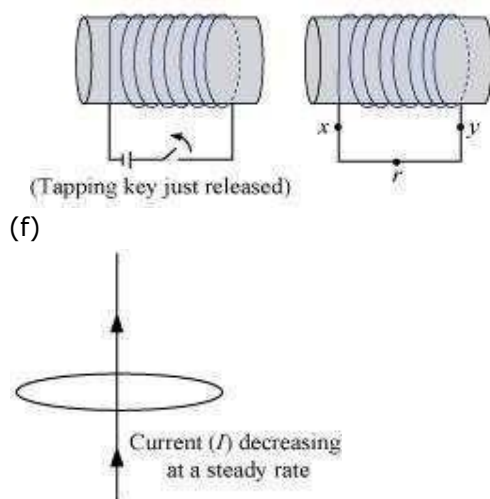
(c)



(d)

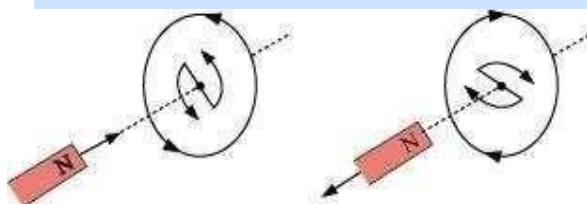


(e)



Answer

The direction of the induced current in a closed loop is given by Lenz's law. The given pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively.



Using Lenz's rule, the direction of the induced current in the given situations can be predicted as follows:

- (a) The direction of the induced current is along qrpq.
- (b) The direction of the induced current is along prqp.
- (c) The direction of the induced current is along yzxy.
- (d) The direction of the induced current is along zyxz.
- (e) The direction of the induced current is along xryx.
- (f) No current is induced since the field lines are lying in the plane of the closed loop.

Question 6.2:

A 1.0 m long metallic rod is rotated with an angular frequency of  $400 \text{ rad s}^{-1}$  about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Answer

Length of the rod,  $l = 1 \text{ m}$

Angular frequency,  $\omega = 400 \text{ rad/s}$

Magnetic field strength,  $B = 0.5 \text{ T}$

One end of the rod has zero linear velocity, while the other end has a linear velocity of  $l\omega$ .

Average linear velocity of the rod,  $v = \frac{l\omega + 0}{2} = \frac{l\omega}{2}$   
Emf developed between the centre and the ring,

$$e = Blv = Bl \left( \frac{l\omega}{2} \right) = \frac{Bl^2\omega}{2}$$

$$= \frac{0.5 \times (1)^2 \times 400}{2} = 100 \text{ V}$$

Hence, the emf developed between the centre and the ring is 100 V.

Question 6.3:

A long solenoid with 15 turns per cm has a small loop of area  $2.0 \text{ cm}^2$  placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Answer

Number of turns on the solenoid = 15 turns/cm = 1500 turns/m

Number of turns per unit length,  $n = 1500 \text{ turns}$

The solenoid has a small loop of area,  $A = 2.0 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$

Current carried by the solenoid changes from 2 A to 4 A.

Change in current in the solenoid,  $di = 4 - 2 = 2 \text{ A}$

Change in time,  $dt = 0.1 \text{ s}$

Induced emf in the solenoid is given by Faraday's law as:

$$e = \frac{d\phi}{dt} \quad \dots (i)$$

Where,

$\phi$  = Induced flux through the small loop

$$= BA \quad \dots (ii)$$

$B$  = Magnetic field

$$= \mu_0 ni \quad \dots (iii)$$

$\mu_0$  = Permeability of free space

$$= 4\pi \times 10^{-7} \text{ H/m}$$

Hence, equation (i) reduces to:

$$\begin{aligned} e &= \frac{d}{dt}(BA) \\ &= A\mu_0 n \times \left(\frac{di}{dt}\right) \\ &= 2 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1} \\ &= 7.54 \times 10^{-6} \text{ V} \end{aligned}$$

Hence, the induced voltage in the loop is  $7.54 \times 10^{-6} \text{ V}$ .

Question 6.4:

A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is  $1 \text{ cm s}^{-1}$  in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Answer

Length of the rectangular wire,  $l = 8 \text{ cm} = 0.08 \text{ m}$

Width of the rectangular wire,  $b = 2 \text{ cm} = 0.02 \text{ m}$

Hence, area of the rectangular loop,



$$\begin{aligned} A &= lb \\ &= 0.08 \times 0.02 \\ &= 16 \times 10^{-4} \text{ m}^2 \end{aligned}$$

Magnetic field strength,  $B = 0.3 \text{ T}$  Velocity  
of the loop,  $v = 1 \text{ cm/s} = 0.01 \text{ m/s}$  (a)

Emf developed in the loop is given as:

$$\begin{aligned} e &= Blv \\ &= 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4} \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Time taken to travel along the width, } t &= \frac{\text{Distance travelled}}{\text{Velocity}} = \frac{b}{v} \\ &= \frac{0.02}{0.01} = 2 \text{ s} \end{aligned}$$

Hence, the induced voltage is  $2.4 \times 10^{-4} \text{ V}$  which lasts for 2 s.

(b) Emf developed,  $e = Bbv$

$$= 0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4} \text{ V}$$

$$\begin{aligned} \text{Time taken to travel along the length, } t &= \frac{\text{Distance traveled}}{\text{Velocity}} = \frac{l}{v} \\ &= \frac{0.08}{0.01} = 8 \text{ s} \end{aligned}$$

Hence, the induced voltage is  $0.6 \times 10^{-4} \text{ V}$  which lasts for 8 s.

Question 6.6:

A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of  $50 \text{ rad s}^{-1}$  in a uniform horizontal magnetic field of magnitude  $3.0 \times 10^{-2} \text{ T}$ . Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance  $10 \Omega$ , calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Answer

$$\text{Max induced emf} = 0.603 \text{ V}$$

$$\text{Average induced emf} = 0 \text{ V}$$

$$\text{Max current in the coil} = 0.0603 \text{ A}$$

Average power loss = 0.018 W

(Power comes from the external rotor)

Radius of the circular coil,  $r = 8 \text{ cm} = 0.08 \text{ m}$

Area of the coil,  $A = \pi r^2 = \pi \times (0.08)^2 \text{ m}^2$

Number of turns on the coil,  $N = 20$

Angular speed,  $\omega = 50 \text{ rad/s}$

Magnetic field strength,  $B = 3 \times 10^{-2} \text{ T}$

Resistance of the loop,  $R = 10 \Omega$

Maximum induced emf is given as:

$$\begin{aligned} e &= N\omega AB \\ &= 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} \\ &= 0.603 \text{ V} \end{aligned}$$

The maximum emf induced in the coil is 0.603 V.

Over a full cycle, the average emf induced in the coil is zero.

Maximum current is given as:

$$\begin{aligned} I &= \frac{e}{R} \\ &= \frac{0.603}{10} = 0.0603 \text{ A} \end{aligned}$$

Average power loss due to joule heating:

$$\begin{aligned} P &= \frac{eI}{2} \\ &= \frac{0.603 \times 0.0603}{2} = 0.018 \text{ W} \end{aligned}$$

The current induced in the coil produces a torque opposing the rotation of the coil. The rotor is an external agent. It must supply a torque to counter this torque in order to keep the coil rotating uniformly. Hence, dissipated power comes from the external rotor.

Question 6.7:

A horizontal straight wire 10 m long extending from east to west is falling with a speed of  $5.0 \text{ m s}^{-1}$ , at right angles to the horizontal component of the earth's magnetic field,  $0.30 \times 10^{-4} \text{ Wb m}^{-2}$ .

- (a) What is the instantaneous value of the emf induced in the wire?
- (b) What is the direction of the emf?
- (c) Which end of the wire is at the higher electrical potential?

Answer

Length of the wire,  $l = 10 \text{ m}$

Falling speed of the wire,  $v = 5.0 \text{ m/s}$

Magnetic field strength,  $B = 0.3 \times 10^{-4} \text{ Wb m}^{-2}$

- (a) Emf induced in the wire,  $e = Blv$

$$= 0.3 \times 10^{-4} \times 5 \times 10$$

$$= 1.5 \times 10^{-3} \text{ V}$$

- (b) Using Fleming's right hand rule, it can be inferred that the direction of the induced emf is from West to East.
- (c) The eastern end of the wire is at a higher potential.

Question 6.8:

Current in a circuit falls from  $5.0 \text{ A}$  to  $0.0 \text{ A}$  in  $0.1 \text{ s}$ . If an average emf of  $200 \text{ V}$  induced, give an estimate of the self-inductance of the circuit.

Answer

Initial current,  $I_1 = 5.0 \text{ A}$

Final current,  $I_2 = 0.0 \text{ A}$

Change in current,  $dI = I_1 - I_2 = 5 \text{ A}$

Time taken for the change,  $t = 0.1 \text{ s}$

Average emf,  $e = 200 \text{ V}$

For self-inductance ( $L$ ) of the coil, we have the relation for average emf as:

$$e = L \frac{di}{dt}$$

$$L = \frac{e}{\left(\frac{di}{dt}\right)}$$

$$= \frac{200}{\frac{5}{0.1}} = 4 \text{ H}$$

Hence, the self induction of the coil is 4 H.

Question 6.9:

A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Answer

Mutual inductance of a pair of coils,  $\mu = 1.5 \text{ H}$

Initial current,  $I_1 = 0 \text{ A}$

Final current  $I_2 = 20 \text{ A}$

Change in current,  $dI = I_2 - I_1 = 20 - 0 = 20 \text{ A}$

Time taken for the change,  $t = 0.5 \text{ s}$

Induced emf,  $e = \frac{d\phi}{dt} \dots (1)$

Where  $d\phi$  is the change in the flux linkage with the coil.

Emf is related with mutual inductance as:

$$e = \mu \frac{dI}{dt} \dots (2)$$

Equating equations (1) and (2), we get

$$\frac{d\phi}{dt} = \mu \frac{dI}{dt}$$

$$d\phi = 1.5 \times (20)$$

$$= 30 \text{ Wb}$$



Hence, the change in the flux linkage is 30 Wb.

Question 6.10:

A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of  $5 \times 10^{-4} \text{ T}$  and the dip angle is  $30^\circ$ .

Answer

Speed of the jet plane,  $v = 1800 \text{ km/h} = 500 \text{ m/s}$

Wing span of jet plane,  $l = 25 \text{ m}$

Earth's magnetic field strength,  $B = 5.0 \times 10^{-4} \text{ T}$

Angle of dip,  $\delta = 30^\circ$

Vertical component of Earth's magnetic field,

$$B_v = B \sin \delta$$

$$= 5 \times 10^{-4} \sin 30^\circ$$

$$= 2.5 \times 10^{-4} \text{ T}$$

Voltage difference between the ends of the wing can be calculated as:

$$e = (B_v) \times l \times v$$

$$= 2.5 \times 10^{-4} \times 25 \times 500$$

$$= 3.125 \text{ V}$$

Hence, the voltage difference developed between the ends of the wings is 3.125 V.