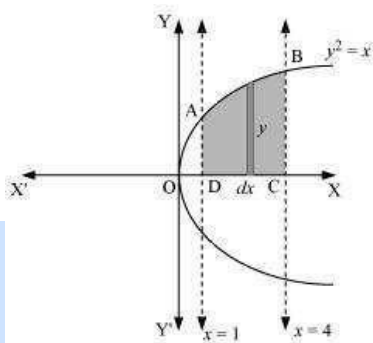


## Exercise 8.1

Question 1:

Find the area of the region bounded by the curve  $y^2 = x$  and the lines  $x = 1$ ,  $x = 4$  and the x-axis.

Answer



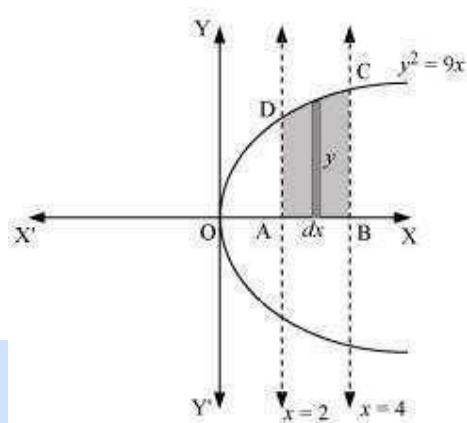
The area of the region bounded by the curve,  $y^2 = x$ , the lines,  $x = 1$  and  $x = 4$ , and the x-axis is the area ABCD.

$$\begin{aligned}\text{Area of ABCD} &= \int_1^4 y \, dx \\ &= \int_1^4 \sqrt{x} \, dx \\ &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\ &= \frac{2}{3} \left[ (4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] \\ &= \frac{2}{3} [8 - 1] \\ &= \frac{14}{3} \text{ units}\end{aligned}$$

Question 2:

Find the area of the region bounded by  $y^2 = 9x$ ,  $x = 2$ ,  $x = 4$  and the x-axis in the first quadrant.

Answer



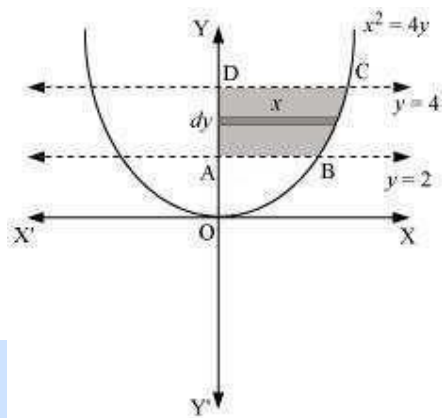
The area of the region bounded by the curve,  $y^2 = 9x$ ,  $x = 2$ , and  $x = 4$ , and the x-axis is the area ABCD.

$$\begin{aligned}\text{Area of ABCD} &= \int_2^4 y \, dx \\ &= \int_2^4 3\sqrt{x} \, dx \\ &= 3 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\ &= 2 \left[ x^{\frac{3}{2}} \right]_2^4 \\ &= 2 \left[ (4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\ &= 2 \left[ 8 - 2\sqrt{2} \right] \\ &= (16 - 4\sqrt{2}) \text{ units}\end{aligned}$$

Question 3:

Find the area of the region bounded by  $x^2 = 4y$ ,  $y = 2$ ,  $y = 4$  and the y-axis in the first quadrant.

Answer



The area of the region bounded by the curve,  $x^2 = 4y$ ,  $y = 2$ , and  $y = 4$ , and the y-axis is the area ABCD.

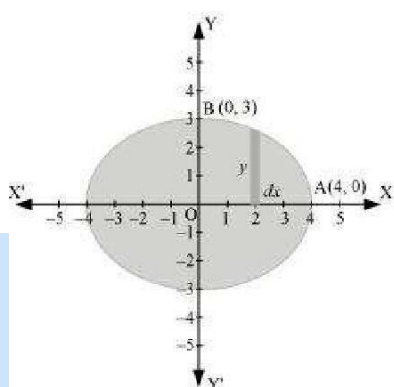
$$\begin{aligned}\text{Area of ABCD} &= \int_2^4 x \, dy \\ &= \int_2^4 2\sqrt{y} \, dy \\ &= 2 \int_2^4 \sqrt{y} \, dy \\ &= 2 \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\ &= \frac{4}{3} \left[ (4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\ &= \frac{4}{3} [8 - 2\sqrt{2}] \\ &= \left( \frac{32 - 8\sqrt{2}}{3} \right) \text{ units}\end{aligned}$$

Question 4:

Find the area of the region bounded by the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse,  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , can be represented as



It can be observed that the ellipse is symmetrical about x-axis and y-axis.

∴ Area bounded by ellipse = 4 × Area of OAB

$$\begin{aligned}
 \text{Area of OAB} &= \int_0^4 y \, dx \\
 &= \int_0^4 3\sqrt{1 - \frac{x^2}{16}} \, dx \\
 &= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} \, dx \\
 &= \frac{3}{4} \left[ \frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4 \\
 &= \frac{3}{4} \left[ 2\sqrt{16 - 16} + 8 \sin^{-1}(1) - 0 - 8 \sin^{-1}(0) \right] \\
 &= \frac{3}{4} \left[ \frac{8\pi}{2} \right] \\
 &= \frac{3}{4} [4\pi] \\
 &= 3\pi
 \end{aligned}$$

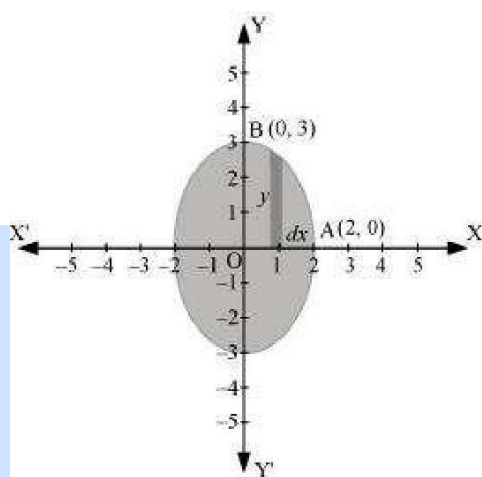
Therefore, area bounded by the ellipse =  $4 \times 3\pi = 12\pi$  units

Question 5:

Find the area of the region bounded by the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse can be represented as



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}} \quad \dots(1)$$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.  
 $\therefore$  Area bounded by ellipse =  $4 \times$  Area OAB

$$\begin{aligned}
 \therefore \text{Area of OAB} &= \int_0^2 y \, dx \\
 &= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} \, dx \quad [\text{Using (1)}] \\
 &= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} \, dx \\
 &= \frac{3}{2} \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\
 &= \frac{3}{2} \left[ \frac{2\pi}{2} \right] \\
 &= \frac{3\pi}{2}
 \end{aligned}$$

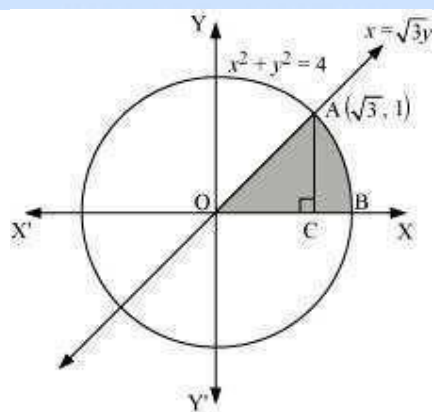
Therefore, area bounded by the ellipse =  $4 \times \frac{3\pi}{2} = 6\pi$  units

Question 6:

Find the area of the region in the first quadrant enclosed by x-axis, line  $x = \sqrt{3}y$  and the circle  $x^2 + y^2 = 4$

Answer

The area of the region bounded by the circle,  $x^2 + y^2 = 4, x = \sqrt{3}y$ , and the x-axis is the area OAB.



The point of intersection of the line and the circle in the first quadrant is

$(\sqrt{3}, 1)$ . Area OAB = Area  $\Delta$ OAC + Area ACB

$$\text{Area of OAC} = \frac{1}{2} \times \text{OC} \times \text{AC} = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \quad \dots(1)$$

$$\text{Area of ABC} = \int_{\sqrt{3}}^2 y \, dx$$

$$= \int_{\sqrt{3}}^2 \sqrt{4-x^2} \, dx$$

$$= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2$$

$$= \left[ 2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4-3} - 2 \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right]$$

$$= \left[ \pi - \frac{\sqrt{3}\pi}{2} - 2 \left( \frac{\pi}{3} \right) \right]$$

$$= \left[ \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \right]$$

$$= \left[ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \quad \dots(2)$$

Therefore, area enclosed by x-axis, the line  $x = \sqrt{3}y$ , and the circle  $x^2 + y^2 = 4$  in the first

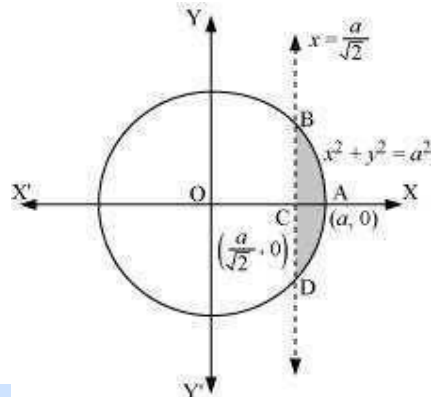
$$\text{quadrant} = \frac{\sqrt{3}\pi}{2} + \frac{3\sqrt{3}}{2} - \frac{\pi}{3} = \frac{3\sqrt{3}}{2} \text{ units}$$

Question 7:

Find the area of the smaller part of the circle  $x^2 + y^2 = a^2$  cut off by the line Answer

$$x = \frac{a}{\sqrt{2}}$$

The area of the smaller part of the circle,  $x^2 + y^2 = a^2$ , cut off by the line,  $x = \frac{a}{\sqrt{2}}$ , is the area ABCDA.



It can be observed that the area ABCD is symmetrical about x-axis.  
 $\therefore \text{Area ABCD} = 2 \times \text{Area ABC}$



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$$\begin{aligned}
 \text{Area of } ABC &= \int_{\frac{a}{\sqrt{2}}}^a y \, dx \\
 &= \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} \, dx \\
 &= \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_{\frac{a}{\sqrt{2}}}^a \\
 &= \left[ \frac{a^2}{2} \left( \frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2} \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \right] \\
 &= \frac{a^2 \pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2} \left( \frac{\pi}{4} \right) \\
 &= \frac{a^2 \pi}{4} - \frac{a^2}{4} - \frac{a^2 \pi}{8} \\
 &= \frac{a^2}{4} \left[ \pi - 1 - \frac{\pi}{2} \right] \\
 &= \frac{a^2}{4} \left[ \frac{\pi}{2} - 1 \right] \\
 \Rightarrow \text{Area } ABCD &= 2 \left[ \frac{a^2}{4} \left( \frac{\pi}{2} - 1 \right) \right] = \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)
 \end{aligned}$$

Therefore, the area of smaller part of the circle,  $x^2 + y^2 = a^2$ , cut off by the line,  $x = \frac{a}{\sqrt{2}}$ , is  $\frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right)$  units.

Question 8:

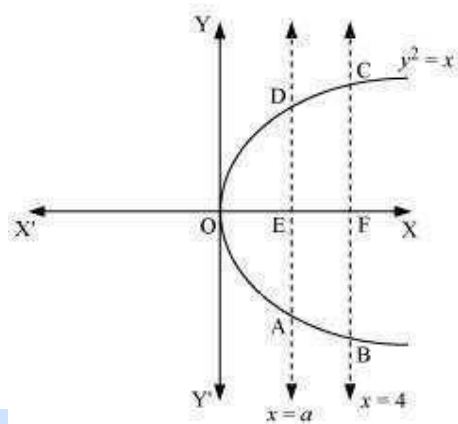
The area between  $x = y^2$  and  $x = 4$  is divided into two equal parts by the line  $x = a$ , find the value of  $a$ .

Answer

The line,  $x = a$ , divides the area bounded by the parabola and  $x = 4$  into two equal parts.

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$$\therefore \text{Area OAD} = \text{Area ABCD}$$



It can be observed that the given area is symmetrical about x-axis.  
 $\Rightarrow \text{Area OED} = \text{Area EFCD}$



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$$\begin{aligned}
 \text{Area } OED &= \int_0^a y \, dx \\
 &= \int_0^a \sqrt{x} \, dx \\
 &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\
 &= \frac{2}{3}(a)^{\frac{3}{2}} \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } EFCD &= \int_0^4 \sqrt{x} \, dx \\
 &= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\
 &= \frac{2}{3} \left[ 8 - a^{\frac{3}{2}} \right] \quad \dots(2)
 \end{aligned}$$

From (1) and (2), we obtain

$$\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3} \left[ 8 - (a)^{\frac{3}{2}} \right]$$

$$\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

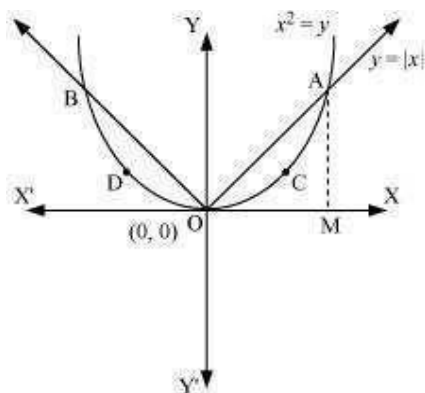
Therefore, the value of  $a$  is  $(4)^{\frac{2}{3}}$ .

Question 9:

Find the area of the region bounded by the parabola  $y = x^2$  and

$y = |x|$  Answer

The area bounded by the parabola,  $x^2 = y$ , and the line,  $y = |x|$ , can be represented as



The given area is symmetrical about y-axis.

$\therefore$  Area OACO = Area ODBO

The point of intersection of parabola,  $x^2 = y$ , and line,  $y = x$ , is A (1, 1). Area of OACO = Area  $\Delta$ OAB – Area OBACO

$$\therefore \text{Area of } \Delta\text{OAB} = \frac{1}{2} \times \text{OB} \times \text{AB} = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

$$\begin{aligned} \text{Area of OBACO} &= \int_0^1 y \, dx = \int_0^1 x^2 \, dx = \left[ \frac{x^3}{3} \right]_0^1 = \frac{1}{3} \\ \Rightarrow \text{Area of OACO} &= \text{Area of } \Delta\text{OAB} - \text{Area of OBACO} \end{aligned}$$

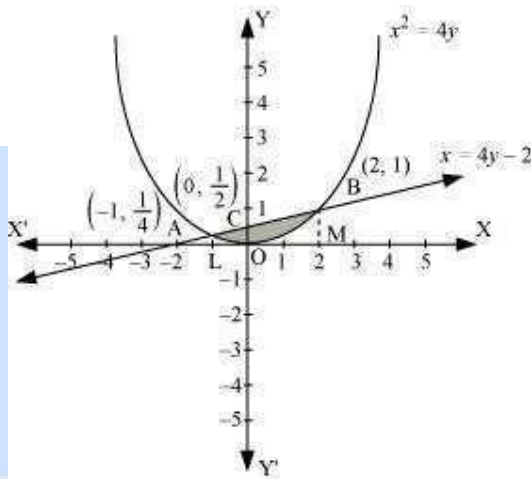
$$\begin{aligned} &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

$$\text{Therefore, required area} = 2 \left[ \frac{1}{6} \right] = \frac{1}{3} \text{ units}$$

Question 10:

Find the area bounded by the curve  $x^2 = 4y$  and the line  $x = 4y - 2$ .  
 2 Answer

The area bounded by the curve,  $x^2 = 4y$ , and line,  $x = 4y - 2$ , is represented by the shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

A are  $\left(-1, \frac{1}{4}\right)$ .

Coordinates of point

Coordinates of point B are (2, 1).

We draw AL and BM perpendicular to x-axis.

It can be observed that,

$$\text{Area OBAO} = \text{Area OBCO} + \text{Area OACO} \dots (1)$$

$$\text{Then, Area OBCO} = \text{Area OMBC} - \text{Area OMBO}$$

$$\begin{aligned}
 &= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx \\
 &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_0^2 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^2 \\
 &= \frac{1}{4} [2+4] - \frac{1}{4} \left[ \frac{8}{3} \right] \\
 &= \frac{3}{2} - \frac{2}{3} \\
 &= \frac{5}{6}
 \end{aligned}$$

Similarly, Area OACO = Area OLAC – Area OLAO

$$\begin{aligned}
 &= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx \\
 &= \frac{1}{4} \left[ \frac{x^2}{2} + 2x \right]_{-1}^0 - \frac{1}{4} \left[ \frac{x^3}{3} \right]_{-1}^0 \\
 &= -\frac{1}{4} \left[ \frac{(-1)^2}{2} + 2(-1) \right] - \left[ -\frac{1}{4} \left( \frac{(-1)^3}{3} \right) \right] \\
 &= -\frac{1}{4} \left[ \frac{1}{2} - 2 \right] - \frac{1}{12} \\
 &= \frac{1}{2} - \frac{1}{8} - \frac{1}{12} \\
 &= \frac{7}{24}
 \end{aligned}$$

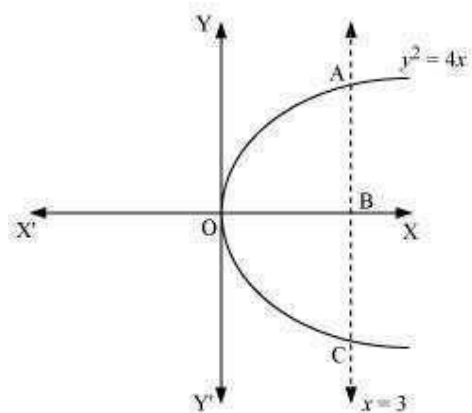
Therefore, required area =  $\left( \frac{5}{6} + \frac{7}{24} \right) = \frac{9}{8}$  units

Question 11:

Find the area of the region bounded by the curve  $y^2 = 4x$  and the line  $x = 3$

3 Answer

The region bounded by the parabola,  $y^2 = 4x$ , and the line,  $x = 3$ , is the area OACO.



The area OACO is symmetrical about x-axis.  
 $\therefore$  Area of OACO = 2 (Area of OAB)

$$\begin{aligned}
 \text{Area OACO} &= 2 \left[ \int_0^3 y \, dx \right] \\
 &= 2 \int_0^3 2\sqrt{x} \, dx \\
 &= 4 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3 \\
 &= \frac{8}{3} \left[ (3)^{\frac{3}{2}} \right] \\
 &= 8\sqrt{3}
 \end{aligned}$$

Therefore, the required area is  $8\sqrt{3}$  units.

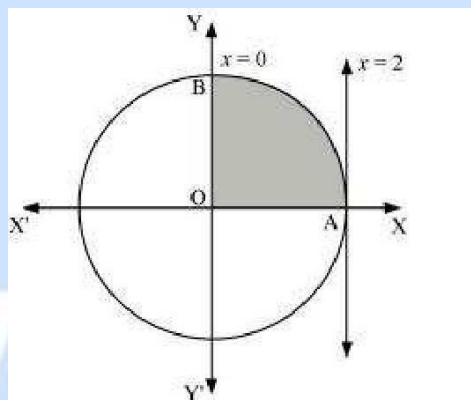
Question 12:

Area lying in the first quadrant and bounded by the circle  $x^2 + y^2 = 4$  and the lines  $x = 0$  and  $x = 2$  is

A.  $\pi$ B.  $\frac{\pi}{2}$ C.  $\frac{\pi}{3}$ D.  $\frac{\pi}{4}$ 

Answer

The area bounded by the circle and the lines,  $x = 0$  and  $x = 2$ , in the first quadrant is represented as



$$\begin{aligned}\therefore \text{Area OAB} &= \int_0^2 y \, dx \\ &= \int_0^2 \sqrt{4-x^2} \, dx \\ &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= 2 \left( \frac{\pi}{2} \right) \\ &= \pi \text{ units}\end{aligned}$$

Thus, the correct answer is A.



Question 13:

Area of the region bounded by the curve  $y^2 = 4x$ , y-axis and the line  $y = 3$  is

A. 2

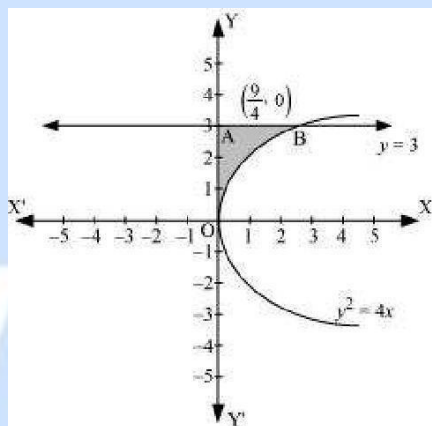
B.  $\frac{9}{4}$

C.  $\frac{9}{2}$

D.  $\frac{3}{2}$

Answer

The area bounded by the curve,  $y^2 = 4x$ , y-axis, and  $y = 3$  is represented as



$$\begin{aligned}\therefore \text{Area OAB} &= \int_0^3 x \, dy \\ &= \int_0^3 \frac{y^2}{4} \, dy \\ &= \frac{1}{4} \left[ \frac{y^3}{3} \right]_0^3 \\ &= \frac{1}{12} (27) \\ &= \frac{9}{4} \text{ units}\end{aligned}$$

Thus, the correct answer is B.

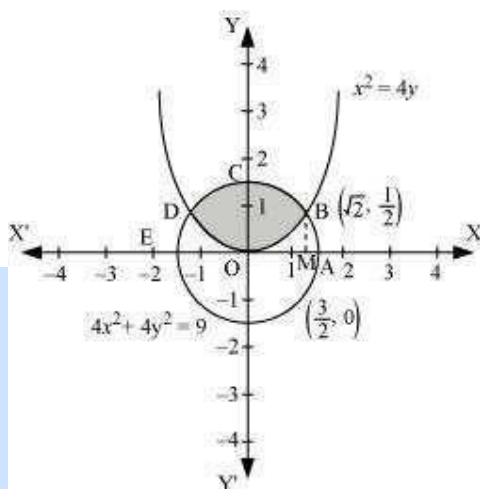
## Exercise 8.2

Question 1:

Find the area of the circle  $4x^2 + 4y^2 = 9$  which is interior to the parabola  $x^2 = 4y$ 

Answer

The required area is represented by the shaded area OBCDO.

Solving the given equation of circle,  $4x^2 + 4y^2 = 9$ , and parabola,  $x^2 = 4y$ , we obtain thepoint of intersection as  $B\left(\sqrt{2}, \frac{1}{2}\right)$  and  $D\left(-\sqrt{2}, \frac{1}{2}\right)$ .

It can be observed that the required area is symmetrical about y-axis.

 $\therefore \text{Area OBCDO} = 2 \times \text{Area OBCO}$ 

We draw BM perpendicular to OA.

Therefore, the coordinates of M are  $(\sqrt{2}, 0)$ .Therefore,  $\text{Area OBCO} = \text{Area OMBCO} - \text{Area OMBO}$

$$\begin{aligned}
 &= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}} dx \\
 &= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\
 &= \frac{1}{4} \left[ x\sqrt{9-4x^2} + \frac{9}{2} \sin^{-1} \frac{2x}{3} \right]_0^{\sqrt{2}} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^{\sqrt{2}} \\
 &= \frac{1}{4} \left[ \sqrt{2}\sqrt{9-8} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} (\sqrt{2})^3 \\
 &= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} \\
 &= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} \\
 &= \frac{1}{2} \left( \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right)
 \end{aligned}$$

Therefore, the required area OBCDO is

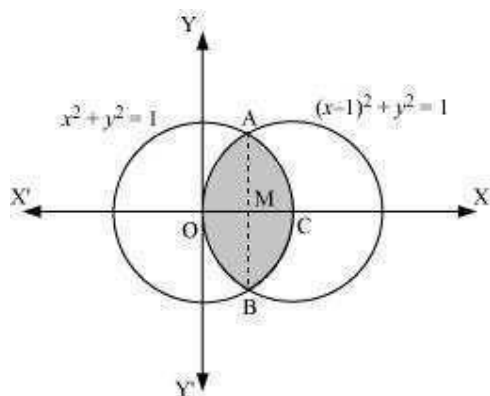
$$\left( 2 \times \frac{1}{2} \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \text{ units}$$

Question 2:

Find the area bounded by curves  $(x-1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$

1 Answer

The area bounded by the curves,  $(x-1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , is represented by the shaded area as



On solving the equations,  $(x - 1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , we obtain the point of

intersection as A  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and B  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ .

It can be observed that the required area is symmetrical about x-axis.

$\therefore \text{Area OBCAO} = 2 \times \text{Area OCAO}$

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are  $\left(\frac{1}{2}, 0\right)$ .

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$$\Rightarrow \text{Area } OCAO = \text{Area } OMAO + \text{Area } MCAM$$

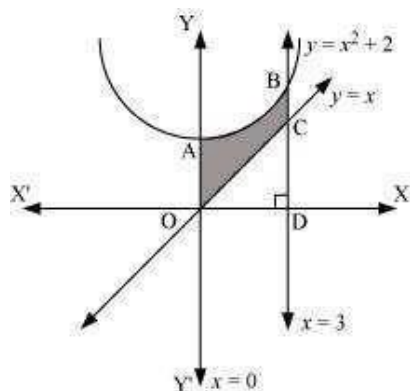
$$\begin{aligned}
 &= \left[ \int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx \right] \\
 &= \left[ \frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{\frac{1}{2}} + \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1 \\
 &= \left[ -\frac{1}{4} \sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2} \sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2} \sin^{-1}(-1) \right] + \\
 &\quad \left[ \frac{1}{2} \sin^{-1}(1) - \frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2} \sin^{-1}\left(\frac{1}{2}\right) \right] \\
 &= \left[ -\frac{\sqrt{3}}{8} + \frac{1}{2} \left(-\frac{\pi}{6}\right) - \frac{1}{2} \left(-\frac{\pi}{2}\right) \right] + \left[ \frac{1}{2} \left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left(\frac{\pi}{6}\right) \right] \\
 &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right] \\
 &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right] \\
 &= \left[ \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right]
 \end{aligned}$$

$$\text{Therefore, required area } OBCAO = 2 \times \left( \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right) = \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{ units}$$

Question 3:

Find the area of the region bounded by the curves  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$  and  $x = 3$   
**Answer**

The area bounded by the curves,  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$ , and  $x = 3$ , is represented by the shaded area  $OCBAO$  as



Then, Area OCB AO = Area ODB AO – Area ODC O

$$\begin{aligned}
 &= \int_0^3 (x^2 + 2) dx - \int_0^3 x dx \\
 &= \left[ \frac{x^3}{3} + 2x \right]_0^3 - \left[ \frac{x^2}{2} \right]_0^3 \\
 &= [9 + 6] - \left[ \frac{9}{2} \right] \\
 &= 15 - \frac{9}{2} \\
 &= \frac{21}{2} \text{ units}
 \end{aligned}$$

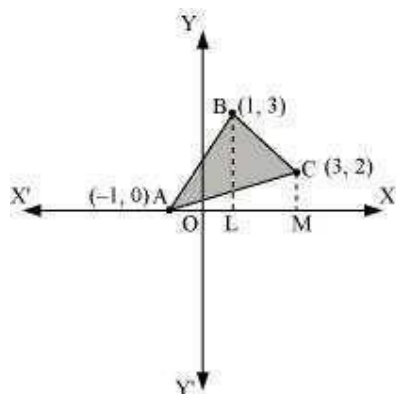
Question 4:

Using integration finds the area of the region bounded by the triangle whose vertices are  $(-1, 0)$ ,  $(1, 3)$  and  $(3, 2)$ .

Answer

BL and CM are drawn perpendicular to x-axis. It can be observed in the following figure that,

$$\text{Area } (\Delta ACB) = \text{Area } (ALBA) + \text{Area } (BLMCB) - \text{Area } (AMCA) \dots (1)$$



Equation of line segment AB is

$$y - 0 = \frac{3 - 0}{1 + 1}(x + 1)$$

$$y = \frac{3}{2}(x + 1)$$

$$\therefore \text{Area (ALBA)} = \int_{-1}^1 \frac{3}{2}(x + 1) dx = \frac{3}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^1 = \frac{3}{2} \left[ \frac{1}{2} + 1 - \frac{1}{2} + 1 \right] = 3 \text{ units}$$

Equation of line segment BC is

$$y - 3 = \frac{2 - 3}{3 - 1}(x - 1)$$

$$y = \frac{1}{2}(-x + 7)$$

$$\therefore \text{Area (BLMCB)} = \int_1^3 \frac{1}{2}(-x + 7) dx = \frac{1}{2} \left[ -\frac{x^2}{2} + 7x \right]_1^3 = \frac{1}{2} \left[ -\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] = 5 \text{ units}$$

Equation of line segment AC is

$$y - 0 = \frac{2 - 0}{3 + 1}(x + 1)$$

$$y = \frac{1}{2}(x + 1)$$

$$\therefore \text{Area (AMCA)} = \frac{1}{2} \int_{-1}^3 (x + 1) dx = \frac{1}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^3 = \frac{1}{2} \left[ \frac{9}{2} + 3 - \frac{1}{2} + 1 \right] = 4 \text{ units}$$

Therefore, from equation (1), we obtain

$$\text{Area } (\triangle ABC) = (3 + 5 - 4) = 4 \text{ units}$$

Question 5:

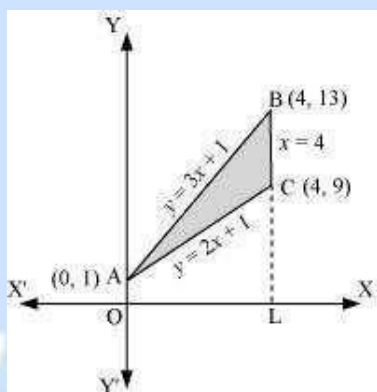
Using integration find the area of the triangular region whose sides have the equations  $y = 2x + 1$ ,  $y = 3x + 1$  and  $x = 4$ .

$$y = 2x + 1, y = 3x + 1 \text{ and } x = 4.$$

4. Answer

The equations of sides of the triangle are  $y = 2x + 1$ ,  $y = 3x + 1$ , and  $x = 4$ .

On solving these equations, we obtain the vertices of triangle as  $A(0, 1)$ ,  $B(4, 13)$ , and  $C(4, 9)$ .



It can be observed that,

$$\text{Area } (\triangle ACB) = \text{Area } (OLBAO) - \text{Area } (OLCAO)$$

$$= \int_0^4 (3x + 1) dx - \int_0^4 (2x + 1) dx$$

$$= \left[ \frac{3x^2}{2} + x \right]_0^4 - \left[ \frac{2x^2}{2} + x \right]_0^4$$

$$= (24 + 4) - (16 + 4)$$

$$= 28 - 20$$

$$= 8 \text{ units}$$

Question 6:

Smaller area enclosed by the circle  $x^2 + y^2 = 4$  and the line  $x + y = 2$  is

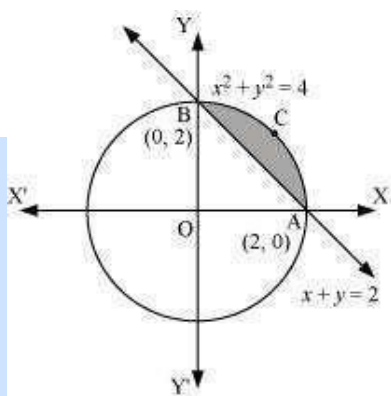
$$A. 2(\pi - 2)$$



B.  $\pi - 2$ C.  $2\pi - 1$ D.  $2(\pi +$ 

2) Answer

The smaller area enclosed by the circle,  $x^2 + y^2 = 4$ , and the line,  $x + y = 2$ , is represented by the shaded area ACBA as



It can be observed that,

Area ACBA = Area OACBO – Area ( $\Delta$ OAB)

$$\begin{aligned}
 &= \int_0^2 \sqrt{4-x^2} \, dx - \int_0^2 (2-x) \, dx \\
 &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2 \\
 &= \left[ 2 \cdot \frac{\pi}{2} \right] - [4-2] \\
 &= (\pi - 2) \text{ units}
 \end{aligned}$$

Thus, the correct answer is B.

Question 7:

Area lying between the curve  $y^2 = 4x$  and  $y = 2x$  is

A.  $\frac{2}{3}$

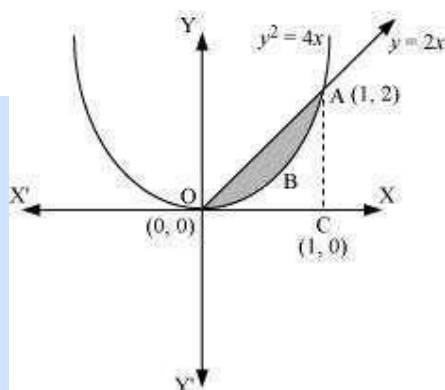
B.  $\frac{1}{3}$

C.  $\frac{1}{4}$

D.  $\frac{3}{4}$

Answer

The area lying between the curve,  $y^2 = 4x$  and  $y = 2x$ , is represented by the shaded area OBAO as



The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

$\therefore$  Area OBAO = Area ( $\Delta OCA$ ) – Area (OCABO)

$$\begin{aligned}&= \int_0^1 2x \, dx - \int_0^1 2\sqrt{x} \, dx \\&= 2 \left[ \frac{x^2}{2} \right]_0^1 - 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 \\&= \left| 1 - \frac{4}{3} \right| \\&= \left| -\frac{1}{3} \right| \\&= \frac{1}{3} \text{ units}\end{aligned}$$

Thus, the correct answer is B.



# Vidya Champ

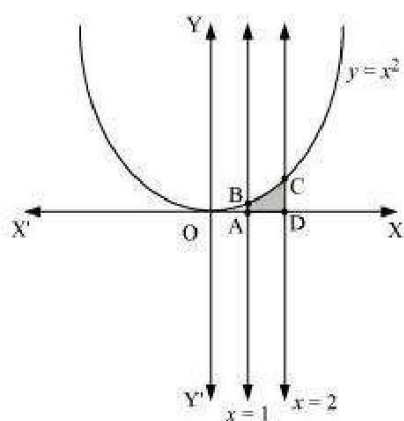
## Miscellaneous Solutions

Question 1:

Find the area under the given curves and given lines:

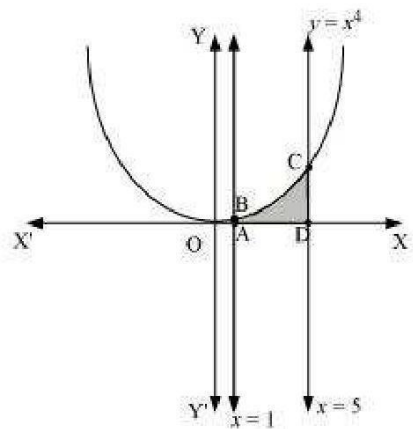
(i)  $y = x^2$ ,  $x = 1$ ,  $x = 2$  and x-axis(ii)  $y = x^4$ ,  $x = 1$ ,  $x = 5$  and x - axis Answer

i. The required area is represented by the shaded area ADCBA as



$$\begin{aligned}\text{Area ADCBA} &= \int_1^2 y dx \\ &= \int_1^2 x^2 dx \\ &= \left[ \frac{x^3}{3} \right]_1^2 \\ &= \frac{8}{3} - \frac{1}{3} \\ &= \frac{7}{3} \text{ units}\end{aligned}$$

ii. The required area is represented by the shaded area ADCBA as



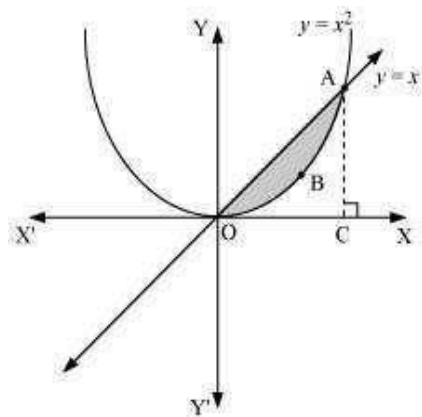
$$\begin{aligned}
 \text{Area ADCBA} &= \int_1^5 x^4 dx \\
 &= \left[ \frac{x^5}{5} \right]_1^5 \\
 &= \frac{(5)^5}{5} - \frac{1}{5} \\
 &= (5)^4 - \frac{1}{5} \\
 &= 625 - \frac{1}{5} \\
 &= 624.8 \text{ units}
 \end{aligned}$$

Question 2:

Find the area between the curves  $y = x$  and  $y = x^2$

Answer

The required area is represented by the shaded area OBAO as



The points of intersection of the curves,  $y = x$  and  $y = x^2$ , is A (1, 1). We draw AC perpendicular to x-axis.

$\therefore \text{Area (OBAO)} = \text{Area } (\Delta OCA) - \text{Area (OCABO)} \dots (1)$

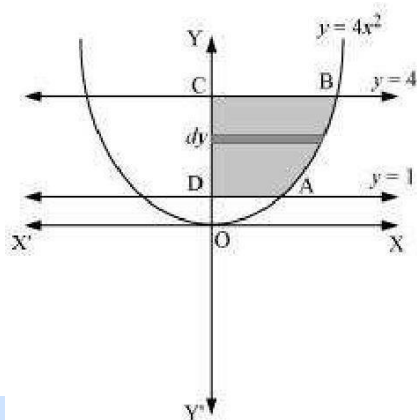
$$\begin{aligned}
 &= \int_0^1 x \, dx - \int_0^1 x^2 \, dx \\
 &= \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 \\
 &= \frac{1}{2} - \frac{1}{3} \\
 &= \frac{1}{6} \text{ units}
 \end{aligned}$$

Question 3:

Find the area of the region lying in the first quadrant and bounded by  $y = 4x^2$ ,  $x = 0$ ,  $y = 1$  and  $y = 4$

Answer

The area in the first quadrant bounded by  $y = 4x^2$ ,  $x = 0$ ,  $y = 1$ , and  $y = 4$  is represented by the shaded area ABCDA as



$$\begin{aligned}
 \therefore \text{Area ABCD} &= \int_1^4 x \, dx \\
 &= \int_1^4 \frac{\sqrt{y}}{2} \, dy \\
 &= \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\
 &= \frac{1}{3} \left[ (4)^{\frac{3}{2}} - 1 \right] \\
 &= \frac{1}{3} [8 - 1] \\
 &= \frac{7}{3} \text{ units}
 \end{aligned}$$

Question 4:

Sketch the graph of  $y = |x+3|$  and evaluate  $\int_{-6}^0 |x+3| \, dx$

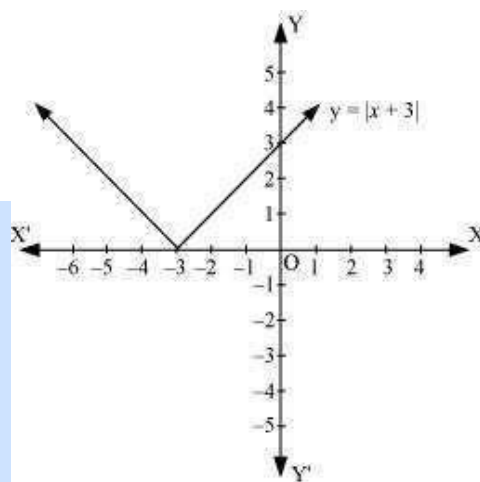
Answer

The given equation is  $y = |x + 3|$

The corresponding values of  $x$  and  $y$  are given in the following table.

<b>x</b>	- 6	- 5	- 4	- 3	- 2	- 1	0
<b>y</b>	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of  $y = |x + 3|$  as follows.



It is known that,  $(x + 3) \leq 0$  for  $-6 \leq x \leq -3$  and  $(x + 3) \geq 0$  for  $-3 \leq x \leq 0$

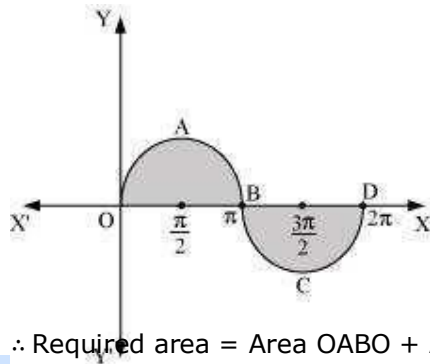
$$\begin{aligned}
 \therefore \int_{-6}^0 |(x + 3)| dx &= -\int_{-6}^{-3} (x + 3) dx + \int_{-3}^0 (x + 3) dx \\
 &= -\left[ \frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[ \frac{x^2}{2} + 3x \right]_{-3}^0 \\
 &= -\left[ \left( \frac{(-3)^2}{2} + 3(-3) \right) - \left( \frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[ 0 - \left( \frac{(-3)^2}{2} + 3(-3) \right) \right] \\
 &= -\left[ -\frac{9}{2} \right] - \left[ -\frac{9}{2} \right] \\
 &= 9
 \end{aligned}$$



Question 5:

Find the area bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = 2\pi$  Answer

The graph of  $y = \sin x$  can be drawn as



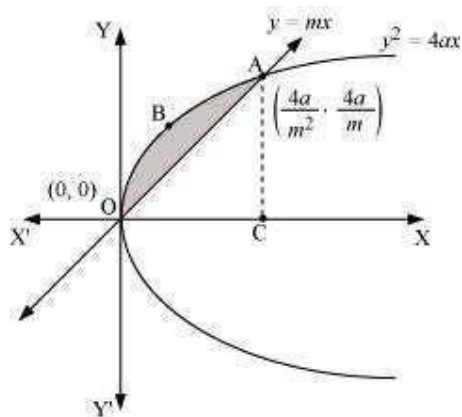
$\therefore$  Required area = Area OABO + Area BCDB

$$\begin{aligned}
 &= \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right| \\
 &= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right| \\
 &= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi| \\
 &= 1 + 1 + |(-1 - 1)| \\
 &= 2 + |-2| \\
 &= 2 + 2 = 4 \text{ units}
 \end{aligned}$$

Question 6:

Find the area enclosed between the parabola  $y^2 = 4ax$  and the line  $y = mx$   
Answer

The area enclosed between the parabola,  $y^2 = 4ax$ , and the line,  $y = mx$ , is represented by the shaded area OABO as



The points of intersection of both the curves are  $(0, 0)$  and

$\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$ . We draw AC perpendicular to x-axis.

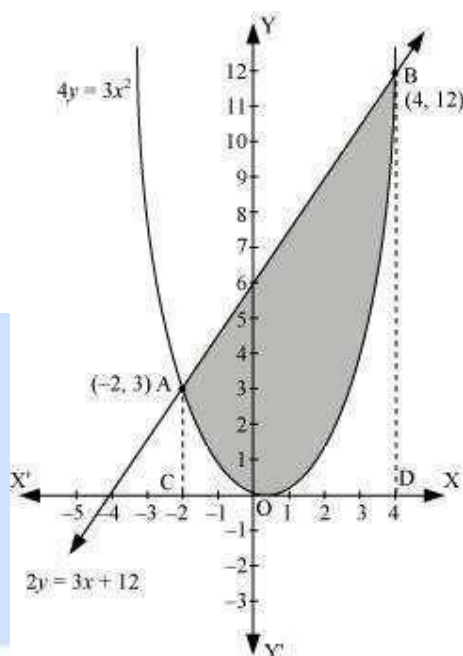
$\therefore \text{Area OABO} = \text{Area OCABO} - \text{Area } (\Delta OCA)$

$$\begin{aligned}
 &= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax} \, dx - \int_0^{\frac{4a}{m^2}} mx \, dx \\
 &= 2\sqrt{a} \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{4a}{m^2}} - m \left[ \frac{x^2}{2} \right]_0^{\frac{4a}{m^2}} \\
 &= \frac{4}{3} \sqrt{a} \left( \frac{4a}{m^2} \right)^{\frac{3}{2}} - \frac{m}{2} \left[ \left( \frac{4a}{m^2} \right)^2 \right] \\
 &= \frac{32a^2}{3m^3} - \frac{m}{2} \left( \frac{16a^2}{m^4} \right) \\
 &= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} \\
 &= \frac{8a^2}{3m^3} \text{ units}
 \end{aligned}$$

Question 7:

Find the area enclosed by the parabola  $4y = 3x^2$  and the line  $2y = 3x + 12$   
12 Answer

The area enclosed between the parabola,  $4y = 3x^2$ , and the line,  $2y = 3x + 12$ , is represented by the shaded area OBAO as



The points of intersection of the given curves are A (-2, 3) and (4, 12). We draw AC and BD perpendicular to x-axis.

$\therefore$  Area OBAO = Area CDBA - (Area ODBO + Area OACO)

$$\begin{aligned} &= \int_{-2}^4 \frac{1}{2}(3x+12)dx - \int_{-2}^4 \frac{3x^2}{4}dx \\ &= \frac{1}{2} \left[ \frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[ \frac{x^3}{3} \right]_{-2}^4 \\ &= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{1}{4} [64 + 8] \\ &= \frac{1}{2} [90] - \frac{1}{4} [72] \\ &= 45 - 18 \\ &= 27 \text{ units} \end{aligned}$$

Question 8:

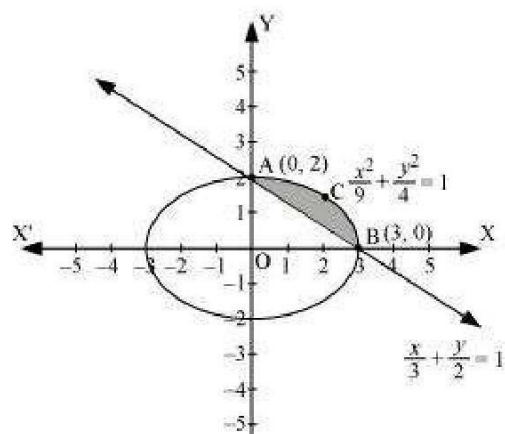
Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line

$$\frac{x}{3} + \frac{y}{2} = 1$$

Answer

The area of the smaller region bounded by the ellipse,  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ , and the line,

$\frac{x}{3} + \frac{y}{2} = 1$ , is represented by the shaded region BCAB as



$\therefore \text{Area BCAB} = \text{Area}_1(\text{OBCAO}) - \text{Area}(\text{OBAO})$

$$\begin{aligned}
 &= \int_0^3 2\sqrt{1 - \frac{x^2}{9}} dx - \int_0^3 2\left(1 - \frac{x}{3}\right) dx \\
 &= \frac{2}{3} \left[ \int_0^3 \sqrt{9 - x^2} dx \right] - \frac{2}{3} \int_0^3 (3 - x) dx \\
 &= \frac{2}{3} \left[ \frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[ 3x - \frac{x^2}{2} \right]_0^3 \\
 &= \frac{2}{3} \left[ \frac{9}{2} \left( \frac{\pi}{2} \right) \right] - \frac{2}{3} \left[ 9 - \frac{9}{2} \right] \\
 &= \frac{2}{3} \left[ \frac{9\pi}{4} - \frac{9}{2} \right] \\
 &= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \\
 &= \frac{3}{2} (\pi - 2) \text{ units}
 \end{aligned}$$

Question 9:

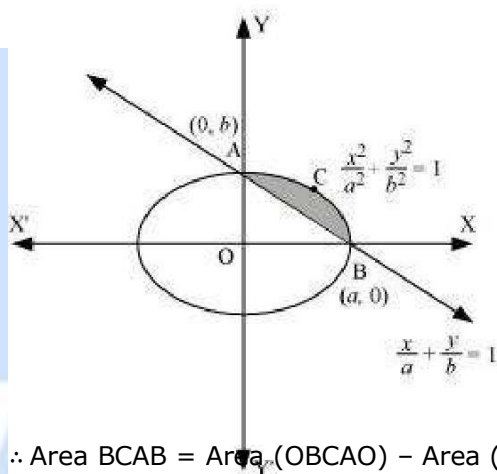
Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

Answer

The area of the smaller region bounded by the ellipse,  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , and the line,

$\frac{x}{a} + \frac{y}{b} = 1$ , is represented by the shaded region BCAB as



$\therefore \text{Area BCAB} = \text{Area (OBCAO)} - \text{Area (OBAO)}$

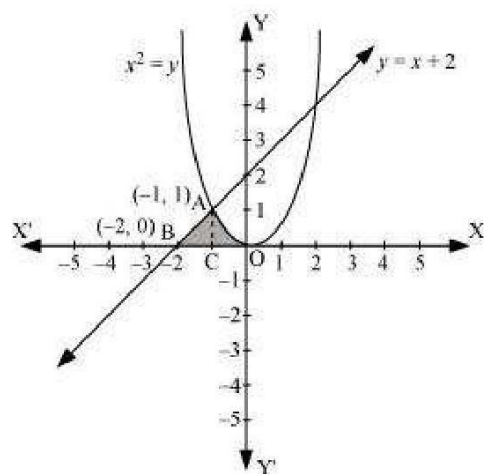
$$\begin{aligned}
 &= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a}\right) dx \\
 &= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx - \frac{b}{a} \int_0^a (a - x) dx \\
 &= \frac{b}{a} \left[ \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^a - \left\{ ax - \frac{x^2}{2} \right\}_0^a \right] \\
 &= \frac{b}{a} \left[ \left\{ \frac{a^2}{2} \left( \frac{\pi}{2} \right) \right\} - \left\{ a^2 - \frac{a^2}{2} \right\} \right] \\
 &= \frac{b}{a} \left[ \frac{a^2 \pi}{4} - \frac{a^2}{2} \right] \\
 &= \frac{ba^2}{2a} \left[ \frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{2} \left[ \frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{4} (\pi - 2)
 \end{aligned}$$

Question 10:

Find the area of the region enclosed by the parabola  $x^2 = y$ , the line  $y = x + 2$  and x-axis

Answer

The area of the region enclosed by the parabola,  $x^2 = y$ , the line,  $y = x + 2$ , and x-axis is represented by the shaded region OABCO as



The point of intersection of the parabola,  $x^2 = y$ , and the line,  $y = x + 2$ , is A  $(-1, 1)$ .

$\therefore$  Area OABCO = Area (BCA) + Area COAC

$$\begin{aligned} &= \int_{-2}^{-1} (x+2) dx + \int_{-1}^0 x^2 dx \\ &= \left[ \frac{x^2}{2} + 2x \right]_{-2}^{-1} + \left[ \frac{x^3}{3} \right]_{-1}^0 \\ &= \left[ \frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2) \right] + \left[ -\frac{(-1)^3}{3} \right] \\ &= \left[ \frac{1}{2} - 2 - 2 + 4 + \frac{1}{3} \right] \\ &= \frac{5}{6} \text{ units} \end{aligned}$$



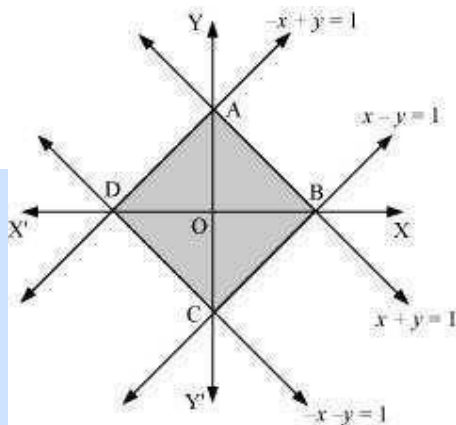
Question 11:

Using the method of integration find the area bounded by the curve  $|x| + |y| = 1$

[Hint: the required region is bounded by lines  $x + y = 1$ ,  $x - y = 1$ ,  $-x + y = 1$  and  $-x - y = 1$ ]

Answer

The area bounded by the curve,  $|x| + |y| = 1$ , is represented by the shaded region ADCB as



The curve intersects the axes at points A (0, 1), B (1, 0), C (0, -1), and D (-1, 0). It can be observed that the given curve is symmetrical about x-axis and y-axis.

$\therefore$  Area ADCB = 4  $\times$  Area OBAO

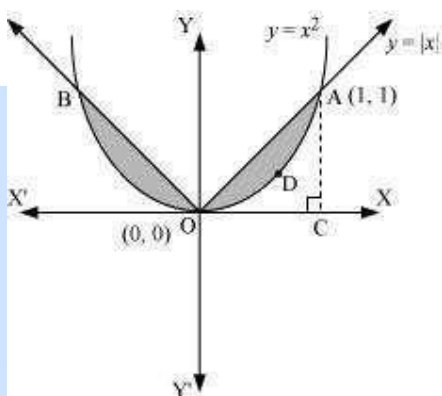
$$\begin{aligned}
 &= 4 \int_0^1 (1-x) dx \\
 &= 4 \left( x - \frac{x^2}{2} \right)_0^1 \\
 &= 4 \left[ 1 - \frac{1}{2} \right] \\
 &= 4 \left( \frac{1}{2} \right) \\
 &= 2 \text{ units}
 \end{aligned}$$

Question 12:

Find the area bounded by curves  $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Answer

The area bounded by the curves,  $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$ , is represented by the shaded region as



It can be observed that the required area is symmetrical about y-axis.

$$\text{Required area} = 2 \left[ \text{Area}(\text{OCAO}) - \text{Area}(\text{OCADO}) \right]$$

$$= 2 \left[ \int_0^1 x \, dx - \int_0^1 x^2 \, dx \right]$$

$$= 2 \left[ \left[ \frac{x^2}{2} \right]_0^1 - \left[ \frac{x^3}{3} \right]_0^1 \right]$$

$$= 2 \left[ \frac{1}{2} - \frac{1}{3} \right]$$

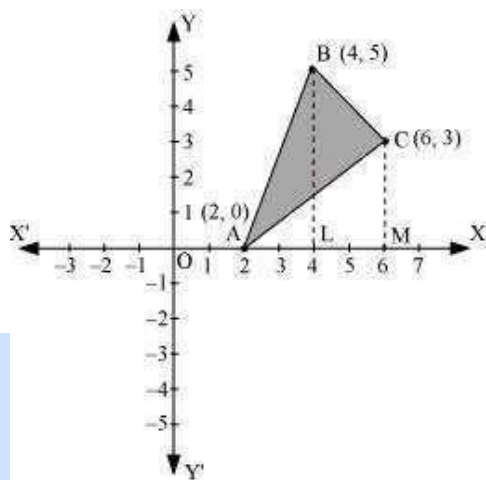
$$= 2 \left[ \frac{1}{6} \right] = \frac{1}{3} \text{ units}$$

Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3)

Answer

The vertices of  $\Delta ABC$  are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$$\begin{aligned}y - 0 &= \frac{5-0}{4-2}(x-2) \\2y &= 5x - 10 \\y &= \frac{5}{2}(x-2) \quad \dots(1)\end{aligned}$$

Equation of line segment BC is

$$\begin{aligned}y - 5 &= \frac{3-5}{6-4}(x-4) \\2y - 10 &= -2x + 8 \\2y &= -2x + 18 \\y &= -x + 9 \quad \dots(2)\end{aligned}$$

Equation of line segment CA is

$$\begin{aligned}y - 3 &= \frac{0-3}{2-6}(x-6) \\-4y + 12 &= -3x + 18 \\4y &= 3x - 6 \\y &= \frac{3}{4}(x-2) \quad \dots(3)\end{aligned}$$

Area ( $\Delta ABC$ ) = Area (ABLA) + Area (BLMCB) – Area (ACMA)

$$\begin{aligned}
 &= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (-x+9)dx - \int_2^6 \frac{3}{4}(x-2)dx \\
 &= \frac{5}{2} \left[ \frac{x^2}{2} - 2x \right]_2^4 + \left[ \frac{-x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[ \frac{x^2}{2} - 2x \right]_2^6 \\
 &= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4] \\
 &= 5 + 8 - \frac{3}{4}(8) \\
 &= 13 - 6 \\
 &= 7 \text{ units}
 \end{aligned}$$

Question 14:

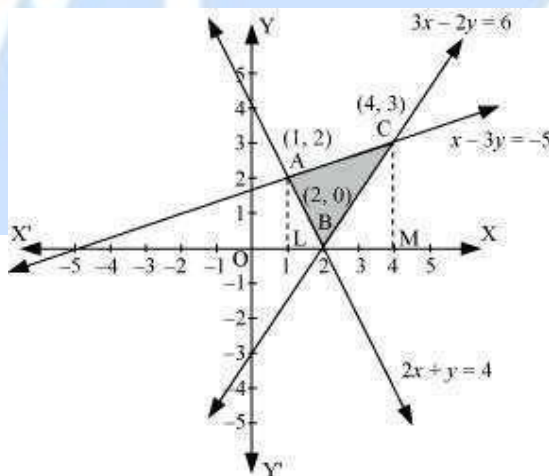
Using the method of integration find the area of the region bounded by lines:  $2x + y = 4$ ,  $3x - 2y = 6$  and  $x - 3y + 5 = 0$

Answer

The given equations of lines are  $2x + y = 4$  ... (1)

$3x - 2y = 6$  ... (2)

And,  $x - 3y + 5 = 0$  ... (3)



The area of the region bounded by the lines is the area of  $\Delta ABC$ . AL and CM are the perpendiculars on x-axis.

Area ( $\Delta ABC$ ) = Area (ALMCA) – Area (ALB) – Area (CMB)

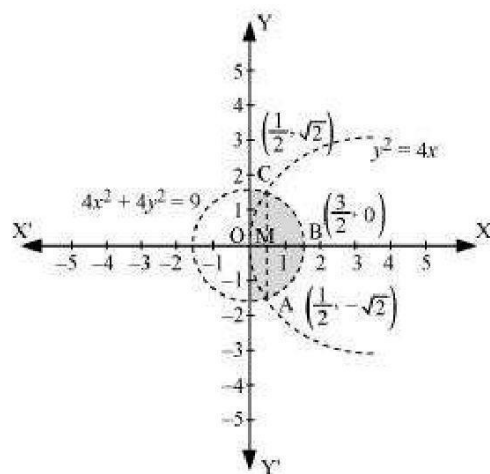
$$\begin{aligned}
 &= \int_1^4 \left( \frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left( \frac{3x-6}{2} \right) dx \\
 &= \frac{1}{3} \left[ \frac{x^2}{2} + 5x \right]_1^4 - \left[ 4x - x^2 \right]_1^2 - \frac{1}{2} \left[ \frac{3x^2}{2} - 6x \right]_2^4 \\
 &= \frac{1}{3} \left[ 8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\
 &= \left( \frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2} (6) \\
 &= \frac{15}{2} - 1 - 3 \\
 &= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ units}
 \end{aligned}$$

Question 15:

Find the area of the region  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

Answer

The area bounded by the curves,  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ , is represented as



The points of intersection of both the curves

are  $\left(\frac{1}{2}, \sqrt{2}\right)$  and  $\left(\frac{1}{2}, -\sqrt{2}\right)$ . The required area is given by OABCO.

It can be observed that area OABCO is symmetrical about x-axis.  
 $\therefore$  Area OABCO = 2  $\times$  Area OBC

Area OBCO = Area OMC + Area MBC

$$\begin{aligned}
 &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{9-4x^2} \, dx \\
 &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{(3)^2 - (2x)^2} \, dx
 \end{aligned}$$

Question 16:

Area bounded by the curve  $y = x^3$ , the x-axis and the ordinates  $x = -2$  and  $x = 1$  is

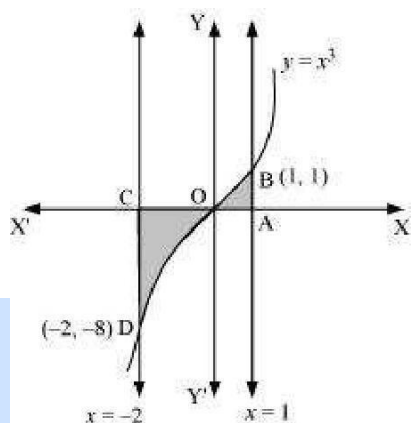
A. - 9

B.  $-\frac{15}{4}$

C.  $\frac{15}{4}$

D.  $\frac{17}{4}$

Answer



$$\text{Required area} = \int_{-2}^1 y dx$$

$$= \int_{-2}^1 x^3 dx$$

$$= \left[ \frac{x^4}{4} \right]_{-2}^1$$

$$= \left[ \frac{1}{4} - \frac{(-2)^4}{4} \right]$$

$$= \left( \frac{1}{4} - 4 \right) = -\frac{15}{4} \text{ units}$$

Thus, the correct answer is B.

Question 17:

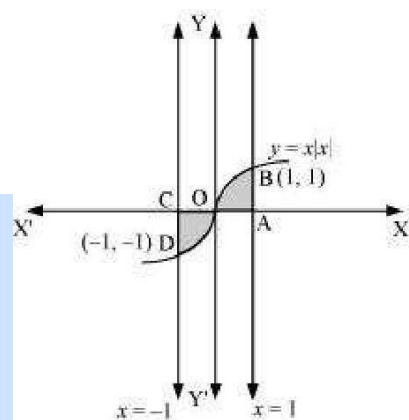
The area bounded by the curve  $y = x|x|$ , x-axis and the ordinates  $x = -1$  and  $x = 1$  is given by

[Hint:  $y = x^2$  if  $x > 0$  and  $y = -x^2$  if  $x < 0$ ]

A. 0

B.  $\frac{1}{3}$ C.  $\frac{2}{3}$ D.  $\frac{4}{3}$ 

Answer



$$\text{Required area} = \int_{-1}^1 y dx$$

$$= \int_{-1}^1 x|x| dx$$

$$= \int_{-1}^0 x^2 dx + \int_0^1 x^2 dx$$

$$= \left[ \frac{x^3}{3} \right]_{-1}^0 + \left[ \frac{x^3}{3} \right]_0^1$$

$$= -\left( -\frac{1}{3} \right) + \frac{1}{3}$$

$$= \frac{2}{3} \text{ units}$$

Thus, the correct answer is C.



Question 18:

The area of the circle  $x^2 + y^2 = 16$  exterior to the parabola  $y^2 = 6x$  is

A.  $\frac{4}{3}(4\pi - \sqrt{3})$

B.  $\frac{4}{3}(4\pi + \sqrt{3})$

C.  $\frac{4}{3}(8\pi - \sqrt{3})$

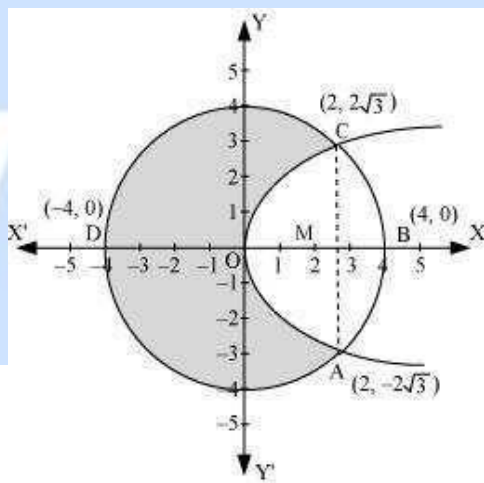
D.  $\frac{4}{3}(4\pi + \sqrt{3})$

Answer

The given equations

are  $x^2 + y^2 = 16$  ... (1)

$y^2 = 6x$  ... (2)



Area bounded by the circle and parabola

$$\begin{aligned}
 &= 2 \left[ \text{Area}(\text{OADO}) + \text{Area}(\text{ADBA}) \right] \\
 &= 2 \left[ \int_0^2 \sqrt{16x} dx + \int_2^4 \sqrt{16-x^2} dx \right] \\
 &= 2 \left[ \sqrt{6} \left\{ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right\}_0^2 \right] + 2 \left[ \frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4 \\
 &= 2\sqrt{6} \times \frac{2}{3} \left[ x^{\frac{3}{2}} \right]_0^2 + 2 \left[ 8 \cdot \frac{\pi}{2} - \sqrt{16-4} - 8 \sin^{-1} \left( \frac{1}{2} \right) \right] \\
 &= \frac{4\sqrt{6}}{3} (2\sqrt{2}) + 2 \left[ 4\pi - \sqrt{12} - 8 \frac{\pi}{6} \right] \\
 &= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi \\
 &= \frac{4}{3} [4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi] \\
 &= \frac{4}{3} [\sqrt{3} + 4\pi] \\
 &= \frac{4}{3} [4\pi + \sqrt{3}] \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of circle} &= \pi (r)^2 \\
 &= \pi (4)^2 \\
 &= 16\pi \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Required area} &= 16\pi - \frac{4}{3} [4\pi + \sqrt{3}] \\
 &= \frac{4}{3} [4 \times 3\pi - 4\pi - \sqrt{3}] \\
 &= \frac{4}{3} (8\pi - \sqrt{3}) \text{ units}
 \end{aligned}$$

Thus, the correct answer is C.

Question 19:

The area bounded by the y-axis,  $y = \cos x$  and  $y = \sin x$  when  $0 \leq x \leq \frac{\pi}{2}$

A.  $2(\sqrt{2}-1)$

B.  $\sqrt{2}-1$

C.  $\sqrt{2}+1$

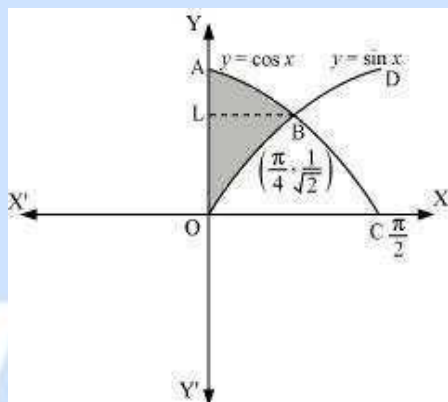
D.  $\sqrt{2}$

Answer

The given equations

are  $y = \cos x$  ... (1)

And,  $y = \sin x$  ... (2)



Required area = Area (ABLA) + area (OBLO)

$$= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_0^{\frac{1}{\sqrt{2}}} x dy$$

$$= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1} x dy$$

Integrating by parts, we obtain

$$\begin{aligned}
 &= \left[ y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[ x \sin^{-1} x + \sqrt{1-x^2} \right]_0^{\frac{1}{\sqrt{2}}} \\
 &= \left[ \cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[ \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right] \\
 &= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\
 &= \frac{2}{\sqrt{2}} - 1 \\
 &= \sqrt{2} - 1 \text{ units}
 \end{aligned}$$

Thus, the correct answer is B.

$$\text{Put } 2x = t \Rightarrow dx = \frac{dt}{2}$$

$$\text{When } x = \frac{3}{2}, t = 3 \text{ and when } x = \frac{1}{2}, t = 1$$

$$\begin{aligned}
 &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} \, dt \\
 &= 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[ \frac{t}{2} \sqrt{9-t^2} + \frac{9}{2} \sin^{-1}\left(\frac{t}{3}\right) \right]_1^3 \\
 &= 2 \left[ \frac{2}{3} \left(\frac{1}{2}\right)^{\frac{3}{2}} \right] + \frac{1}{4} \left[ \left\{ \frac{3}{2} \sqrt{9-(3)^2} + \frac{9}{2} \sin^{-1}\left(\frac{3}{3}\right) \right\} - \left\{ \frac{1}{2} \sqrt{9-(1)^2} + \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right\} \right] \\
 &= \frac{2}{3\sqrt{2}} + \frac{1}{4} \left[ \left\{ 0 + \frac{9}{2} \sin^{-1}(1) \right\} - \left\{ \frac{1}{2} \sqrt{8} + \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right\} \right] \\
 &= \frac{\sqrt{2}}{3} + \frac{1}{4} \left[ \frac{9\pi}{4} - \sqrt{2} - \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right] \\
 &= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) \\
 &= \frac{9\pi}{16} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}
 \end{aligned}$$

Therefore, the required area is  $\left[ 2 \times \left( \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left( \frac{1}{3} \right) + \frac{\sqrt{2}}{12} \right) \right] = \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left( \frac{1}{3} \right) + \frac{1}{3\sqrt{2}}$  units



# Vidya Champ