

# Mathematics

(Chapter – 8) (Binomial Theorem)

(Class – XI)

## Exercise 8.1

### Question 1:

Expand the expression  $(1 - 2x)^5$

### Answer 1:

By using Binomial Theorem, the expression  $(1 - 2x)^5$  can be expanded as

$$\begin{aligned} (1 - 2x)^5 &= {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)(2x)^4 - {}^5C_5(2x)^5 \\ &= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - (32x^5) \\ &= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5 \end{aligned}$$

### Question 2:

Expand the expression  $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

### Answer 2:

By using Binomial Theorem, the expression  $\left(\frac{2}{x} - \frac{x}{2}\right)^5$  can be expanded as

$$\begin{aligned} \left(\frac{2}{x} - \frac{x}{2}\right)^5 &= {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 \\ &\quad - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5 \\ &= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32} \\ &= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32} \end{aligned}$$

### Question 3:

Expand the expression  $(2x - 3)^6$

### Answer 3:

By using Binomial Theorem, the expression  $(2x - 3)^6$  can be expanded as

$$\begin{aligned}
 (2x-3)^6 &= {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 \\
 &\quad + {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6 \\
 &= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) \\
 &\quad + 15(4x^2)(81) - 6(2x)(243) + 729 \\
 &= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729
 \end{aligned}$$

**Question 4:**

Expand the expression  $\left(\frac{x}{3} + \frac{1}{x}\right)^5$

**Answer 4:**

By using Binomial Theorem, the expression  $\left(\frac{x}{3} + \frac{1}{x}\right)^5$  can be expanded as

**Question 5:**

$$\begin{aligned}
 \left(\frac{x}{3} + \frac{1}{x}\right)^5 &= {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 \\
 &\quad + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5 \\
 &= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5} \\
 &= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}
 \end{aligned}$$

Expand  $\left(x + \frac{1}{x}\right)^6$

**Answer 5:**

By using Binomial Theorem, the expression  $\left(x + \frac{1}{x}\right)^6$  can be expanded as

$$\begin{aligned} \left(x + \frac{1}{x}\right)^6 &= {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 \\ &\quad + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6 \\ &= x^6 + 6(x)^5\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x}\right)^2 + 20(x)^3\left(\frac{1}{x}\right)^3 + 15(x)^2\left(\frac{1}{x}\right)^4 + 6(x)\left(\frac{1}{x}\right)^5 + \frac{1}{x^6} \\ &= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6} \end{aligned}$$

**Question 6:**

Using Binomial Theorem, evaluate  $(96)^3$

**Answer 6:**

96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that,  $96 = 100 - 4$

$$\begin{aligned} \therefore (96)^3 &= (100 - 4)^3 \\ &= {}^3C_0(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3 \\ &= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 884736 \end{aligned}$$

**Question 7:**

Using Binomial Theorem, evaluate  $(102)^5$

**Answer 7:**

102 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that,  $102 = 100 + 2$

$$\begin{aligned}
 \therefore (102)^5 &= (100+2)^5 \\
 &= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 \\
 &\quad + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5 \\
 &= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5 \\
 &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 \\
 &= 11040808032
 \end{aligned}$$

**Question 8:**

Using Binomial Theorem, evaluate  $(101)^4$

**Answer 8:**

101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that,  $101 = 100 + 1$

$$\begin{aligned}
 \therefore (101)^4 &= (100+1)^4 \\
 &= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4 \\
 &= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4 \\
 &= 100000000 + 4000000 + 60000 + 400 + 1 \\
 &= 104060401
 \end{aligned}$$

**Question 9:**

Using Binomial Theorem, evaluate  $(99)^5$

**Answer 9:**

99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that,  $99 = 100 - 1$

$$\begin{aligned}
 \therefore (99)^5 &= (100-1)^5 \\
 &= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3 \\
 &\quad + {}^5C_4(100)(1)^4 - {}^5C_5(1)^5 \\
 &= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1 \\
 &= 10000000000 - 5000000000 + 100000000 - 1000000 + 500 - 1 \\
 &= 10010000500 - 500100001 \\
 &= 9509900499
 \end{aligned}$$

**Question 10:**

Using Binomial Theorem, indicate which number is larger  $(1.1)^{10000}$  or 1000.

**Answer 10:**

By splitting 1.1 and then applying Binomial Theorem, the first few terms of  $(1.1)^{10000}$  can be obtained as

$$\begin{aligned}
 (1.1)^{10000} &= (1+0.1)^{10000} \\
 &= {}^{10000}C_0 + {}^{10000}C_1(1.1) + \text{Other positive terms} \\
 &= 1 + 10000 \times 1.1 + \text{Other positive terms} \\
 &= 1 + 11000 + \text{Other positive terms} \\
 &> 1000
 \end{aligned}$$

Hence,  $(1.1)^{10000} > 1000$

**Question 11:**

Find  $(a + b)^4 - (a - b)^4$ . Hence, evaluate.  $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$

**Answer 11:**

Using Binomial Theorem, the expressions,  $(a + b)^4$  and  $(a - b)^4$ , can be expanded as

$$\begin{aligned}(a+b)^4 &= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4 \\(a-b)^4 &= {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4 \\ \therefore (a+b)^4 - (a-b)^4 &= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4 \\ &\quad - [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4] \\ &= 2({}^4C_1a^3b + {}^4C_3ab^3) = 2(4a^3b + 4ab^3) \\ &= 8ab(a^2 + b^2)\end{aligned}$$

By putting  $a = \sqrt{3}$  and  $b = \sqrt{2}$ , we obtain

$$\begin{aligned}(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8(\sqrt{3})(\sqrt{2})\{(\sqrt{3})^2 + (\sqrt{2})^2\} \\ &= 8(\sqrt{6})\{3+2\} = 40\sqrt{6}\end{aligned}$$

**Question 12:**

Find  $(x + 1)^6 + (x - 1)^6$ . Hence or otherwise evaluate.  $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$

**Answer 12:**

Using Binomial Theorem, the expressions,  $(x + 1)^6$  and  $(x - 1)^6$ , can be expanded as

$$\begin{aligned}(x+1)^6 &= {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 \\(x-1)^6 &= {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6 \\ \therefore (x+1)^6 + (x-1)^6 &= 2[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6] \\ &= 2[x^6 + 15x^4 + 15x^2 + 1]\end{aligned}$$

By putting  $x = \sqrt{2}$ , we obtain

$$\begin{aligned}(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 &= 2\left[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1\right] \\ &= 2(8 + 15 \times 4 + 15 \times 2 + 1) \\ &= 2(8 + 60 + 30 + 1) \\ &= 2(99) = 198\end{aligned}$$

**Question 13:**

Show that  $9^{n+1} - 8n - 9$  is divisible by 64, whenever  $n$  is a positive integer.

**Answer 13:**

In order to show that  $9^{n+1} - 8n - 9$  is divisible by 64, it has to be proved that,  $9^{n+1} - 8n - 9 = 64k$ , where  $k$  is some natural number

By Binomial Theorem,

$$(1+a)^m = {}^m C_0 + {}^m C_1 a + {}^m C_2 a^2 + \dots + {}^m C_m a^m$$

For  $a = 8$  and  $m = n + 1$ , we obtain

$$\begin{aligned} (1+8)^{n+1} &= {}^{n+1} C_0 + {}^{n+1} C_1 (8) + {}^{n+1} C_2 (8)^2 + \dots + {}^{n+1} C_{n+1} (8)^{n+1} \\ \Rightarrow 9^{n+1} &= 1 + (n+1)(8) + 8^2 \left[ {}^{n+1} C_2 + {}^{n+1} C_3 \times 8 + \dots + {}^{n+1} C_{n+1} (8)^{n-1} \right] \\ \Rightarrow 9^{n+1} &= 9 + 8n + 64 \left[ {}^{n+1} C_2 + {}^{n+1} C_3 \times 8 + \dots + {}^{n+1} C_{n+1} (8)^{n-1} \right] \\ \Rightarrow 9^{n+1} - 8n - 9 &= 64k, \text{ where } k = {}^{n+1} C_2 + {}^{n+1} C_3 \times 8 + \dots + {}^{n+1} C_{n+1} (8)^{n-1} \text{ is a natural number} \end{aligned}$$

Thus,  $9^{n+1} - 8n - 9$  is divisible by 64, whenever  $n$  is a positive integer.

**Question 14:**

Prove that.  $\sum_{r=0}^n 3^r {}^n C_r = 4^n$

**Answer 14:**

By Binomial Theorem,

$$\sum_{r=0}^n {}^n C_r a^{n-r} b^r = (a+b)^n$$

By putting  $b = 3$  and  $a = 1$  in the above equation, we obtain

$$\begin{aligned} \sum_{r=0}^n {}^n C_r (1)^{n-r} (3)^r &= (1+3)^n \\ \Rightarrow \sum_{r=0}^n 3^r {}^n C_r &= 4^n \end{aligned}$$

Hence, proved.



# Mathematics

## (Chapter – 8) (Binomial Theorem)

(Class – XI)

### Exercise 8.2

**Question 1:**

Find the coefficient of  $x^5$  in  $(x + 3)^8$

**Answer 1:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Assuming that  $x^5$  occurs in the  $(r + 1)^{\text{th}}$  term of the expansion  $(x + 3)^8$ , we obtain

$$T_{r+1} = {}^8 C_r (x)^{8-r} (3)^r$$

Comparing the indices of  $x$  in  $x^5$  and in  $T_{r+1}$ ,

we obtain  $r = 3$

Thus, the coefficient of  $x^5$  is  ${}^8 C_3 (3)^3 = \frac{8!}{3!5!} \times 3^3 = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 5!} \cdot 3^3 = 1512$

**Question 2:**

Find the coefficient of  $a^5 b^7$  in  $(a - 2b)^{12}$

**Answer 2:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Assuming that  $a^5 b^7$  occurs in the  $(r + 1)^{\text{th}}$  term of the expansion  $(a - 2b)^{12}$ , we obtain

$$T_{r+1} = {}^{12} C_r (a)^{12-r} (-2b)^r = {}^{12} C_r (-2)^r (a)^{12-r} (b)^r$$

Comparing the indices of  $a$  and  $b$  in  $a^5 b^7$  and in  $T_{r+1}$ ,

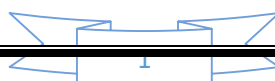
we obtain  $r = 7$

Thus, the coefficient of  $a^5 b^7$  is

$${}^{12} C_7 (-2)^7 = -\frac{12!}{7!5!} \cdot 2^7 = -\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 7!} \cdot 2^7 = -(792)(128) = -101376$$

**Question 3:**

Write the general term in the expansion of  $(x^2 - y)^6$





**Answer 3:**

It is known that the general term  $T_{r+1}$  {which is the  $(r + 1)^{\text{th}}$  term} in the binomial expansion of  $(a + b)^n$  is given by  $T_{r+1} = {}^n C_r \cdot a^{n-r} b^r$ .

Thus, the general term in the expansion of  $(x^2 - y^6)$  is

$$T_{r+1} = {}^6 C_r (x^2)^{6-r} (-y)^r = (-1)^r {}^6 C_r x^{12-2r} \cdot y^r$$

**Question 4:**

Write the general term in the expansion of  $(x^2 - yx)^{12}$ ,  $x \neq 0$

**Answer 4:**

It is known that the general term  $T_{r+1}$  {which is the  $(r + 1)^{\text{th}}$  term} in the binomial expansion of  $(a + b)^n$  is given by  $T_{r+1} = {}^n C_r \cdot a^{n-r} b^r$ .

Thus, the general term in the expansion of  $(x^2 - yx)^{12}$  is

$$T_{r+1} = {}^{12} C_r (x^2)^{12-r} (-yx)^r = (-1)^r {}^{12} C_r x^{24-2r} \cdot y^r \cdot x^r = (-1)^r {}^{12} C_r x^{24-r} \cdot y^r$$

**Question 5:**

Find the 4<sup>th</sup> term in the expansion of  $(x - 2y)^{12}$ .

**Answer 5:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Thus, the 4<sup>th</sup> term in the expansion of  $(x - 2y)^{12}$  is

$$T_4 = T_{3+1} = {}^{12} C_3 (x)^{12-3} (-2y)^3 = (-1)^3 \cdot \frac{12!}{3!9!} \cdot x^9 \cdot (2)^3 \cdot y^3 = -\frac{12 \cdot 11 \cdot 10}{3 \cdot 2} \cdot (2)^3 x^9 y^3 = -1760x^9 y^3$$

**Question 6:**

Find the 13<sup>th</sup> term in the expansion of  $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ ,  $x \neq 0$

**Answer 6:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Thus, 13<sup>th</sup> term in the expansion of  $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$  is

$$\begin{aligned} T_{13} = T_{12+1} &= {}^{18}C_{12} (9x)^{18-12} \left(-\frac{1}{3\sqrt{x}}\right)^{12} \\ &= (-1)^{12} \frac{18!}{12!6!} (9)^6 (x)^6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{\sqrt{x}}\right)^{12} \\ &= \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \cdot x^6 \cdot \left(\frac{1}{x^6}\right) \cdot 3^{12} \left(\frac{1}{3^{12}}\right) \quad \left[9^6 = (3^2)^6 = 3^{12}\right] \\ &= 18564 \end{aligned}$$

**Question 7:**

Find the middle terms in the expansions of  $\left(3 - \frac{x^3}{6}\right)^7$

**Answer 7:**

It is known that in the expansion of  $(a + b)^n$ , if  $n$  is odd, then there are two middle terms,

namely  $\left(\frac{n+1}{2}\right)^{\text{th}}$  term and  $\left(\frac{n+1}{2} + 1\right)^{\text{th}}$  term.

Therefore, the middle terms in the expansion  $\left(3 - \frac{x^3}{6}\right)^7$  are  $\left(\frac{7+1}{2}\right)^{\text{th}} = 4^{\text{th}}$  term and  $\left(\frac{7+1}{2} + 1\right)^{\text{th}} = 5^{\text{th}}$  term

$$\begin{aligned} T_4 = T_{3+1} &= {}^7C_3 (3)^{7-3} \left(-\frac{x^3}{6}\right)^3 = (-1)^3 \frac{7!}{3!4!} \cdot 3^4 \cdot \frac{x^9}{6^3} \\ &= -\frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 4!} \cdot 3^4 \cdot \frac{1}{2^3 \cdot 3^3} \cdot x^9 = -\frac{105}{8} x^9 \end{aligned}$$

$$\begin{aligned} T_5 = T_{4+1} &= {}^7C_4 (3)^{7-4} \left(-\frac{x^3}{6}\right)^4 = (-1)^4 \frac{7!}{4!3!} (3)^3 \cdot \frac{x^{12}}{6^4} \\ &= \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2} \cdot \frac{3^3}{2^4 \cdot 3^4} \cdot x^{12} = \frac{35}{48} x^{12} \end{aligned}$$

Thus, the middle terms in the expansion of  $\left(3 - \frac{x^3}{6}\right)^7$  are  $-\frac{105}{8}x^9$  and  $\frac{35}{48}x^{12}$

**Question 8:**

Find the middle terms in the expansions of  $\left(\frac{x}{3} + 9y\right)^{10}$

**Answer 8:**

It is known that in the expansion  $(a + b)^n$ , if  $n$  is even, then the middle term is  $\left(\frac{n}{2} + 1\right)^{\text{th}}$  term.

Therefore, the middle term in the expansion of  $\left(\frac{x}{3} + 9y\right)^{10}$  is  $\left(\frac{10}{2} + 1\right)^{\text{th}} = 6^{\text{th}}$

$$\begin{aligned} T_6 = T_{5+1} &= {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5 = \frac{10!}{5!5!} \cdot \frac{x^5}{3^5} \cdot 9^5 \cdot y^5 \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 5!} \cdot \frac{1}{3^5} \cdot 3^{10} \cdot x^5 y^5 && [9^5 = (3^2)^5 = 3^{10}] \\ &= 252 \times 3^5 \cdot x^5 \cdot y^5 = 61236x^5y^5 \end{aligned}$$

Thus, the middle term in the expansion of  $\left(\frac{x}{3} + 9y\right)^{10}$  is  $61236 x^5y^5$ .

**Question 9:**

In the expansion of  $(1 + a)^{m+n}$ , prove that coefficients of  $a^m$  and  $a^n$  are equal.

**Answer 9:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Assuming that  $a^m$  occurs in the  $(r + 1)^{\text{th}}$  term of the expansion  $(1 + a)^{m+n}$ , we obtain

$$T_{r+1} = {}^{m+n} C_r (1)^{m+n-r} (a)^r = {}^{m+n} C_r a^r$$

Comparing the indices of  $a$  in  $a^m$  and in  $T_{r+1}$ ,

we obtain

$$r = m$$

Therefore, the coefficient of  $a^m$  is

$${}^{m+n}C_m = \frac{(m+n)!}{m!(m+n-m)!} = \frac{(m+n)!}{m!n!} \quad \dots(1)$$

Assuming that  $a^n$  occurs in the  $(k + 1)^{\text{th}}$  term of the expansion  $(1 + a)^{m+n}$ , we obtain

$$T_{k+1} = {}^{m+n}C_k (1)^{m+n-k} (a)^k = {}^{m+n}C_k (a)^k$$

Comparing the indices of  $a$  in  $a^n$  and in  $T_{k+1}$ , we obtain

$$k = n$$

Therefore, the coefficient of  $a^n$  is

$${}^{m+n}C_n = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!} \quad \dots(2)$$

Thus, from (1) and (2), it can be observed that the coefficients of  $a^m$  and  $a^n$  in the expansion of  $(1 + a)^{m+n}$  are equal.

**Question 10:**

The coefficients of the  $(r - 1)^{\text{th}}$ ,  $r^{\text{th}}$  and  $(r + 1)^{\text{th}}$  terms in the expansion of  $(x + 1)^n$  are in the ratio 1:3:5. Find  $n$  and  $r$ .

**Answer 10:**

It is known that  $(k + 1)^{\text{th}}$  term,  $(T_{k+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{k+1} = {}^nC_k a^{n-k} b^k$$

Therefore,  $(r - 1)^{\text{th}}$  term in the expansion of  $(x + 1)^n$  is

$$T_{r-1} = {}^nC_{r-2} (x)^{n-(r-2)} (1)^{(r-2)} = {}^nC_{r-2} x^{n-r+2}$$

$(r + 1)$  term in the expansion of  $(x + 1)^n$  is

$$T_{r+1} = {}^nC_r (x)^{n-r} (1)^r = {}^nC_r x^{n-r}$$

$r^{\text{th}}$  term in the expansion of  $(x + 1)^n$  is

$$T_r = {}^n C_{r-1} (x)^{n-(r-1)} (1)^{(r-1)} = {}^n C_{r-1} x^{n-r+1}$$

Therefore, the coefficients of the  $(r - 1)^{\text{th}}$ ,  $r^{\text{th}}$ , and  $(r + 1)^{\text{th}}$  terms in the expansion of  $(x + 1)^n$   ${}^n C_{r-2}$ ,  ${}^n C_{r-1}$ , and  ${}^n C_r$  are respectively. Since these coefficients are in the ratio 1:3:5, we obtain

$$\frac{{}^n C_{r-2}}{{}^n C_{r-1}} = \frac{1}{3} \quad \text{and} \quad \frac{{}^n C_{r-1}}{{}^n C_r} = \frac{3}{5}$$

$$\begin{aligned} \frac{{}^n C_{r-2}}{{}^n C_{r-1}} &= \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!} \\ &= \frac{r-1}{n-r+2} \end{aligned}$$

$$\begin{aligned} \therefore \frac{r-1}{n-r+2} &= \frac{1}{3} \\ \Rightarrow 3r-3 &= n-r+2 \\ \Rightarrow n-4r+5 &= 0 \end{aligned}$$

$$\begin{aligned} \frac{{}^n C_{r-1}}{{}^n C_r} &= \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!} \\ &= \frac{r}{n-r+1} \end{aligned} \quad \dots(1)$$

$$\begin{aligned} \therefore \frac{r}{n-r+1} &= \frac{3}{5} \\ \Rightarrow 5r &= 3n-3r+3 \\ \Rightarrow 3n-8r+3 &= 0 \end{aligned} \quad \dots(2)$$

Multiplying (1) by 3 and subtracting it from (2), we obtain

$$4r - 12 = 0$$

$$\Rightarrow r = 3$$

Putting the value of  $r$  in (1), we obtain  $n$

$$-12 + 5 = 0$$

$$\Rightarrow n = 7$$

Thus,  $n = 7$  and  $r = 3$

**Question 11:**

Prove that the coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n}$  is twice the coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n-1}$ .

**Answer 11:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Assuming that  $x^n$  occurs in the  $(r + 1)^{\text{th}}$  term of the expansion of  $(1 + x)^{2n}$ , we obtain

$$T_{r+1} = {}^{2n} C_r (1)^{2n-r} (x)^r = {}^{2n} C_r (x)^r$$

Comparing the indices of  $x$  in  $x^n$  and in  $T_{r+1}$ , we obtain  $r = n$

Therefore, the coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n}$  is

$${}^{2n} C_n = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!} = \frac{(2n)!}{(n!)^2} \quad \dots(1)$$

Assuming that  $x^n$  occurs in the  $(k + 1)^{\text{th}}$  term of the expansion  $(1 + x)^{2n-1}$ , we obtain

$$T_{k+1} = {}^{2n-1} C_k (1)^{2n-1-k} (x)^k = {}^{2n-1} C_k (x)^k$$

Comparing the indices of  $x$  in  $x^n$  and  $T_{k+1}$ , we obtain  $k = n$

Therefore, the coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n-1}$  is

$$\begin{aligned} {}^{2n-1} C_n &= \frac{(2n-1)!}{n!(2n-1-n)!} = \frac{(2n-1)!}{n!(n-1)!} \\ &= \frac{2n \cdot (2n-1)!}{2n \cdot n!(n-1)!} = \frac{(2n)!}{2 \cdot n!n!} = \frac{1}{2} \left[ \frac{(2n)!}{(n!)^2} \right] \quad \dots(2) \end{aligned}$$

From (1) and (2), it is observed that

$$\begin{aligned} \frac{1}{2} ({}^{2n} C_n) &= {}^{2n-1} C_n \\ \Rightarrow {}^{2n} C_n &= 2 ({}^{2n-1} C_n) \end{aligned}$$

Therefore,

the coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n}$  is twice the coefficient of  $x^n$  in the expansion of  $(1 + x)^{2n-1}$ .

Hence, proved.

**Question 12:**

Find a positive value of  $m$  for which the coefficient of  $x^2$  in the expansion  $(1 + x)^m$  is 6.

**Answer 12:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Assuming that  $x^2$  occurs in the  $(r + 1)^{\text{th}}$  term of the expansion  $(1 + x)^m$ , we obtain

$$T_{r+1} = {}^m C_r (1)^{m-r} (x)^r = {}^m C_r (x)^r$$

Comparing the indices of  $x$  in  $x^2$  and in  $T_{r+1}$ , we obtain  $r = 2$

Therefore, the coefficient of  $x^2$  is  ${}^m C_2$

It is given that the coefficient of  $x^2$  in the expansion  $(1 + x)^m$  is 6.



$$\begin{aligned} \therefore {}^m C_2 &= 6 \\ \Rightarrow \frac{m!}{2!(m-2)!} &= 6 \\ \Rightarrow \frac{m(m-1)(m-2)!}{2 \times (m-2)!} &= 6 \\ \Rightarrow m(m-1) &= 12 \\ \Rightarrow m^2 - m - 12 &= 0 \\ \Rightarrow m^2 - 4m + 3m - 12 &= 0 \\ \Rightarrow m(m-4) + 3(m-4) &= 0 \\ \Rightarrow (m-4)(m+3) &= 0 \\ \Rightarrow (m-4) = 0 \text{ or } (m+3) &= 0 \\ \Rightarrow m = 4 \text{ or } m = -3 \end{aligned}$$

Thus, the positive value of  $m$ , for which the coefficient of  $x^2$  in the expansion  $(1 + x)^m$  is 6, is 4.





# Mathematics

(Chapter – 8) (Binomial Theorem)

(Class – XI)

## Miscellaneous Exercise on Chapter 8

### Question 1:

Find  $a$ ,  $b$  and  $n$  in the expansion of  $(a + b)^n$  if the first three terms of the expansion are 729, 7290 and 30375, respectively.

### Answer 1:

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

The first three terms of the expansion are given as 729, 7290, and 30375 respectively.

Therefore, we obtain

$$T_1 = {}^n C_0 a^{n-0} b^0 = a^n = 729 \quad \dots(1)$$

$$T_2 = {}^n C_1 a^{n-1} b^1 = na^{n-1}b = 7290 \quad \dots(2)$$

$$T_3 = {}^n C_2 a^{n-2} b^2 = \frac{n(n-1)}{2} a^{n-2} b^2 = 30375 \quad \dots(3)$$

Dividing (2) by (1), we obtain

$$\begin{aligned} \frac{na^{n-1}b}{a^n} &= \frac{7290}{729} \\ \Rightarrow \frac{nb}{a} &= 10 \quad \dots(4) \end{aligned}$$

Dividing (3) by (2), we obtain

$$\begin{aligned} \frac{n(n-1)a^{n-2}b^2}{2na^{n-1}b} &= \frac{30375}{7290} \\ \Rightarrow \frac{(n-1)b}{2a} &= \frac{30375}{7290} \\ \Rightarrow \frac{(n-1)b}{a} &= \frac{30375 \times 2}{7290} = \frac{25}{3} \end{aligned}$$

$$\Rightarrow \frac{nb}{a} - \frac{b}{a} = \frac{25}{3}$$

$$\Rightarrow 10 - \frac{b}{a} = \frac{25}{3} \quad [\text{Using (4)}]$$

$$\Rightarrow \frac{b}{a} = 10 - \frac{25}{3} = \frac{5}{3} \quad \dots(5)$$

From (4) and (5), we obtain

$$n \cdot \frac{5}{3} = 10$$

$$\Rightarrow n = 6$$

Substituting  $n = 6$  in equation (1), we obtain  $a^6$

$$= 729$$

$$\Rightarrow a = \sqrt[6]{729} = 3$$

From (5), we obtain

$$\frac{b}{3} = \frac{5}{3} \Rightarrow b = 5$$

Thus,  $a = 3$ ,  $b = 5$ , and  $n = 6$ .

**Question 2:**

Find  $a$  if the coefficients of  $x^2$  and  $x^3$  in the expansion of  $(3 + ax)^9$  are equal.

**Answer 2:**

It is known that  $(r + 1)^{\text{th}}$  term,  $(T_{r+1})$ , in the binomial expansion of  $(a + b)^n$  is given by

$$T_{r+1} = {}^n C_r a^{n-r} b^r$$

Assuming that  $x^2$  occurs in the  $(r + 1)^{\text{th}}$  term in the expansion of  $(3 + ax)^9$ , we obtain

$$T_{r+1} = {}^9 C_r (3)^{9-r} (ax)^r = {}^9 C_r (3)^{9-r} a^r x^r$$

Comparing the indices of  $x$  in  $x^2$  and in  $T_{r+1}$ , we obtain

$$r = 2$$

Thus, the coefficient of  $x^2$  is

$${}^9 C_2 (3)^{9-2} a^2 = \frac{9!}{2!7!} (3)^7 a^2 = 36(3)^7 a^2$$

Assuming that  $x^3$  occurs in the  $(k + 1)^{\text{th}}$  term in the expansion of  $(3 + ax)^9$ , we obtain

$$T_{k+1} = {}^9 C_k (3)^{9-k} (ax)^k = {}^9 C_k (3)^{9-k} a^k x^k$$

Comparing the indices of  $x$  in  $x^3$  and in  $T_{k+1}$ , we obtain  $k = 3$

Thus, the coefficient of  $x^3$  is

$${}^9 C_3 (3)^{9-3} a^3 = \frac{9!}{3!6!} (3)^6 a^3 = 84(3)^6 a^3$$

It is given that the coefficients of  $x^2$  and  $x^3$  are the same.

$$84(3)^6 a^3 = 36(3)^7 a^2$$

$$\Rightarrow 84a = 36 \times 3$$

$$\Rightarrow a = \frac{36 \times 3}{84} = \frac{104}{84}$$

$$\Rightarrow a = \frac{9}{7}$$

Thus, the required value of  $a$  is  $9/7$ .

**Question 3:**

Find the coefficient of  $x^5$  in the product  $(1 + 2x)^6 (1 - x)^7$  using binomial theorem.

**Answer 3:**

Using Binomial Theorem, the expressions,  $(1 + 2x)^6$  and  $(1 - x)^7$ , can be expanded as

$$(1+2x)^6 = {}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$

$$= 1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + 15(2x)^4 + 6(2x)^5 + (2x)^6$$

$$= 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6$$

$$(1-x)^7 = {}^7C_0 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7$$

$$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

$$\therefore (1+2x)^6 (1-x)^7$$

$$= (1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6)(1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7)$$

The complete multiplication of the two brackets is not required to be carried out. Only those terms, which involve  $x^5$ , are required.

The terms containing  $x^5$  are

$$1(-21x^5) + (12x)(35x^4) + (60x^2)(-35x^3) + (160x^3)(21x^2) + (240x^4)(-7x) + (192x^5)(1) = 171x^5$$

Thus, the coefficient of  $x^5$  in the given product is 171.

**Question 4:**

If  $a$  and  $b$  are distinct integers, prove that  $a - b$  is a factor of  $a^n - b^n$ , whenever  $n$  is a positive integer. [Hint: write  $a^n = (a - b + b)^n$  and expand]

**Answer 4:**

In order to prove that  $(a - b)$  is a factor of  $(a^n - b^n)$ , it has to be proved that

$a^n - b^n = k(a - b)$ , where  $k$  is some natural number

It can be written that,  $a = a - b + b$

$$\begin{aligned} \therefore a^n &= (a - b + b)^n = [(a - b) + b]^n \\ &= {}^n C_0 (a - b)^n + {}^n C_1 (a - b)^{n-1} b + \dots + {}^n C_{n-1} (a - b) b^{n-1} + {}^n C_n b^n \\ &= (a - b)^n + {}^n C_1 (a - b)^{n-1} b + \dots + {}^n C_{n-1} (a - b) b^{n-1} + b^n \\ \Rightarrow a^n - b^n &= (a - b) [(a - b)^{n-1} + {}^n C_1 (a - b)^{n-2} b + \dots + {}^n C_{n-1} b^{n-1}] \\ \Rightarrow a^n - b^n &= k(a - b) \\ \text{where, } k &= [(a - b)^{n-1} + {}^n C_1 (a - b)^{n-2} b + \dots + {}^n C_{n-1} b^{n-1}] \text{ is a natural number} \end{aligned}$$

This shows that  $(a - b)$  is a factor of  $(a^n - b^n)$ , where  $n$  is a positive integer.

**Question 5:**

Evaluate.  $(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$

**Answer 5:**

Firstly, the expression  $(a + b)^6 - (a - b)^6$  is simplified by using Binomial Theorem. This can be done as

$$\begin{aligned} (a + b)^6 &= {}^6 C_0 a^6 + {}^6 C_1 a^5 b + {}^6 C_2 a^4 b^2 + {}^6 C_3 a^3 b^3 + {}^6 C_4 a^2 b^4 + {}^6 C_5 a b^5 + {}^6 C_6 b^6 \\ &= a^6 + 6a^5 b + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6ab^5 + b^6 \\ (a - b)^6 &= {}^6 C_0 a^6 - {}^6 C_1 a^5 b + {}^6 C_2 a^4 b^2 - {}^6 C_3 a^3 b^3 + {}^6 C_4 a^2 b^4 - {}^6 C_5 a b^5 + {}^6 C_6 b^6 \\ &= a^6 - 6a^5 b + 15a^4 b^2 - 20a^3 b^3 + 15a^2 b^4 - 6ab^5 + b^6 \\ \therefore (a + b)^6 - (a - b)^6 &= 2[6a^5 b + 20a^3 b^3 + 6ab^5] \end{aligned}$$

Putting  $a = \sqrt{3}$  and  $b = \sqrt{2}$ , we obtain

$$\begin{aligned} (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6 &= 2[6(\sqrt{3})^5 (\sqrt{2}) + 20(\sqrt{3})^3 (\sqrt{2})^3 + 6(\sqrt{3})(\sqrt{2})^5] \\ &= 2[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}] \\ &= 2 \times 198\sqrt{6} \\ &= 396\sqrt{6} \end{aligned}$$

**Question 6:**

Find the value of  $(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4$

**Answer 6:**

Firstly, the expression  $(x + y)^4 + (x - y)^4$  is simplified by using Binomial Theorem.

This can be done as

$$\begin{aligned}(x + y)^4 &= {}^4C_0x^4 + {}^4C_1x^3y + {}^4C_2x^2y^2 + {}^4C_3xy^3 + {}^4C_4y^4 \\ &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4\end{aligned}$$

$$\begin{aligned}(x - y)^4 &= {}^4C_0x^4 - {}^4C_1x^3y + {}^4C_2x^2y^2 - {}^4C_3xy^3 + {}^4C_4y^4 \\ &= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4\end{aligned}$$

$$\therefore (x + y)^4 + (x - y)^4 = 2(x^4 + 6x^2y^2 + y^4)$$

Putting  $x = a^2$  and  $y = \sqrt{a^2 - 1}$ , we obtain

$$\begin{aligned}(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4 &= 2 \left[ (a^2)^4 + 6(a^2)^2(\sqrt{a^2 - 1})^2 + (\sqrt{a^2 - 1})^4 \right] \\ &= 2 \left[ a^8 + 6a^4(a^2 - 1) + (a^2 - 1)^2 \right] \\ &= 2 \left[ a^8 + 6a^6 - 6a^4 + a^4 - 2a^2 + 1 \right] \\ &= 2 \left[ a^8 + 6a^6 - 5a^4 - 2a^2 + 1 \right] \\ &= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2\end{aligned}$$

**Question 7:**

Find an approximation of  $(0.99)^5$  using the first three terms of its expansion.

**Answer 7:**

$$0.99 = 1 - 0.01$$

$$\begin{aligned}\therefore (0.99)^5 &= (1 - 0.01)^5 \\ &= {}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) + {}^5C_2(1)^3(0.01)^2 \quad \text{(Approximately)} \\ &= 1 - 5(0.01) + 10(0.01)^2 \\ &= 1 - 0.05 + 0.001 \\ &= 1.001 - 0.05 \\ &= 0.951\end{aligned}$$

Thus, the value of  $(0.99)^5$  is approximately 0.951.

**Question 8:**

Find  $n$ , if the ratio of the fifth term from the beginning to the fifth term from the end in

the expansion of  $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$  is  $\sqrt{6} : 1$

**Answer 8:**

In the expansion,  $(a + b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n$

Fifth term from the beginning =  ${}^nC_4 a^{n-4} b^4$

Fifth term from the end =  ${}^nC_{n-4} a^4 b^{n-4}$

Therefore, it is evident that in the expansion of  $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$  the fifth term from the

beginning is  ${}^nC_4 (\sqrt[4]{2})^{n-4} \left(\frac{1}{\sqrt[4]{3}}\right)^4$  and the fifth term from the end is  ${}^nC_{n-4} (\sqrt[4]{2})^4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$

$${}^nC_4 (\sqrt[4]{2})^{n-4} \left(\frac{1}{\sqrt[4]{3}}\right)^4 = {}^nC_4 \frac{(\sqrt[4]{2})^n}{(\sqrt[4]{2})^4} \cdot \frac{1}{3} = {}^nC_4 \frac{(\sqrt[4]{2})^n}{2} \cdot \frac{1}{3} = \frac{n!}{6 \cdot 4! (n-4)!} (\sqrt[4]{2})^n \quad \dots(1)$$

$${}^nC_{n-4} (\sqrt[4]{2})^4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4} = {}^nC_{n-4} \cdot 2 \cdot \frac{(\sqrt[4]{3})^4}{(\sqrt[4]{3})^n} = {}^nC_{n-4} \cdot 2 \cdot \frac{3}{(\sqrt[4]{3})^n} = \frac{6n!}{(n-4)! 4!} \cdot \frac{1}{(\sqrt[4]{3})^n} \quad \dots(2)$$

It is given that the ratio of the fifth term from the beginning to the fifth term from the end is  $\sqrt{6} : 1$ . Therefore, from (1) and (2), we obtain

$$\frac{n!}{6 \cdot 4! (n-4)!} (\sqrt[4]{2})^n : \frac{6n!}{(n-4)! 4!} \cdot \frac{1}{(\sqrt[4]{3})^n} = \sqrt{6} : 1$$

$$\Rightarrow \frac{(\sqrt[4]{2})^n}{6} : \frac{6}{(\sqrt[4]{3})^n} = \sqrt{6} : 1$$

$$\Rightarrow \frac{(\sqrt[4]{2})^n}{6} \times \frac{(\sqrt[4]{3})^n}{6} = \sqrt{6}$$

$$\Rightarrow (\sqrt[4]{6})^n = 36\sqrt{6}$$

$$\Rightarrow 6^{\frac{n}{4}} = 6^{\frac{5}{2}}$$

$$\Rightarrow \frac{n}{4} = \frac{5}{2}$$

$$\Rightarrow n = 4 \times \frac{5}{2} = 10$$

Thus, the value of  $n$  is 10.





**Question 9:**

Expand using Binomial Theorem  $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0$

**Answer 9:**

Using Binomial Theorem, the given expression  $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4$  can be expanded as

$$\begin{aligned} & \left[ \left(1 + \frac{x}{2}\right) - \frac{2}{x} \right]^4 \\ &= {}^4C_0 \left(1 + \frac{x}{2}\right)^4 - {}^4C_1 \left(1 + \frac{x}{2}\right)^3 \left(\frac{2}{x}\right) + {}^4C_2 \left(1 + \frac{x}{2}\right)^2 \left(\frac{2}{x}\right)^2 - {}^4C_3 \left(1 + \frac{x}{2}\right) \left(\frac{2}{x}\right)^3 + {}^4C_4 \left(\frac{2}{x}\right)^4 \\ &= \left(1 + \frac{x}{2}\right)^4 - 4 \left(1 + \frac{x}{2}\right)^3 \left(\frac{2}{x}\right) + 6 \left(1 + \frac{x}{2}\right)^2 \left(\frac{4}{x^2}\right) - 4 \left(1 + \frac{x}{2}\right) \left(\frac{8}{x^3}\right) + \frac{16}{x^4} \\ &= \left(1 + \frac{x}{2}\right)^4 - \frac{8}{x} \left(1 + \frac{x}{2}\right)^3 + \frac{24}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} - \frac{16}{x^2} + \frac{16}{x^4} \\ &= \left(1 + \frac{x}{2}\right)^4 - \frac{8}{x} \left(1 + \frac{x}{2}\right)^3 + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \quad \dots(1) \end{aligned}$$

Again by using Binomial Theorem, we obtain

$$\begin{aligned} \left(1 + \frac{x}{2}\right)^4 &= {}^4C_0 (1)^4 + {}^4C_1 (1)^3 \left(\frac{x}{2}\right) + {}^4C_2 (1)^2 \left(\frac{x}{2}\right)^2 + {}^4C_3 (1) \left(\frac{x}{2}\right)^3 + {}^4C_4 \left(\frac{x}{2}\right)^4 \\ &= 1 + 4 \times \frac{x}{2} + 6 \times \frac{x^2}{4} + 4 \times \frac{x^3}{8} + \frac{x^4}{16} \\ &= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} \quad \dots(2) \end{aligned}$$

$$\begin{aligned} \left(1 + \frac{x}{2}\right)^3 &= {}^3C_0 (1)^3 + {}^3C_1 (1)^2 \left(\frac{x}{2}\right) + {}^3C_2 (1) \left(\frac{x}{2}\right)^2 + {}^3C_3 \left(\frac{x}{2}\right)^3 \\ &= 1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8} \quad \dots(3) \end{aligned}$$

From (1), (2), and (3), we obtain

$$\begin{aligned} & \left[ \left( 1 + \frac{x}{2} \right) - \frac{2}{x} \right]^4 \\ &= 1 + 2x + \frac{3x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - \frac{8}{x} \left( 1 + \frac{3x}{2} + \frac{3x^2}{4} + \frac{x^3}{8} \right) + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \\ &= 1 + 2x + \frac{3}{2}x^2 + \frac{x^3}{2} + \frac{x^4}{16} - \frac{8}{x} - 12 - 6x - x^2 + \frac{8}{x^2} + \frac{24}{x} + 6 - \frac{32}{x^3} + \frac{16}{x^4} \\ &= \frac{16}{x} + \frac{8}{x^2} - \frac{32}{x^3} + \frac{16}{x^4} - 4x + \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^4}{16} - 5 \end{aligned}$$

**Question 10:**

Find the expansion of  $(3x^2 - 2ax + 3a^2)^3$  using binomial theorem.

**Answer 10:**

Using Binomial Theorem, the given expression  $(3x^2 - 2ax + 3a^2)^3$  can be expanded as

$$\begin{aligned} & \left[ (3x^2 - 2ax) + 3a^2 \right]^3 \\ &= {}^3C_0 (3x^2 - 2ax)^3 + {}^3C_1 (3x^2 - 2ax)^2 (3a^2) + {}^3C_2 (3x^2 - 2ax)(3a^2)^2 + {}^3C_3 (3a^2)^3 \\ &= (3x^2 - 2ax)^3 + 3(9x^4 - 12ax^3 + 4a^2x^2)(3a^2) + 3(3x^2 - 2ax)(9a^4) + 27a^6 \\ &= (3x^2 - 2ax)^3 + 81a^2x^4 - 108a^3x^3 + 36a^4x^2 + 81a^4x^2 - 54a^5x + 27a^6 \\ &= (3x^2 - 2ax)^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6 \quad \dots(1) \end{aligned}$$

Again by using Binomial Theorem, we obtain

$$\begin{aligned} & (3x^2 - 2ax)^3 \\ &= {}^3C_0 (3x^2)^3 - {}^3C_1 (3x^2)^2 (2ax) + {}^3C_2 (3x^2)(2ax)^2 - {}^3C_3 (2ax)^3 \\ &= 27x^6 - 3(9x^4)(2ax) + 3(3x^2)(4a^2x^2) - 8a^3x^3 \\ &= 27x^6 - 54ax^5 + 36a^2x^4 - 8a^3x^3 \quad \dots(2) \end{aligned}$$

From (1) and (2), we obtain

$$\begin{aligned} & (3x^2 - 2ax + 3a^2)^3 \\ &= 27x^6 - 54ax^5 + 36a^2x^4 - 8a^3x^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6 \\ &= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6 \end{aligned}$$