

# Mathematics

## (Chapter – 2) (Relations and Functions)

(Class – XI)

### Exercise 2.1

#### Question 1:

If

, find the values of  $x$  and  $y$ .

#### Answer 1:

It is given that

$$\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right).$$

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, and

$$\therefore x = 2 \text{ and } y = 1$$

#### Question 2:

If the set  $A$  has 3 elements and the set  $B = \{3, 4, 5\}$ , then find the number of elements in  $(A \times B)$ ?

#### Answer 2:

It is given that set  $A$  has 3 elements and the elements of set  $B$  are 3, 4, and 5.

$\Rightarrow$  Number of elements in set  $B = 3$

Number of elements in  $(A \times B)$   
 $=$  (Number of elements in  $A$ )  $\times$  (Number of elements in  $B$ )  
 $= 3 \times 3 = 9$

Thus, the number of elements in  $(A \times B)$  is 9.

**Question 3:**

If  $G = \{7, 8\}$  and  $H = \{5, 4, 2\}$ , find  $G \times H$  and  $H \times G$ .

**Answer 3:**

$G = \{7, 8\}$  and  $H = \{5, 4, 2\}$

We know that the Cartesian product  $P \times Q$  of two non-empty sets  $P$  and  $Q$  is defined as  $P \times Q = \{(p, q) : p \in P, q \in Q\}$

$\therefore G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$

$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

**Question 4:**

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

- (i) If  $P = \{m, n\}$  and  $Q = \{n, m\}$ , then  $P \times Q = \{(m, n), (n, m)\}$ .
- (ii) If  $A$  and  $B$  are non-empty sets, then  $A \times B$  is a non-empty set of ordered pairs  $(x, y)$  such that  $x \in A$  and  $y \in B$ .

(iii) If  $A = \{1, 2\}$ ,  $B = \{3, 4\}$ , then  $A \times (B \cap \Phi) = \Phi$ .

**Answer 4:**

- (i) False  
 If  $P = \{m, n\}$  and  $Q = \{n, m\}$ , then  
 $P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$
- (ii) True
- (iii) True

**Question 5:**

If  $A = \{-1, 1\}$ , find  $A \times A \times A$ .

**Answer 5:**

It is known that for any non-empty set  $A$ ,  $A \times A \times A$  is defined as  
 $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$

It is given that  $A = \{-1, 1\}$

$$\therefore A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$$

**Question 6:**

If  $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$ . Find  $A$  and  $B$ .

**Answer 6:**

It is given that  $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets  $P$  and  $Q$  is defined as  $P \times Q = \{(p, q) : p \in P, q \in Q\}$

$\therefore A$  is the set of all first elements and  $B$  is the set of all second elements.

Thus,  $A = \{a, b\}$  and  $B = \{x, y\}$

**Question 7:**

Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$ . Verify that

- (i)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- (ii)  $A \times C$  is a subset of  $B \times D$

**Answer 7:**

(i) To verify:  $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have  $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

$\therefore$  L.H.S. =  $A \times (B \cap C) = A \times \Phi = \Phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$$\therefore \text{R.H.S.} = (A \times B) \cap (A \times C) = \Phi$$

$$\therefore \text{L.H.S.} = \text{R.H.S}$$

$$\text{Hence, } A \times (B \cap C) = (A \times B) \cap (A \times C)$$

**(ii)** To verify:  $A \times C$  is a subset of  $B \times D$

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$A \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), \\ (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

We can observe that all the elements of set  $A \times C$  are the elements of set  $B \times D$ . Therefore,  $A \times C$  is a subset of  $B \times D$ .

**Question 8:**

Let  $A = \{1, 2\}$  and  $B = \{3, 4\}$ . Write  $A \times B$ . How many subsets will  $A \times B$  have? List them.

**Answer 8:**

$$A = \{1, 2\} \text{ and } B = \{3, 4\}$$

$$\therefore A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$\Rightarrow n(A \times B) = 4$$

We know that if  $C$  is a set with  $n(C) = m$ , then  $n[P(C)] = 2^m$ .

Therefore, the set  $A \times B$  has  $2^4 = 16$  subsets. These are

$$\Phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\}, \{(1, 3), (1, 4)\}, \{(1, 3), (2, 3)\}, \\ \{(1, 3), (2, 4)\}, \{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3), (2, 4)\}, \\ \{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4), (2, 4)\}, \{(1, 3), (2, 3), \\ (2, 4)\}, \{(1, 4), (2, 3), (2, 4)\}, \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

**Question 9:**

Let A and B be two sets such that  $n(A) = 3$  and  $n(B) = 2$ . If  $(x, 1)$ ,  $(y, 2)$ ,  $(z, 1)$  are in  $A \times B$ , find A and B, where x, y and z are distinct elements.

**Answer 9:**

It is given that  $n(A) = 3$  and  $n(B) = 2$ ; and  $(x, 1)$ ,  $(y, 2)$ ,  $(z, 1)$  are in  $A \times B$ .

We know that

A = Set of first elements of the ordered pair elements of  $A \times B$

B = Set of second elements of the ordered pair elements of  $A \times B$ .

$\therefore x, y,$  and  $z$  are the elements of A; and 1 and 2 are the elements of B.

Since  $n(A) = 3$  and  $n(B) = 2$ ,

it is clear that  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ .

**Question 10:**

The Cartesian product  $A \times A$  has 9 elements among which are found  $(-1, 0)$  and  $(0, 1)$ . Find the set A and the remaining elements of  $A \times A$ .

**Answer 10:**

We know that if  $n(A) = p$  and  $n(B) = q$ , then  $n(A \times B) = pq$ .

$$\therefore n(A \times A) = n(A) \times n(A)$$

It is given that  $n(A \times A) = 9$

$$\therefore n(A) \times n(A) = 9$$

$$\Rightarrow n(A) = 3$$

The ordered pairs  $(-1, 0)$  and  $(0, 1)$  are two of the nine elements of  $A \times A$ .

We know that  $A \times A = \{(a, a) : a \in A\}$ . Therefore,  $-1, 0,$  and  $1$  are elements of A.

Since  $n(A) = 3$ , it is clear that  $A = \{-1, 0, 1\}$ .

The remaining elements of set  $A \times A$  are  $(-1, -1)$ ,  $(-1, 1)$ ,  $(0, -1)$ ,  $(0, 0)$ ,  $(1, -1)$ ,  $(1, 0)$ , and  $(1, 1)$ .

# Mathematics

## (Chapter – 2) (Relations and Functions)

(Class – XI)

### Exercise 2.2

**Question 1:**

Let  $A = \{1, 2, 3, \dots, 14\}$ . Define a relation  $R$  from  $A$  to  $A$  by  $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$ . Write down its domain, codomain and range.

**Answer 1:**

The relation  $R$  from  $A$  to  $A$  is given as  $R = \{(x, y) : 3x - y = 0, \text{ where } x, y \in A\}$

i.e.,  $R = \{(x, y) : 3x = y, \text{ where } x, y \in A\}$   
 $\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

The domain of  $R$  is the set of all first elements of the ordered pairs in the relation.

$\therefore$  Domain of  $R = \{1, 2, 3, 4\}$

The whole set  $A$  is the codomain of the relation  $R$ .

$\therefore$  Codomain of  $R = A = \{1, 2, 3, \dots, 14\}$

The range of  $R$  is the set of all second elements of the ordered pairs in the relation.

$\therefore$  Range of  $R = \{3, 6, 9, 12\}$

**Question 2:**

Define a relation  $R$  on the set  $\mathbf{N}$  of natural numbers by  $R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4; x, y \in \mathbf{N}\}$ . Depict this relationship using roster form. Write down the domain and the range.

**Answer 2:**

$R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbf{N}\}$

The natural numbers less than 4 are 1, 2, and 3.

$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$

The domain of  $R$  is the set of all first elements of the ordered pairs in the relation.

$\therefore$  Domain of  $R = \{1, 2, 3\}$

The range of  $R$  is the set of all second elements of the ordered pairs in the relation.  $\therefore$  Range of  $R = \{6, 7, 8\}$

**Question 3:**

$A = \{1, 2, 3, 5\}$  and  $B = \{4, 6, 9\}$ . Define a relation  $R$  from  $A$  to  $B$  by  $R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$ . Write  $R$  in roster form.

**Answer 3:**

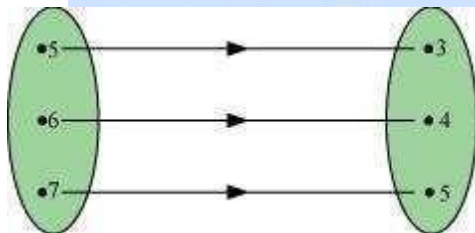
$A = \{1, 2, 3, 5\}$  and  $B = \{4, 6, 9\}$

$R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$

$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

**Question 4:**

The given figure shows a relationship between the sets  $P$  and  $Q$ . write this relation **(i)** in set-builder form **(ii)** in roster form. What is its domain and range?



**Answer 4:**

According to the given figure,  $P = \{5, 6, 7\}$ ,  $Q = \{3, 4, 5\}$

**(i)**  $R = \{(x, y): y = x - 2; x \in P\}$  or  $R = \{(x, y): y = x - 2 \text{ for } x = 5, 6, 7\}$

**(ii)**  $R = \{(5, 3), (6, 4), (7, 5)\}$

Domain of  $R = \{5, 6, 7\}$

Range of  $R = \{3, 4, 5\}$

n)

**Question 5:**

Let  $A = \{1, 2, 3, 4, 6\}$ . Let  $R$  be the relation on  $A$  defined by  $\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$ .

- (i) Write  $R$  in roster form
- (ii) Find the domain of  $R$
- (iii) Find the range of  $R$ .

**Answer 5:**

$A = \{1, 2, 3, 4, 6\}$ ,  $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

- (i)  $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$
- (ii) Domain of  $R = \{1, 2, 3, 4, 6\}$
- (iii) Range of  $R = \{1, 2, 3, 4, 6\}$

**Question 6:**

Determine the domain and range of the relation  $R$  defined by  $R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$ .

**Answer 6:**

$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$   
 $\therefore R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$   
 $\therefore$  Domain of  $R = \{0, 1, 2, 3, 4, 5\}$   
 Range of  $R = \{5, 6, 7, 8, 9, 10\}$

**Question 7:**

Write the relation  $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$  in roster form.

**Answer 7:**

$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$  The prime numbers less than 10 are 2, 3, 5, and 7.

$\therefore R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$



**Question 8:**

Let  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ . Find the number of relations from A to B.

**Answer 8:**

It is given that  $A = \{x, y, z\}$  and  $B = \{1, 2\}$ .

$$\therefore A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$$

Since  $n(A \times B) = 6$ , the number of subsets of  $A \times B$  is  $2^6$ .

Therefore, the number of relations from A to B is  $2^6$ .

**Question 9:**

Let R be the relation on  $\mathbf{Z}$  defined by  $R = \{(a, b) : a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$ . Find the domain and range of R.

**Answer 9:**

$$R = \{(a, b) : a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$$

It is known that the difference between any two integers is always an integer.

$$\therefore \text{Domain of } R = \mathbf{Z}$$

$$\text{Range of } R = \mathbf{Z}$$

# Mathematics

## (Chapter – 2) (Relations and Functions)

(Class – XI)

### Exercise 2.3

#### Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

- (i)  $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$   
 (ii)  $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$   
 (iii)  $\{(1, 3), (1, 5), (2, 5)\}$

#### Answer 1:

- (i)  $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function. Here, domain =  $\{2, 5, 8, 11, 14, 17\}$  and range =  $\{1\}$

- (ii)  $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain =  $\{2, 4, 6, 8, 10, 12, 14\}$  and range =  $\{1, 2, 3, 4, 5, 6, 7\}$

- (iii)  $\{(1, 3), (1, 5), (2, 5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

#### Question 2:

Find the domain and range of the following real function:

- (i)  $f(x) = -|x|$   
 (ii)  $f(x) = \sqrt{9 - x^2}$

#### Answer 2:

- (i)  $f(x) = -|x|, x \in \mathbf{R}$

We know that  $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & \text{if } x \geq 0 \\ x, & \text{if } x < 0 \end{cases}$$

Since  $f(x)$  is defined for  $x \in \mathbf{R}$ , the domain of  $f$  is  $\mathbf{R}$ .

It can be observed that the range of  $f(x) = -|x|$  is all real numbers except positive real numbers.

$\therefore$  The range of  $f$  is  $(-\infty, 0]$ .

(ii)  $f(x) = \sqrt{9 - x^2}$

Since  $\sqrt{9 - x^2}$  is defined for all real numbers that are greater than or equal to  $-3$  and less than or equal to  $3$ , the domain of  $f(x)$  is  $\{x : -3 \leq x \leq 3\}$  or  $[-3, 3]$ .

For any value of  $x$  such that  $-3 \leq x \leq 3$ , the value of  $f(x)$  will lie between  $0$  and  $3$ .  $\therefore$  The range of  $f(x)$  is  $\{x : 0 \leq x \leq 3\}$  or  $[0, 3]$ .

**Question 3:**

A function  $f$  is defined by  $f(x) = 2x - 5$ . Write down the values of

- (i)  $f(0)$ ,
- (ii)  $f(7)$ ,
- (iii)  $f(-3)$

**Answer 3:**

The given function is  $f(x) = 2x - 5$ .

Therefore,

- (i)  $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$
- (ii)  $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$
- (iii)  $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

**Question 4:**

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by  $f(C) = \frac{9C}{5} + 32$ . Find

- (i)  $t(0)$                                       (ii)  $t(28)$                                       (iii)  $t(-10)$
- (iv) The value of  $C$ , when  $t(C) = 212$

**Answer 4:**

The given function is  $f(C) = \frac{9C}{5} + 32$ .

Therefore,

$$(i) \quad t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$$

$$(ii) \quad t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$$

$$(iii) \quad t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$$

(iv) It is given that  $t(C) = 212$

$$\therefore 212 = \frac{9C}{5} + 32$$

$$\Rightarrow \frac{9C}{5} = 212 - 32$$

$$\Rightarrow \frac{9C}{5} = 180$$

$$\Rightarrow 9C = 180 \times 5$$

$$\Rightarrow C = \frac{180 \times 5}{9} = 100$$

Thus, the value of  $t$ , when  $t(C) = 212$ , is 100.

### Question 5:

Find the range of each of the following functions.

(i)  $f(x) = 2 - 3x$ ,  $x \in \mathbf{R}$ ,  $x > 0$ .

(ii)  $f(x) = x^2 + 2$ ,  $x$ , is a real number.

(iii)  $f(x) = x$ ,  $x$  is a real number

### Answer 5:

(i)  $f(x) = 2 - 3x$ ,  $x \in \mathbf{R}$ ,  $x > 0$

The values of  $f(x)$  for various values of real numbers  $x > 0$  can be written in the tabular form as

<b>x</b>	<b>0.01</b>	<b>0.1</b>	<b>0.9</b>	<b>1</b>	<b>2</b>	<b>2.5</b>	<b>4</b>	<b>5</b>	<b>...</b>
<b>f(x)</b>	1.97	1.7	- 0.7	- 1	- 4	- 5.5	- 10	- 13	...

Thus, it can be clearly observed that the range of  $f$  is the set of all real numbers less than 2.

i.e., range of  $f = (-\infty, 2)$

**Alter:**

Let  $x > 0$

$$\Rightarrow 3x > 0$$

$$\Rightarrow 2 - 3x < 2$$

$$\Rightarrow f(x) < 2$$

$\therefore$  Range of  $f = (-\infty, 2)$

(ii)  $f(x) = x^2 + 2$ ,  $x$ , is a real number

The values of  $f(x)$  for various values of real numbers  $x$  can be written in the tabular form as

<b>x</b>	<b>0</b>	<b><math>\pm 0.3</math></b>	<b><math>\pm 0.8</math></b>	<b><math>\pm 1</math></b>	<b><math>\pm 2</math></b>	<b><math>\pm 3</math></b>	<b>...</b>
<b>f(x)</b>	2	2.09	2.64	3	6	11	.....

Thus, it can be clearly observed that the range of  $f$  is the set of all real numbers greater than 2.

i.e., range of  $f = [2, \infty)$

**Alter:**

Let  $x$  be any real number. Accordingly,

$$x^2 \geq 0$$

$$\Rightarrow x^2 + 2 \geq 0 + 2$$

$$\Rightarrow x^2 + 2 \geq 2$$

$$\Rightarrow f(x) \geq 2$$

$\therefore$  Range of  $f = [2, \infty)$

(iii)  $f(x) = x$ ,  $x$  is a real number

It is clear that the range of  $f$  is the set of all real numbers.  $\therefore$  Range of  $f = \mathbf{R}$

# Mathematics

## (Chapter – 2) (Relations and Functions)

(Class – XI)

### Miscellaneous Exercise on Chapter 2

#### Question 1:

The relation  $f$  is defined by

The relation  $g$  is defined by

Show that  $f$  is a function and  $g$  is not a function.

#### Answer 1:

The relation  $f$  is defined as

It is observed that for

$$0 \leq x < 3, \quad f(x) = x^2$$

$$3 < x \leq 10, \quad f(x) = 3x$$

Also, at  $x = 3$ ,  $f(x) = 3^2 = 9$  or  $f(x) = 3 \times 3 = 9$  i.e., at  $x = 3$ ,  $f(x) = 9$

Therefore, for  $0 \leq x \leq 10$ , the images of  $f(x)$  are unique. Thus, the given relation is a function.

The relation  $g$  is defined as

It can be observed that for  $x = 2$ ,  $g(x) = 2^2 = 4$  and  $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation  $g$  corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

**Question 2:**

If  $f(x) = x^2$ , find.  $\frac{f(1.1) - f(1)}{(1.1 - 1)}$

**Answer 2:**

$$f(x) = x^2$$

$$\therefore \frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

**Question 3:**

Find the domain of the function

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$

**Answer 3:**

The given function is  $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It can be seen that function  $f$  is defined for all real numbers except at  $x = 6$  and  $x = 2$ . Hence, the domain of  $f$  is  $\mathbf{R} - \{2, 6\}$ .

**Question 4:**

Find the domain and the range of the real function  $f$  defined by

$$f(x) = \sqrt{x - 1}$$

**Answer 4:**

The given real function is  $f(x) = \sqrt{x - 1}$

It can be seen that  $\sqrt{x - 1}$  is defined for  $x \geq 1$ .

Therefore, the domain of  $f$  is the set of all real numbers greater than or equal to 1 i.e., the domain of  $f = [1, \infty)$ .

$$\text{As } x \geq 1 \Rightarrow (x - 1) \geq 0 \Rightarrow \sqrt{(x - 1)} \geq 0$$

Therefore, the range of  $f$  is the set of all real numbers greater than or equal to 0 i.e., the range of  $f = [0, \infty)$ .

**Question 5:**

Find the domain and the range of the real function  $f$  defined by  $f(x) = |x - 1|$ .

**Answer 5:**

The given real function is  $f(x) = |x - 1|$ .

It is clear that  $|x - 1|$  is defined for all real numbers.

$\therefore$  Domain of  $f = \mathbf{R}$

Also, for  $x \in \mathbf{R}$ ,  $|x - 1|$  assumes all real numbers.

Hence, the range of  $f$  is the set of all non-negative real numbers.

**Question 6:**

$$\text{Let } f = \left\{ \left( x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$$

be a function from  $\mathbf{R}$  into  $\mathbf{R}$ . Determine the range of  $f$ .

**Answer 6:**

$$\begin{aligned} f &= \left\{ \left( x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\} \\ &= \left\{ (0, 0), \left( \pm 0.5, \frac{1}{5} \right), \left( \pm 1, \frac{1}{2} \right), \left( \pm 1.5, \frac{9}{13} \right), \left( \pm 2, \frac{4}{5} \right), \left( 3, \frac{9}{10} \right), \left( 4, \frac{16}{17} \right), \dots \right\} \end{aligned}$$

The range of  $f$  is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator] Thus, range of  $f = [0, 1)$



**Question 7:**

Let  $f, g: \mathbf{R} \rightarrow \mathbf{R}$  be defined, respectively by  $f(x) = x + 1$ ,  $g(x) = 2x - 3$ . Find  $f + g$ ,  $f - g$  and  $\frac{f}{g}$ .

**Answer 7:**

$f, g: \mathbf{R} \rightarrow \mathbf{R}$  is defined as  $f(x) = x + 1$ ,  $g(x) = 2x - 3$

$$(f + g)(x) = f(x) + g(x) = (x + 1) + (2x - 3) = 3x - 2$$

$$\therefore (f + g)(x) = 3x - 2$$

$$(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = x + 1 - 2x + 3 = -x + 4$$

$$\therefore (f - g)(x) = -x + 4$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, 2x-3 \neq 0 \text{ or } 2x \neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$$

**Question 8:**

Let  $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$  be a function from  $\mathbf{Z}$  to  $\mathbf{Z}$  defined by  $f(x) = ax + b$ , for some integers  $a, b$ . Determine  $a, b$ .

**Answer 8:**

$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$  and  $f(x) = ax + b$

$$(1, 1) \in f \Rightarrow f(1) = 1 \Rightarrow a \times 1 + b = 1 \\ \Rightarrow a + b = 1$$

$$(0, -1) \in f \Rightarrow f(0) = -1 \Rightarrow a \times 0 + b = -1 \\ \Rightarrow b = -1$$

On substituting  $b = -1$  in  $a + b = 1$ ,

We obtain  $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$ . Thus, the respective values of  $a$  and  $b$  are 2 and -1.

**Question 9:**

Let  $R$  be a relation from  $\mathbf{N}$  to  $\mathbf{N}$  defined by  $R = \{(a, b) : a, b \in \mathbf{N} \text{ and } a = b^2\}$ . Are the following true?

- (i)  $(a, a) \in R$ , for all  $a \in \mathbf{N}$
- (ii)  $(a, b) \in R$ , implies  $(b, a) \in R$
- (iii)  $(a, b) \in R, (b, c) \in R$  implies  $(a, c) \in R$ .

Justify your answer in each case.

**Answer 9:**

$R = \{(a, b) : a, b \in \mathbf{N} \text{ and } a = b^2\}$

(i) It can be seen that  $2 \in \mathbf{N}$ ; however,  $2 \neq 2^2 = 4$ .

Therefore, the statement " $(a, a) \in R$ , for all  $a \in \mathbf{N}$ " is not true.

(ii) It can be seen that  $(9, 3) \in R$  because  $9, 3 \in \mathbf{N}$  and  $9 = 3^2$ . Now,  $3 \neq 9^2 = 81$ ; therefore,  $(3, 9) \notin R$ .

Therefore, the statement " $(a, b) \in R$ , implies  $(b, a) \in R$ " is not true.

(iii) It can be seen that  $(9, 3) \in R, (16, 4) \in R$  because  $9, 3, 16, 4 \in \mathbf{N}$  and  $9 = 3^2$  and  $16 = 4^2$ .

Now,  $9 \neq 4^2 = 16$ ; therefore,  $(9, 4) \notin R$ .

Therefore, the statement " $(a, b) \in R, (b, c) \in R$  implies  $(a, c) \in R$ " is not true.

**Question 10:**

Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{1, 5, 9, 11, 15, 16\}$  and  $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ . Are the following true?

- (i)  $f$  is a relation from  $A$  to  $B$
- (ii)  $f$  is a function from  $A$  to  $B$ .

Justify your answer in each case.

**Answer 10:**

$A = \{1, 2, 3, 4\}$  and  $B = \{1, 5, 9, 11, 15, 16\}$

$\therefore A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$

It is given that  $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) A relation from a non-empty set  $A$  to a non-empty set  $B$  is a subset of the Cartesian product  $A \times B$ .

It is observed that  $f$  is a subset of  $A \times B$ .

Thus,  $f$  is a relation from  $A$  to  $B$ .

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation  $f$  is not a function.

**Question 11:**

Let  $f$  be the subset of  $\mathbf{Z} \times \mathbf{Z}$  defined by  $f = \{(ab, a + b) : a, b \in \mathbf{Z}\}$ . Is  $f$  a function from  $\mathbf{Z}$  to  $\mathbf{Z}$ : justify your answer.

**Answer 11:**

The relation  $f$  is defined as  $f = \{(ab, a + b) : a, b \in \mathbf{Z}\}$

We know that a relation  $f$  from a set  $A$  to a set  $B$  is said to be a function if every element of set  $A$  has unique images in set  $B$ .

Since  $2, 6, -2, -6 \in \mathbf{Z}$ ,  $(2 \times 6, 2 + 6)$ ,  $(-2 \times -6, -2 + (-6)) \in f$  i.e.,  $(12, 8)$ ,  $(12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation  $f$  is not a function.

**Question 12:**

Let  $A = \{9, 10, 11, 12, 13\}$  and let  $f: A \rightarrow \mathbf{N}$  be defined by  $f(n) =$  the highest prime factor of  $n$ . Find the range of  $f$ .

**Answer 12:**

$A = \{9, 10, 11, 12, 13\}$   $f: A \rightarrow \mathbf{N}$  is defined as  $f(n) =$  The highest prime factor of  $n$

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

$\therefore f(9) =$  The highest prime factor of 9 = 3

$f(10) =$  The highest prime factor of 10 = 5

$f(11) =$  The highest prime factor of 11 = 11

$f(12) =$  The highest prime factor of 12 = 3

$f(13) =$  The highest prime factor of 13 = 13

The range of  $f$  is the set of all  $f(n)$ , where  $n \in A$ .

$\therefore$  Range of  $f = \{3, 5, 11, 13\}$

