CBSE Test Paper 02

Chapter 9 Differential Equations

1. Forming a differential equation representing the given family of curves by eliminating arbitrary constants a and b from $\frac{x}{a} + \frac{y}{b} = 1$ yields the differential equation.
   a. y'' = 0
   b. y'' = y
   c. $y'' = y^3$
   d. y'' = 2y

2. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that y = 0 and x = 0.
   a. $4e^{3x} + 3e^{-4y} + 7 = 0$
   b. $4e^{3x} - 3e^{-4y} - 7 = 0$
   c. $4e^{3x} + 3e^{-4y} - 7 = 1$
   d. $4e^{3x} + 3e^{-4y} - 7 = 0$

3. Order of a differential equation is defined as
   a. the number of derivative terms
   b. the order of the lowest order derivative of the dependent variable
   c. the order of the highest order derivative of the dependent variable
   d. the number of constant terms

4. A function f(x,y) is said to be homogenous function of degree n if
   a. $f(\lambda x, \lambda y) = \lambda^3 f(x, y)$
   b. None of these
   c. $f(\lambda x, \lambda y) = \lambda^n f(x, y)$
   d. $f(\lambda x, y) = \lambda^n f(x, y)$

5. Determine order and degree (if defined) of $\frac{d^4y}{dx^4}$ +sin(y’’’) = 0.

a. 2, degree undefined
b. 1, degree undefined
c. 4, degree undefined
d. 3, degree undefined

6. The degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 = 0$ is ________.
7. The differential equation representing the family of curves y = A sinx + B cosx is ________.
8. The solution of the differential equation $\frac{xdy}{dx} + 2y = x^2$ is ________.
9. Verify that the function is a solution of the corresponding differential equation. $y = x^2 + 2x + c$; $y^1 - 2x - 2 = 0$.
10. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.
11. Write the degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 3x\frac{d^2y}{dx^2} = 0$.
12. Find the differential equation of all non-vertical lines in a plane.
13. Solve the diff. equ. $\sec^2 x . \tan y \, dx + \sec^2 y \tan x \, dy = 0$.
14. Find the general solution of $\frac{dy}{dx} + y = 1 \, (y \neq 1)$.
15. Solve the following differential equation.
$(x \log|x|)\frac{dy}{dx} + y = 2\log|x|$
16. Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose diff eq. is sinx cosy dx + cosx.siny dy = 0.
17. Solve the following differential equation $\frac{dy}{dx} + y\sec x = \tan x, \left(0 \leq x < \frac{\pi}{2}\right)$.
18. Solve the diff. eq. $\frac{dy}{dx} + 2y\tan x = \sin x$.

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Solution

1. a. y'' = 0, **Explanation:** $\frac{1}{a}+\frac{1}{b}\frac{\mathrm{dy}}{\mathrm{d}x}=0$
   $\frac{dy}{dx}=\frac{-a}{b}$
   $\frac{d^2y}{dx^2}=0$
2. d. $4e^{3x}+3e^{-4y}-7=0$, **Explanation:** $\frac{dy}{dx}=e^{3x}e^{4y}$
   $\int e^{-4y}dy=\int e^{3x}dx$
   $\frac{-e^{-4y}}{4}=\frac{e^{3x}}{3}+c$
   here x = y =0 gives
   $\frac{-1}{4}=\frac{1}{3}+c$
   $\therefore c=\frac{-7}{12}$
   $\therefore\frac{-e^{-4y}}{4}=\frac{e^{3x}}{3}-\frac{7}{12}$
   $4e^{3x}+3e^{-4y}-7=0$
3. c. the order of the highest order derivative of the dependent variable
   Explanation: Order of a differential equation is defined as the order of the highest order derivative of the dependent variable present in the differential equation.
4. c. $f(\lambda x,\lambda y)=\lambda^n f(x,y)$, **Explanation:** A function is homogenous if we can write in the form of $f(\lambda x,\lambda y)=\lambda^n f(x,y)$ where n is an whole number.
5. c. 4, degree undefined, **Explanation:** Order = 4, degree not defined, because the function y’’’ present in the angle of sine function.
6. Two
7. $\frac{d^2y}{dx^2}+y=0$
8. $y=\frac{x^2}{4}+cx^{-2}$
9. y = x^2 + 2x+ c

   y^1 = 2x + 2

   y^1 - 2x - 2 = 0 Proved
10. Given differential equation is $\frac{dy}{dx}=x^3e^{-2y}$

On separating the variables, we get $e^{2y}dy = x^3dx$

On integrating both sides, we get

$\int e^{2y} dy = \int x^3 dx$

$\Rightarrow \quad \frac{e^{2y}}{2} = \frac{x^4}{4} + C_1$

$\Rightarrow 2e^{2y} = x^4 + 4C_1$

$\therefore \ 2e^{2y} = x^4 + C$, where $C = 4 C_1$

11. According to the question,the given equation is,

$\left(\frac{dy}{dx}\right)^4 + 3x\frac{d^2y}{dx^2} = 0.$

Here, the highest order derivative is d^2y/dx^2, whose degree is one. So, the degree of differential equation is 1.

12. Since, the family of all non-vertical line is y = mx + c, where $m \neq \tan\frac{\pi}{2}$.

On differentiating w.r.t. x, we get $\frac{dy}{dx} = m$

again, differentiating w.r.t x, we get $\frac{d^2y}{dx^2} = 0$

13. $\sec^2x. \tan y\ dx = -\sec^2y \tan x\ dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dx$

$\log(\tan x) = -\log(\tan y) + logc$

$\log(\tan x . \tan y) = \log c$a

$\tan x \ \ \tan y = c$

14. Given: Differential equation $\frac{dy}{dx} + y = 1$

$\Rightarrow \frac{dy}{dx} = 1 - y$

$\Rightarrow dy = (1 - y)\, dx$

$\Rightarrow dy = -\,(y - 1)\, dx$

$\Rightarrow \frac{dy}{y-1} = -dx$

Integrating both sides,

$\Rightarrow \int \frac{dy}{y-1} dx = -\int 1 dx$

$\Rightarrow \log|y - 1| = -x + c$

$\Rightarrow |y - 1| = e^{-x+c}$ [$\because$ if log x = t, then $x = e^x$]

$\Rightarrow y - 1 = \pm e^{-x+c}$

$\Rightarrow y = 1 \pm e^{-x}e^c$

$\Rightarrow y = 1 \pm e^c e^{-x}$

$\Rightarrow y = 1 + Ae^{-x}$, where $A = \pm e^c$

15. We have,

$x \log |x| \frac{dy}{dx} + y = 2 \log |x|$

On dividing both sides by x log IxI, we get

$\frac{dy}{dx} + \frac{y}{x \log |x|} = \frac{2}{x}$

which is a linear differential equation which is in the form of $\frac{dy}{dx} + Py = Q$,

Where, $P = \frac{1}{x \log |x|}$ and $Q = \frac{2}{x}$

we know that ,

$\text{IF} = e^{\int P dx} = e^{\int \frac{1}{x \log |x|}}$

put $\log |x| = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore \int \frac{1}{x \log |x|} dx = \int \frac{dt}{t} = \log |t| = \log |\log x|$

IF = log |x| [$\because e^{\log |x|} = x$]

The solution of linear differential equation is given by

$y \times \text{IF} = \int (Q \times \text{IF}) dx + C$

$\therefore \quad y \times \log |x| = \int \frac{2}{x} \log |x| dx + C$

put $\log |x| = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore \int \frac{\log |x|}{x} dx = \int t dt = \frac{t^2}{2} = \frac{(\log |x|)^2}{2}$

$\Rightarrow \quad y \log |x| = \frac{2(\log |x|)^2}{2} + C$

On dividing both sides by log |x| , we get

$\therefore \quad y = \log |x| + \frac{C}{\log |x|}$

which is the required solution of differential equation.

16. Given diff eq. is sinx cosy dx + cosx.siny dy = 0.

sinx cosy dx = - cosx.siny dy

$\Rightarrow \int \frac{sinx}{cosx} dx = -\int \frac{siny}{cosy} dy$

$\Rightarrow \int \tan x dx = -\int \tan y dy$

$\Rightarrow$ log(sec x) = -log(sec y) + log c

$\Rightarrow$ log(sec x.sec y) = log c

secx.secy = c ....(1)

when x = 0, $y = \frac{x}{4}$, therefore we get, $c = \sqrt{2}$

put the value of c in (1), we get, $\sec x . \sec y = \sqrt{2}$

17. We have,

$\frac{dy}{dx} + y \sec x = \tan x$

which is a linear differential equation of first order and is of the form

$\frac{dy}{dx} + Py = Q$ ...(i)

Here, P = sec x and Q = tan x

$\therefore \quad \text{IF} = \text{e}^{\int Pdx} = \text{e}^{\int \sec x dx} = e^{\log|\sec x + \tan x|}$

$[\because \int \sec x dx = \log|\sec x + \tan x|]$

$\Rightarrow$ IF = sec x + tan x

The general solution is $y \times \text{IF} = \int (Q \times \text{IF}) dx + C$

$y(\sec x + \tan x) = \int \tan x \cdot (\sec x + \tan x) dx$

$\Rightarrow y(\sec x + \tan x) = \int \sec x \tan x dx + \int \tan^2 x dx$

$\Rightarrow y(\sec x + \tan x) = \sec x + \int (\sec^2 x - 1)\, dx$

$\Rightarrow$ y(sec x + tan x) = sec x + tan x - x + C $\left[\because \int \sec^2 x dx = \tan x\right]$

On dividing both sides by (sec x + tan x), we get

$y = 1 - \frac{x}{\sec x + \tan x} + \frac{C}{\sec x + \tan x}$

18. $\frac{dy}{dx} + 2y \tan x = \sin x$

Given diff eq is the form of

$\frac{dy}{dx} + Py = Q$

P = 2 tan x,Q = sin x

$I.F = e^{\int Pdx}$

$= e^{\int 2 \tan x dx}$

$= e^{2 \log \sec x}$

$= e^{\log \sec^2 x}$

$= \sec^2 x$

Solution is,

$y \times \sec^2 x = \int \sin x \sec^2 x dx + c$

$= \int \sec x . \tan x dx + c$

$y \times \sec^2 x = \sec x + c$

$y = \frac{\sec x + c}{\sec^2 x}$

$y = \frac{1}{\sec x} + \frac{c}{\sec^2 x}$

$y = \cos x + c . \cos^2 x$