

CBSE Test Paper 02
Chapter 8 Application of Integrals

1. The area bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ is
 - a. $\frac{5\pi-5}{4}$ sq. units
 - b. $\frac{5\pi+2}{4}$ sq. units
 - c. $\frac{5\pi-2}{4}$ sq. units
 - d. $\frac{\pi-5}{4}$ sq. units
2. The area of the loop between the curve $y=a \sin x$ and the x – axis and $x= 0, x=\pi$ is
 - a. $3a$
 - b. $2a$ square units
 - c. none of these
 - d. a
3. The area bounded by the curves $x^2 = 4y$ and $x = 4y - 2$ is
 - a. $\frac{9}{2}$ sq. units
 - b. $\frac{9}{8}$ sq. units
 - c. $\frac{9}{4}$ sq. units
 - d. 9 sq. units
4. The area bounded by the ellipse $x^2 + 9y^2 = 9$ and the straight line $x + 3y = 3$ is
 - a. 4π
 - b. $\Rightarrow \frac{3}{4}(\pi - 2)$
 - c. 6π
 - d. 9π
5. The area lying in the first quadrant and bounded by the curve $y = x^3$, the x - axis and the ordinates at $x = - 2$ and $x = 1$ is
 - a. 2
 - b. $\frac{15}{4}$

- c. 3
d. 6
6. Find the area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis.
7. Find the value of c for which the area of figure bounded by the curve $y = 3$, the straight lines $x=1$ and $x=c$ and the x-axis is equal to $\frac{16}{3}$.
8. Find the area of the region enclosed by the lines $y=x$, $x=e$, and the curve $y = \frac{1}{x}$ and the positive x-axis.
9. Integrate the following function $\frac{1}{\sqrt{(x-1)(x-2)}}.$
10. Integrate the function $\frac{(1+\log x)^2}{x}.$
11. Integrate the functions $(4x + 2) \sqrt{x^2 + x + 1}.$
12. $\int_0^{\pi/4} \log(1 + \tan x) dx.$
13. Find $\int \frac{x^3}{x^4+3x^2+2} dx.$
14. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}.$
15. Using integration, find the area of the region $\{(x, y): x^2 + y^2 \leq 16, x^2 \leq 6y\}.$

CBSE Test Paper 02
Chapter 8 Application of Integrals

Solution

1. (c) $\frac{5\pi-2}{4}$ sq. units

Explanation: Required area:

$$\begin{aligned} & \int_{-1}^2 \sqrt{5-x^2} dx - \int_{-1}^1 (1-x) dx - \int_1^2 (x-1) dx \\ &= \left[\frac{x\sqrt{5-x^2}}{2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 \\ &\quad - \left[x - \frac{x^2}{2} \right]_{-1}^1 - \left[\frac{x^2}{2} - x \right]_1^2 \\ &= \left(\frac{5\pi-2}{4} \right) \text{sq. units} \end{aligned}$$

2. (b) 2a square units

Explanation: Required area = $\int_0^\pi a \sin x dx$

$$\begin{aligned} &= a[-\cos x]_0^\pi \\ &= a(-\cos \pi + \cos 0) = a(1+1) = 2a \end{aligned}$$

3. (b) $\frac{9}{8}$ sq. units

Explanation: Eliminating y, we get :

$$x^2 - x - 2 = 0 \Rightarrow x = -1, 2$$

$$\begin{aligned} \text{Required area} &:= \int_{-1}^2 \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4} \right) dx = \frac{1}{8}(4-1) + \frac{3}{2} - \frac{1}{12}(8+1) \\ &= \frac{3}{8} + \frac{3}{2} - \frac{3}{4} = \frac{9}{8} \text{sq. units} \end{aligned}$$

4. (b) $\frac{3}{4}(\pi - 2)$

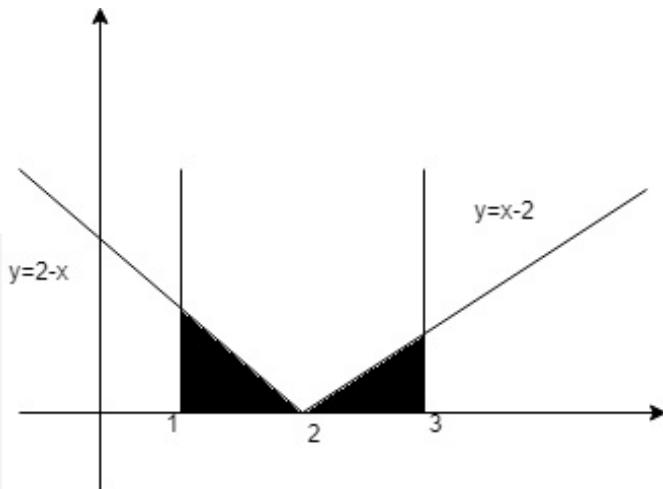
$$\begin{aligned} \text{Explanation: Required area} &:= \int_0^3 \left(\frac{1}{3} \sqrt{3^2 - x^2} - \frac{1}{3}(3-x) \right) dx \\ &= \frac{1}{3} \left[\frac{x\sqrt{3^2-x^2}}{2} + \frac{3^2}{2} \sin^{-1} \frac{x}{3} - 3x + \frac{x^2}{2} \right]_0^3 \\ &= \frac{1}{3} \left[0 + \frac{3^2}{2} \sin^{-1} 1 - 3^2 + \frac{3^2}{2} \right] = \frac{1}{3} \left[0 + \frac{3^2}{2} \sin^{-1} 0 - 0 + 0 \right] \\ &= \frac{1}{3} \left[\frac{3^2}{2} \cdot \frac{\pi}{2} - \frac{3^2}{2} - 0 \right] = 3/4(\pi - 2) \end{aligned}$$

5. (b) $\frac{15}{4}$

Explanation: Required area = $\int_{-2}^1 x^3 dx = \left[\frac{x^4}{4} \right]_{-2}^1 = \frac{15}{4}$

6. We have to find the area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis.

$$\begin{aligned}\text{Required area} &= \int_1^2 (2 - x) dx + \int_2^3 (x - 2) dx \\ &= [2x - x^2]_1^2 + [x^2 - 2x]_2^3 \\ &= [(4 - 4) - (2 - 1)] + [(9 - 6) - (4 - 4)] = -1 + 3 = 2\end{aligned}$$



7. we have, $\int_0^c 3dx = \frac{16}{3}$

$$3(x)_0^c = \frac{16}{3}$$

$$3c = \frac{16}{3}$$

$$c = \frac{16}{9}$$

8. Required area = the area of the region enclosed by the lines $y=x$, $x=e$, and the curve

$y = \frac{1}{x}$ and the positive x-axis

$$= \int_0^1 x dx + \int_1^e \frac{1}{x} dx$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2} \text{ sq units}$$

9. $\int \frac{1}{\sqrt{(x-1)(x-2)}} dx$

$$= \int \frac{1}{\sqrt{x^2 - 2x - x + 2}} dx$$

$$= \int \frac{1}{\sqrt{x^2 - 3x + 2}} dx$$

$$= \int \frac{1}{\sqrt{x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2}} dx$$

$$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

$$= \log \left| \left(x - \frac{3}{2} \right) + \sqrt{\left(x - \frac{3}{2} \right)^2 - \left(\frac{1}{2} \right)^2} \right| + c$$

$$= \log \left| \left(x - \frac{3}{2} \right) + \sqrt{x^2 - 3x + 2} \right| + c$$

10. Let $I = \int \frac{(1+\log x)^2}{x} dx \dots(i)$

Putting $1 + \log x = t$

$$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dx}{x} = dt$$

\therefore From eq. (i), $I = \int t^2 dt$

$$= \frac{t^3}{3} + c$$

$$= \frac{1}{3}(1 + \log x)^3 + c$$

11. Let $I = \int (4x+2) \sqrt{x^2+x+1} dx$

$$= \int 2(2x+1) \sqrt{x^2+x+1} dx$$

$$= \int 2\sqrt{x^2+x+1} (2x+1) dx \dots(i)$$

Putting $x^2 + x + 1 = t$

$$\Rightarrow (2x+1) = \frac{dt}{dx}$$

$$\Rightarrow (2x+1) dx = dt$$

\therefore From eq. (i), $I = \int 2\sqrt{t} dt$

$$= 2 \int t^{\frac{1}{2}} dt$$

$$= 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{4}{3} t^{\frac{3}{2}} + c$$

$$= \frac{4}{3} (x^2 + x + 1)^{\frac{3}{2}} + c$$

12. $I = \int_0^{\pi/4} \log(1 + \tan x) dx \dots(1)$

$$I = \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx$$

$$= \int_0^{\pi/4} \log \left[1 + \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \cdot \tan x} \right] dx$$

$$= \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx$$

$$= \int_0^{\pi/4} \log \left[\frac{1 + \tan x + 1 - \tan x}{1 + \tan x} \right] dx$$

$$= \int_0^{\pi/4} \log \left[\frac{2}{1 + \tan x} \right] dx$$

$$= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx$$

$$= \log 2 [x]_0^{\pi/4} - I$$

$$2I = \log 2 \left[\frac{\pi}{4} - 0 \right]$$

$$I = \log 2 \cdot \frac{\pi}{8}$$

13. According to the question, $I = \int \frac{x^3}{x^4 + 3x^2 + 2} dx$

$$I = \int \frac{x^2 \cdot x}{x^4 + 3x^2 + 2} dx$$

$$\text{Let } x^2 = t \Rightarrow 2xdx = dt$$

$$\Rightarrow xdx = \frac{dt}{2}$$

$$\therefore I = \frac{1}{2} \int \frac{t}{t^2 + 3t + 2} dt$$

$$= \frac{1}{2} \int \frac{t}{(t+2)(t+1)} dt$$

By using partial fractions,

$$\frac{t}{(t+2)(t+1)} = \frac{A}{t+2} + \frac{B}{t+1}$$

$$I = \frac{1}{2} \int \frac{t}{(t+2)(t+1)} dt = \frac{1}{2} \int \frac{A}{t+2} + \frac{B}{t+1} dt \dots (\text{i})$$

$$t = A(t+1) + B(t+2)$$

$$\text{if } t = -2 \Rightarrow -2 = A(-1), \therefore A = 2$$

$$\text{if } t = -1 \Rightarrow -1 = B(1), \therefore B = -1$$

put values of A and B in (i)

$$\begin{aligned} I &= \frac{1}{2} \left[\int \frac{2}{t+2} dt - \int \frac{1}{t+1} dt \right] \\ &= \frac{1}{2} [2 \log |t+2| - \log |t+1|] + C \\ &= \log |t+2| - \frac{1}{2} \log |t+1| + C \\ &= \log |t+2| - \log \sqrt{t+1} + C \\ &= \log \left| \frac{t+2}{\sqrt{t+1}} \right| + C \end{aligned}$$

$$\text{put } t = x^2$$

$$I = \log \left| \frac{x^2 + 2}{\sqrt{x^2 + 1}} \right| + C$$

14. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$

$$\int \frac{dx}{\sqrt{\sin^4 x \cdot \frac{\sin(x+\alpha)}{\sin x}}}$$

$$\int \frac{dx}{\sin^2 x \sqrt{\frac{\sin(x+\alpha)}{\sin x}}} = \int \frac{\cosec^2 x dx}{\sqrt{\frac{\sin(x+\alpha)}{\sin x}}}$$

$$= \int \frac{\cosec^2 x dx}{\sqrt{\frac{\sin x \cos \alpha + \cos x \sin \alpha}{\sin x}}}$$

$$= \int \frac{\cosec^2 x dx}{\sqrt{\cos \alpha + \cot x \sin \alpha}}$$

put $\cos \alpha + \cot x \cdot \sin \alpha = t$

$$\begin{aligned}0 - \cos ec^2 x \cdot \sin \alpha dx &= dt \\&= \int -\frac{1}{\sin \alpha} \frac{dt}{\sqrt{t}} = -\frac{1}{\sin \alpha} \cdot \frac{t^{1/2}}{1/2} + C \\&= \frac{-2}{\sin \alpha} \sqrt{\cos \alpha + \cot x \cdot \sin \alpha} + C\end{aligned}$$

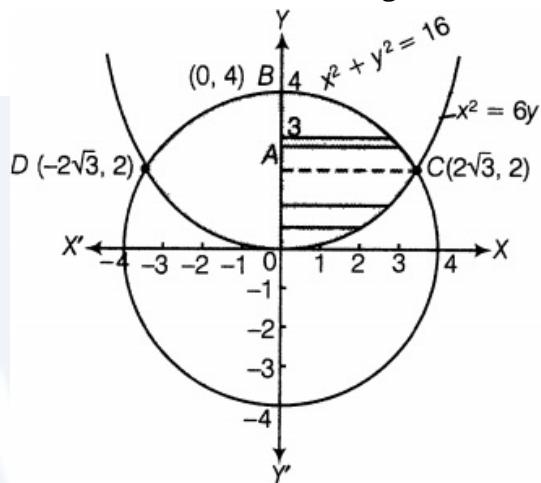
15. According to the question, given region is $(x, y) : x^2 + y^2 \leq 16, x^2 \leq 6y$

Above region consists a parabola whose vertex is $(0, 0)$ and

Axis of parabola is along Y-axis.

Above region also consists a circle $x^2 + y^2 = 16$ whose centre is $(0, 0)$ and
Radius of circle is = 4.

First, let us sketch the region, as shown below:



For finding the points of intersection of two curves, we have

$$x^2 + y^2 = 16 \dots\dots\dots (i)$$

$$\text{and } x^2 = 6y \dots\dots\dots (ii)$$

On putting $x^2 = 6y$ from Eq.(ii) in Eq. (i), we get

$$y^2 + 6y - 16 = 0$$

$$y^2 + 8y - 2y - 16 = 0$$

$$y(y + 8) - 2(y + 8) = 0$$

$$(y - 2)(y + 8) = 0$$

$$y = 2 \text{ or } -8$$

When $y = 2$, then from Eq. (ii), we get

$$x = \pm\sqrt{12} = \pm 2\sqrt{3}$$

When $y = -8$, then from Eq. (ii), we get

$x^2 = -48$ which is not possible [\because square root of negative terms does not exist.]

So, $y = -8$ is rejected.

Here, we consider only one value of y i.e. 2

Thus, the two curves meet at points $C(2\sqrt{3}, 2)$ and $D(-2\sqrt{3}, 2)$.

Now,

Required area = Area of shaded region OCBDO

$$= 2 [\text{Area of region OACO} + \text{Area of region ABCA}]$$

$$= 2 \left[\int_0^2 x_{(\text{parabola})} dy + \int_2^4 x_{(\text{circle})} dy \right]$$

$$= 2 \left[\int_0^2 \sqrt{6y} dy + \int_2^4 \sqrt{16 - y^2} dy \right]$$

$$= 2 \left[\sqrt{6} \int_0^2 \sqrt{y} dy + \int_2^4 \sqrt{16 - y^2} dy \right]$$

$$= 2 \left(\left[\sqrt{6} \cdot y^{3/2} \cdot \frac{2}{3} \right]_0^2 + \left[\frac{y}{2} \sqrt{16 - y^2} + \frac{16}{2} \sin^{-1} \frac{y}{4} \right]_2^4 \right)$$

$$= 2 \left\{ \left[\frac{2\sqrt{6}}{3} y^{3/2} \right]_0^2 + \left[0 + 8 \sin^{-1} 1 - \sqrt{12} - 8 \sin^{-1} \frac{1}{2} \right] \right\}$$

$$= 2 \left[\frac{2 \times \sqrt{2} \times \sqrt{3}}{3} \times [(\sqrt{2})^2]^{3/2} + 8 \sin^{-1} \left(\sin \frac{\pi}{2} \right) - 2\sqrt{3} - 8 \sin^{-1} \left(\sin \frac{\pi}{6} \right) \right]$$

$$= 2 \left[\frac{2\sqrt{2}\sqrt{3}}{3} \times 2\sqrt{2} + 4\pi - 2\sqrt{3} - \frac{8\pi}{6} \right]$$

$$= 2 \left[\frac{8\sqrt{3}}{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3} \right]$$

$$= 2 \left[\frac{2\sqrt{3}}{3} + \frac{8\pi}{3} \right]$$

$$= \left(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3} \right)$$

$$\therefore I = \frac{4\sqrt{3}}{3} + \frac{16\pi}{3} \text{ sq units.}$$