

## CBSE Test Paper 02

### Chapter 7 Integrals

1.  $\int_0^{\pi} \sqrt{1 + \cos x} dx$  is equal to

- a. 2
- b.  $\sqrt{2}$
- c.  $2\sqrt{2}$
- d. 1

2.  $\int \frac{x^2 - 1}{x^4 + 3x^2 + 1} dx$  is equal to

- a.  $\tan\left(x + \frac{1}{x}\right) + C$
- b.  $\tan^{-1}\left(x + \frac{1}{x}\right) + C$
- c.  $\tan^{-1}(3x^2 + 2x) + C$
- d.  $\tan^{-1}(x^2 + 1) + C$

3.  $\int \frac{\cos 4x + 1}{\cot x + \tan x} dx$  is equal to

- a.  $-\frac{1}{6}\cos^3 2x + C$
- b.  $\frac{1}{6}\cos^3 2x + C$
- c.  $-\frac{1}{6}\sin^3 2x + C$
- d.  $-\frac{1}{2}\sin^2 6x + C$

4.  $\int \frac{1}{\sqrt{1-x}} dx$  is equal to

- 1.  $-2\sqrt{1-x} + C$
- 2.  $3\sqrt{x-2} + C$
- 3.  $2\sqrt{1-x} + C$
- 4.  $\sqrt{1-x} + C$

5.  $\int (1 - \cos x) \cos ec^2 x dx$  is equal to

- a.  $\frac{1}{2}\tan x + C$
- b.  $\cot \frac{x}{2} + C$

- c.  $\tan \frac{x}{2} + C$   
d.  $3 \cot \frac{2x}{3} + C$
6. The value of integral  $\int_0^1 \frac{x dx}{\sqrt{1+x^2}}$  is \_\_\_\_\_.
7. The indefinite integral of  $2x^3 + 4$  is \_\_\_\_\_.
8. The definite integral of  $\int_1^3 (x^2 + 3x + 2) dx$  is \_\_\_\_\_.
9. Evaluate  $\int \frac{x^3 - x^2 + x - 1}{x-1} dx$ .
10. Evaluate  $\int_0^{\pi/2} \tan^2 x dx$ .
11. Write the value of  $\int \frac{\sec^2 x}{\cosec^2 x} dx$ .
12. Evaluate  $\int \frac{x^3 + x}{x^4 - 9} dx$ .
13. Integrate the function  $\tan^2(2x - 3)$ .
14. Evaluate  $\int \frac{x}{\sqrt{x+1}} dx$ . (2)
15. Integrate the function  $\sqrt{1 + 3x - x^2}$ .
16. Integrate the function  $\frac{(x-3)e^x}{(x-1)^3}$ .
17. Evaluate  $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$ .
18. Evaluate  $\int_1^3 (3x^2 + 1) dx$  by the method of limit of sum.

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**Solution**

1. c.  $2\sqrt{2}$

**Explanation:**  $= \sqrt{2} \int_0^{\pi} \cos \frac{x}{2} dx$   
 $= \sqrt{2} \left[ \frac{\sin \frac{x}{2}}{\frac{1}{2}} \right]_0^{\pi}$   
 $= 2\sqrt{2} (\sin \frac{\pi}{2} - \sin 0) = 2\sqrt{2}$

2. b.  $\tan^{-1}(x + \frac{1}{x}) + C$

**Explanation:** Divide num. and deno. by  $x^2$

Substitute  $x + \frac{1}{x} = t$  then  $(1 - \frac{1}{x^2})dx = dt$   
 $\Rightarrow \int \frac{dt}{t^2+1}$   
 $\Rightarrow \tan^{-1}(x + \frac{1}{x}) + C$

3. a.  $-\frac{1}{6}\cos^3 2x + C$

**Explanation:**  $\int \frac{\cos 4x+1}{\tan x+\cot x} dx$   
 $= \int \frac{2 \cos^2 2x}{2 \operatorname{cosec} 2x} dx$   
 $= \int \frac{2 \cos^2 2x}{2 \operatorname{cosec} 2x} dx$

substitute  $\cos 2x = t$ , then  $-2 \sin 2x dx = dt$

$$\frac{-1}{2} \int \frac{t^2}{2} dt \Rightarrow = \frac{-1}{2} \left( \frac{\cos^3 2x}{3} \right) + C$$

4. a.  $-2\sqrt{1-x} + C$

**Explanation:**  $\int (1-x)^{-1/2} dx = \frac{(1-x)^{1/2}}{(1/2)(-1)} + C = -2\sqrt{1-x} + C$

5. c.  $\tan \frac{x}{2} + C$

**Explanation:**  $\int (\operatorname{cosec}^2 x - \cot x \operatorname{cosec} x) dx = (-\cot x + \operatorname{cosec} x) + C$   
 $= \frac{1-\cos x}{\sin x} + C = \tan \frac{x}{2} + C$

6.  $\sqrt{2} - 1$

7.  $\frac{1}{2}x^4 + 4x + C$

8.  $\frac{74}{3}$

9.  $\int \frac{x^3 - x^2 + x - 1}{x-1} dx$

$$\begin{aligned}
&= \int \frac{x^2(x-1)+(x-1)}{x-1} dx \\
&= \int \frac{(x-1)(x^2+1)}{(x-1)} dx \\
&= \int (x^2 + 1) dx \\
&= \frac{x^3}{3} + x + c
\end{aligned}$$

10.  $I = \int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$

$$= [\tan x - x]_0^{\pi/4}$$

$$= (\tan \frac{\pi}{4} - \frac{\pi}{4}) - (0 - 0) = 1 - \frac{\pi}{4}$$

11. Let  $I = \int \frac{\sec^2 x}{\cosec^2 x} dx = \int \frac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{1}{\sin^2 x}\right)} dx$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx = \int (\sec^2 x - 1) dx \quad [\because \tan^2 x = \sec^2 x - 1]$$

$$= \int \sec^2 x dx - \int 1 dx = \tan x - x + c$$

12. we have

$$I = \int \frac{x^3+x}{x^4-9} dx = \int \frac{x^3}{x^4-9} dx + \frac{xdx}{x^4-9} = I_1 + I_2$$

$$\text{Now } I_1 = \int \frac{x^3}{x^4-9}$$

Put  $t = x^4 - 9$  so that  $4x^3 dx = dt$ . Therefore

$$I_1 = \frac{1}{4} \int \frac{dt}{t} = \frac{1}{4} \log|t| + C_1 = \frac{1}{4} \log|x^4 - 9| + C_1$$

$$\text{Again, } I_2 = \int \frac{xdx}{x^4-9}$$

Put  $x^2 = u$  so that  $2x dx = du$ . Then

$$I_2 = \frac{1}{2} \int \frac{du}{u^2-(3)^2} = \frac{1}{2 \times 6} \log \left| \frac{u-3}{u+3} \right| + C_2$$

$$= \frac{1}{12} \log \left| \frac{x^2-3}{x^2+3} \right| + C_2$$

Thus  $I = I_1 + I_2$

$$= \frac{1}{4} \log|x^4 - 9| + \frac{1}{12} \log \left| \frac{x^2-3}{x^2+3} \right| + C. \quad [\because C = C_1 + C_2]$$

13.  $\int \tan^2(2x-3) dx$

$$= \int \{\sec^2(2x-3) - 1\} dx$$

$$= \int \sec^2(2x-3) dx - \int 1 dx$$

$$\text{Using } \int \sec^2(ax+b) dx = \frac{\tan(ax+b)}{a} + C$$

$$= \frac{\tan(2x-3)}{2} - x + c$$

14. Let  $I = \int \frac{x}{\sqrt{x+1}} dx$

$$\text{Put } \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\begin{aligned}
&\Rightarrow dx = 2\sqrt{x}dt \\
\therefore I &= 2 \int \left( \frac{x\sqrt{x}}{t+1} \right) dt = 2 \int \frac{t^2 \cdot t}{t+1} dt = 2 \int \frac{t^3}{t+1} dt = 2 \int \frac{t^3+1-1}{t+1} dt \\
&= 2 \int \frac{t^3+1}{t+1} dt - 2 \int \frac{1}{t+1} dt = 2 \int \frac{(t+1)(t^2-t+1)}{t+1} dt - 2 \int \frac{1}{t+1} dt \\
&= 2 \int (t^2 - t + 1) dt - 2 \int \frac{1}{t+1} dt \\
&= 2 \left[ \frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1| \right] + C \\
&= 2 \left[ \frac{x\sqrt{x}}{2} - \frac{x}{2} + \sqrt{x} - \log|\sqrt{x}+1| \right] + C
\end{aligned}$$

15.  $\int \sqrt{1+3x-x^2}dx$

$$\begin{aligned}
&= \int \sqrt{-x^2+3x+1}dx \\
&= \int \sqrt{-(x^2-3x-1)}dx \\
&= \int \sqrt{-\left(x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2-1\right)}dx \\
&= \int \sqrt{-\left[\left(x-\frac{3}{2}\right)^2-\left(\frac{\sqrt{13}}{2}\right)^2\right]}dx \\
&= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}dx \\
&= \frac{x-\frac{3}{2}}{2} \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2} + \frac{\left(\frac{\sqrt{13}}{2}\right)^2}{2} \sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + c \\
\left[\because \int \sqrt{a^2-x^2}dx\right. &= \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{2}] \\
&= \left(\frac{2x-3}{4}\right) \sqrt{1+3x-x^2} + \frac{13}{8} \sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + c
\end{aligned}$$

16. Let  $I = \int \frac{(x-3)e^x}{(x-1)^3}dx$

$$\begin{aligned}
&= \int \frac{(x-1)-2}{(x-1)^3}e^x dx \\
&= \int e^x \left[ \frac{(x-1)}{(x-1)^3} - \frac{2}{(x-1)^3} \right] dx \\
\Rightarrow I &= \int e^x \left[ \frac{1}{(x-1)^2} + \frac{-2}{(x-1)^3} \right] dx
\end{aligned}$$

$$[\int e^x \{f(x) + f'(x)\} dx]$$

It is in the form of  $\int e^x \{f(x) + f'(x)\} dx$  since here  $f(x) = \frac{1}{(x-1)^2}$  and

$$\begin{aligned}
f'(x) &= \frac{d}{dx} \left\{ (x-1)^{-2} \right\} \\
&= -2(x-1)^{-3}
\end{aligned}$$

$$= \frac{-2}{(x-1)^3}.$$

$$\Rightarrow I = \frac{e^x}{(x-1)^2} + c$$

$$[\because \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c]$$

17. Let  $I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

Put  $x = a\tan^2\theta$

$$\Rightarrow dx = 2a\tan\theta\sec^2\theta d\theta$$

$$\therefore I = \int \sin^{-1} \sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}} (2a\tan\theta\sec^2\theta) d\theta$$

$$= 2a \int \sin^{-1} \left( \frac{\tan\theta}{\sec\theta} \right) \tan\theta\sec^2\theta d\theta$$

$$= 2a \int \sin^{-1} (\sin\theta) \tan\theta\sec^2\theta d\theta$$

$$= 2a \int \theta \tan\theta\sec^2\theta d\theta$$

$$= 2a \left[ \theta \cdot \int \tan\theta\sec^2\theta d\theta - \int \left( \frac{d}{d\theta} \theta \cdot \int \tan\theta\sec^2\theta d\theta \right) d\theta \right]$$

Let  $\tan\theta = t$

$$\sec^2\theta d\theta = dt$$

$$\int \tan\theta\sec^2\theta d\theta = \int t dt = \frac{t^2}{2} = \frac{\tan^2\theta}{2}$$

$$I = 2a \left[ \theta \cdot \frac{\tan^2\theta}{2} - \int \frac{\tan^2\theta}{2} d\theta \right]$$

$$= a\theta\tan^2\theta - a \int (\sec^2\theta - 1) d\theta$$

$$= a\theta\tan^2\theta - a\tan\theta + a\theta + C$$

$$= a \left[ \frac{x}{a} \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1} \sqrt{\frac{x}{a}} \right] + C$$

18. According to given question,  $I = \int_1^3 (3x^2 + 1) dx$

On comparing the given integral with  $\int_a^b f(x)dx$ , we get

Here,  $a = 1, b = 3$  and  $f(x) = 3x^2 + 1$

We know that,

$$\int_a^b f(x)dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

where,  $h = \frac{b-a}{n} \Rightarrow nh = b-a$ .

$$f(a) = f(1) = 3(1)^2 + 1 = 4$$

$$f(a+h) = f(1+h) = 3(1+h)^2 + 1 = 4 + 6h + 3h^2$$

$$f(a+2h) = f(1+2h) = 3(1+2h)^2 + 1 = 4 + 6h + 3h^2$$

so on

$$f[a + (n-1)h] = f[1 + (n-1)h] = 3[1 + (n-1)h]^2 + 1 = 4 + 6h(n-1) + 3h^2(n-1)^2$$

$$\text{Now, } \int_1^3 (3x^2 + 1) dx = \lim_{h \rightarrow 0} h [ f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h) ] \\ = \lim_{h \rightarrow 0} [4 + 4 + 6h(1) + 3h^2(1)^2 + 4 + 6h(2) + 3h^2(2)^2 + \dots + 4 + 6h(n-1) + 3h^2(n-1)^2]$$

On rearranging terms , we get

$$= \lim_{h \rightarrow 0} [4 + 4 + 4 + \dots + 4 + 6h\{1 + 2 + 3 + \dots + (n-1)\} + 3h^2\{1^2 + 2^2 + 3^2 + \dots + (n-1)^2\}] \\ = \lim_{h \rightarrow 0} h \left[ 4n + 6h \cdot \frac{n(n-1)}{2} + 3h^2 \frac{n(n-1)(2n-1)}{6} \right] \\ [\because \sum n = \frac{n(n+1)}{2}, \sum n^2 = \frac{n(n+1)(2n+1)}{6}] \\ = \lim_{h \rightarrow 0} \left[ 4nh + \frac{6nh(nh-h)}{2} + \frac{3hn(nh-h)(2nh-h)}{6} \right] \\ = 4(2) + \frac{6(2)(2-0)}{2} + \frac{3 \times 2(2-0)(2 \times 2 - 0)}{6} \\ = 8 + 12 + 8 \\ = 28 \text{ sq units.}$$