

**CBSE Test Paper 01**  
**Chapter 7 Integrals**

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1.  $\int \frac{1}{e^x+1} dx$  is equal to

- a.  $\log(1 + e^{-2x}) + C$
- b.  $\log(e^{-2x} - 2x) + C$
- c.  $-\log(1 + e^{-x}) + C$
- d.  $\log(e^{3x} + x) + C$

2. The function  $f(x) = \int_0^x \log(t + \sqrt{1 + t^2}) dt$  is

- a. an odd function
- b. an even function
- c. Neither odd nor Even
- d. a periodic function

3.  $\int_0^{\pi/2} \log|\cos x| dx$  is equal to

- a.  $-\frac{\pi}{2} \log 2$
- b.  $\pi \log 2$
- c.  $\frac{\pi}{2} \log 3$
- d.  $-\frac{\pi}{3} \log 3$

4.  $\int_a^b \frac{\log x}{x} dx$  is equal to

- a.  $\frac{\log(b-a)}{b-a}$
- b.  $\log(a+b) \cdot \log(b-a)$
- c.  $\log(ab) \cdot \log\left(\frac{b}{a}\right)$
- d.  $\frac{1}{2} \log(ab) \cdot \log\left(\frac{b}{a}\right)$

5. If  $\int f(x) dx = f(x)$ , then

- a.  $f(x) = a^x$   
b.  $f(x) = x$   
c.  $f(x) = 0$   
d.  $f(x) = e^x$
6. The function  $A(x)$  denotes the \_\_\_\_\_ function and is given by  $A(x) = \int_a^x f(x) dx$ .
7. The indefinite integral of  $2x^{\frac{1}{2}}$  is \_\_\_\_\_.
8. The indefinite integral of  $2x^2 + 3$  is \_\_\_\_\_.
9. Show that  $\int \frac{2x+3}{x^2+3x} dx = \log|x^2+3x| + C$ .
10. Evaluate  $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ .
11. Evaluate  $\int \sin^3 x dx$ .
12. Evaluate the definite integral  $\int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) dx$ .
13. Integrate the following function  $\frac{1}{\sqrt{7-6x-x^2}}$ .
14. Integrate the function  $(x^2 + 1) \log x$
15.  $\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$ .
16. Evaluate  $\int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{\frac{-x}{2}} dx$ .
17. Evaluate  $\int_0^1 x \log(1+2x) dx$
18. Evaluate  $\int_1^3 (2x^2 + 5x) dx$  as a limit of a sum.

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**Solution**

1. c.  $-\log(1 + e^{-x}) + C$ , **Explanation:**

$$\int \frac{e^{-x}}{1+e^{-x}} dx = - \int \frac{-e^{-x}}{1+e^{-x}} dx = -\log(1 + e^{-x}) + C$$

2. b. an even function, **Explanation:**  $t = -u$

$$\begin{aligned} f(-x) &= \int_0^x \log(-u + \sqrt{1 + u^2})(-du) \\ &= - \int_0^\pi \log\left(\frac{1+u+u^2}{\sqrt{1+u^2+u}}\right)(-du) \\ &= \int_0^x \log(u + \sqrt{1 + u^2}) du = f(x) \end{aligned}$$

$\Rightarrow f(-x) = f(x) \Rightarrow f$  is an even function

3. a.  $-\frac{\pi}{2}\log 2$ , **Explanation:**  $\int_0^{\pi/2} \log|\cos x| dx$

$$= \int_0^{\pi/2} \log|\cos(\frac{\pi}{2} - x)| dx$$

$$= \int_0^{\pi/2} \log|\sin x| dx$$

$$= -\frac{\pi}{2} \log 2 \text{ (standard result)}$$

4. d.  $\frac{1}{2}\log(ab) \cdot \log\left(\frac{b}{a}\right)$ , **Explanation:**  $= \int_a^b (\log x)^1 \left(\frac{1}{x}\right) dx$

(let  $\log x = t$  then  $\frac{1}{x}dx = dt$ )

$$= \left[ \frac{(\log x)^2}{2} \right]_a^b$$

$$= \frac{1}{2} \left[ (\log b)^2 - (\log a)^2 \right]$$

$$= \frac{1}{2} (\log b + \log a)(\log b - \log a)$$

$$= \frac{1}{2} \log(ab) \log \frac{b}{a}$$

5. d.  $f(x) = e^x$ , **mExplanation:**  $\frac{d}{dx}(f(x)) = f(x)$

It implies that the function remains same after integrating or differentiating it.

So the function must be  $e^x$

6. area

$$7. \frac{4}{3}x^{\frac{3}{2}} + C$$

$$8. = \frac{4}{5}x^{\frac{5}{4}} + C$$

$$9. \text{ Let } I = \int \frac{2x+3}{x^2+3x} dx$$

$$\text{Put } x^2 + 3x = t$$

$$\Rightarrow (2x+3)dx = dt$$

$$\therefore I = \int \frac{1}{t} dt = \log|t| + C$$

$$= \log |(x^2 + 3x)| + C$$

$$10. \text{ According to the question, } I = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

$$= [\sin^{-1} x]_0^1 \quad \left[ \because \int_b^a \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(a) - \sin^{-1}(b) \right]$$

$$= \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \sin^{-1}(\sin \frac{\pi}{2}) - \sin^{-1}(\sin 0)$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$11. \text{ Let } I = \int \sin^3 x dx = \int \frac{3 \sin x - \sin 3x}{4} dx \quad [\because \sin 3x = 3 \sin x - 4 \sin^3 x]$$

$$= \frac{1}{4} \int 3 \sin x dx - \frac{1}{4} \int \sin 3x dx$$

$$= \frac{1}{4} \left( -3 \cos x + \frac{\cos 3x}{3} \right) + C$$

$$12. \int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) dx$$

$$= 2 \int_0^{\frac{\pi}{4}} \sec^2 x dx + \int_0^{\frac{\pi}{4}} x^3 dx + 2 \int_0^{\frac{\pi}{4}} 1 dx$$

$$= 2(\tan x)_0^{\frac{\pi}{4}} + \left( \frac{x^4}{4} \right)_0^{\frac{\pi}{4}} + 2(x)_0^{\frac{\pi}{4}}$$

$$= 2 \left( \tan \frac{\pi}{4} - \tan 0^\circ \right) + \frac{\left( \frac{\pi}{4} \right)^4}{4} - 0 + 2 \left( \frac{\pi}{4} - 0 \right)$$

$$= 2(1 - 0) + \frac{\left( \frac{\pi^4}{256} \right)}{4} + \frac{2\pi}{4}$$

$$= 2 + \frac{\pi^4}{1024} + \frac{\pi}{2}$$

$$= \frac{\pi^4}{1024} + \frac{\pi}{2} + 2$$

$$13. \int \frac{1}{\sqrt{7-6x-x^2}} dx$$

$$\begin{aligned}
&= \int \frac{1}{\sqrt{-x^2 - 6x + 7}} dx \\
&= \int \frac{1}{\sqrt{-(x^2 + 6x - 7)}} dx \\
&= \int \frac{1}{\sqrt{-(x^2 + 6x + 9 - 9 - 7)}} dx \\
&= \int \frac{1}{\sqrt{-\{(x+3)^2 - 16\}}} dx \\
&= \int \frac{1}{\sqrt{(16) - (x+3)^2}} dx \\
&= \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx \\
&= \sin^{-1} \left( \frac{x+3}{4} \right) + c \\
&\quad \left[ \because \int \frac{1}{a^2 - x^2} dx = \sin^{-1} \frac{x}{a} \right]
\end{aligned}$$

14.  $\int (x^2 + 1) \log x dx$   
 $= \int (\log x) (x^2 + 1) dx$

[Applying product rule]

$$\begin{aligned}
&= \log x \left( \frac{x^3}{3} + x \right) - \int \frac{1}{x} \left( \frac{x^3}{3} + x \right) dx \\
&= \left( \frac{x^3}{3} + x \right) \log x - \int \left( \frac{x^2}{3} + 1 \right) dx \\
&= \left( \frac{x^3}{3} + x \right) \log x - \frac{1}{3} \int x^2 dx - \int 1 dx \\
&= \left( \frac{x^3}{3} + x \right) \log x - \frac{1}{3} \frac{x^3}{3} - x + c \\
&= \left( \frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + c
\end{aligned}$$

15.  $I = \int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Dividing Numerator and Denominator by  $\cos^4 x$

$$\begin{aligned}
&= \int_0^{\pi/4} \frac{\frac{\sin x \cos x}{\cos^4 x}}{\frac{\cos^4 x}{\cos^4 x} + \frac{\sin^4 x}{\cos^4 x}} dx \\
&= \int_0^{\pi/4} \frac{\tan x \cdot \sec^2 x}{1 + \tan^4 x} dx \\
&= \int_0^{\pi/4} \frac{\tan x \cdot \sec^2 x}{1 + (\tan^2 x)^2} dx
\end{aligned}$$

put  $\tan^2 x = t$

$$2 \tan x \cdot \sec^2 x dx = dt$$

when  $x=0, t=0$  and when  $x=\frac{\pi}{4}, t=1$

$$\therefore I = \frac{1}{2} \int_0^1 \frac{dt}{1+t^2}$$

$$\begin{aligned}
&= \frac{1}{2} [\tan^{-1} t]_0^1 \\
&= \frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8} \\
16. \text{ Given, } I &= \int \frac{\sqrt{1-\sin x}}{1+\cos x} \cdot e^{\frac{-x}{2}} dx \\
\text{Let } \frac{-x}{2} &= t \Rightarrow dx = -2dt \\
I &= \int \frac{\sqrt{1-\sin(-2t)}}{1+\cos(-2t)} e^t (-2dt) [\because x = -2t] \\
&= -2 \int e^t \frac{\sqrt{1+\sin 2t}}{1+\cos 2t} dt [\because \sin(-\theta) = -\sin \theta \text{ and } \cos(-\theta) = \cos \theta] \\
&= -2 \int e^t \frac{\sqrt{\cos^2 x + \sin^2 x + 2\sin x \cos x}}{1+\cos 2t} dt [\because \cos^2 x + \sin^2 x = 1, \sin 2x = 2\sin x \cos x] \\
&= -2 \int e^t \left( \frac{\sqrt{(\cos t + \sin t)^2}}{2 \cos^2 t} \right) dt [\because (a+b)^2 = a^2 + b^2 + 2ab] \\
&= -2 \int e^t \left( \frac{\cos t + \sin t}{2 \cos^2 t} \right) dt \\
&= - \int e^t (\sec t + \tan t \sec t) dt [\because \frac{1}{\cos x} = \sec x, \frac{\sin x}{\cos x} = \tan x] \\
\text{we know that, } \int e^t [f(t) + f'(t)] dt &= e^t f(t) + C \\
\text{Now, consider } f(t) &= \sec t \\
\text{then } f'(t) &= \sec t \tan t \\
\therefore I &= e^t \sec t + C \\
&= -e^{-x/2} \sec \frac{x}{2} + C [\because t = \frac{-x}{2} \text{ and } \sec(-\theta) = \sec \theta] \\
I &= -e^{-x/2} \sec \frac{x}{2} + C \\
17. \text{ } I &= \int_0^1 x \log(1+2x) dx \\
&= \left[ \log(1+2x) \frac{x^2}{2} \right]_0^1 - \int \frac{1}{1+2x} \cdot 2 \cdot \frac{x^2}{2} dx \\
&= \frac{1}{2} [x^2 \log(1+2x)]_0^1 - \int \frac{x^2}{1+2x} dx \\
&= \frac{1}{2} [\log 3 - 0]_0^1 - \left[ \int_0^1 \left( \frac{x}{2} - \frac{\frac{x^2}{2}}{1+2x} \right) dx \right] \\
&= \frac{1}{2} \log 3 - \frac{1}{2} \int_0^1 x dx + \frac{1}{2} \int_0^1 \frac{x}{1+2x} dx \\
&= \frac{1}{2} \log 3 - \frac{1}{2} \left[ \frac{x^2}{2} \right]_0^1 + \frac{1}{2} \int_0^1 \frac{\frac{1}{2}(2x+1-1)}{(2x+1)} dx \\
&= \frac{1}{2} \log 3 - \frac{1}{2} \left[ \frac{1}{2} - 0 \right] + \frac{1}{4} \int_0^1 dx - \frac{1}{4} \int_0^1 \frac{1}{1+2x} dx \\
&= \frac{1}{2} \log 3 - \frac{1}{4} + \frac{1}{4} [x]_0^1 - \frac{1}{8} [\log|(1+2x)|]_0^1 \\
&= \frac{1}{2} \log 3 - \frac{1}{4} + \frac{1}{4} - \frac{1}{8} [\log 3 - \log 1] \\
&= \frac{1}{2} \log 3 - \frac{1}{8} \log 3 \\
&= \frac{3}{8} \log 3
\end{aligned}$$

18. According to the question ,  $I = \int_1^3 (2x^2 + 5x) dx$

On comparing the given integral with  $\int_a^b f(x)dx$ , we get

$$a = 1, b = 3 \text{ and } f(x) = 2x^2 + 5x$$

We know that ,

$$\int_a^b f(x)dx = \lim_{h \rightarrow 0} h[f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)] \dots(i)$$

$$\text{where, } h = \frac{b-a}{n} \Rightarrow nh = b - a$$

$$= 3 - 1 = 2$$

$$f(a) = f(1) = 2(1)^2 + 5(1) = 2 + 5 = 7$$

$$f(a+h) = f(1+h) = 2(1+h)^2 + 5(1+h) = 2 + 2h^2 + 4h + 5 + 5h = 2h^2 + 9h + 7$$

$$f(a+2h) = 2(1+2h)^2 + 5(1+2h) = 2 + 8h^2 + 8h + 5 + 10h = 8h^2 + 18h + 7$$

so on

$$\begin{aligned} f(a+(n-1)h) &= f\{1 + (n-1)h\} = 2\{1 + (n-1)h\}^2 + 5\{1 + (n-1)h\}^2 = 2 + 2(n-1)2h^2 + 4(n-1)h \\ &+ 5 + 5(n-1)h = 2(n-1)2h^2 + 9(n-1)h + 7 \end{aligned}$$

On putting all above values in (i), we get

$$\int_1^3 (2x^2 + 5x) dx = \lim_{h \rightarrow 0} h[7 + (2h^2 + 9h + 7) + (8h^2 + 18h + 7) + \dots + 2(n-1)2h^2 + 9(n-1)h + 7]$$

On rearranging terms , we get

$$\begin{aligned} &= \lim_{h \rightarrow 0} h[7 + 7 + 7 + \dots + 7] + \lim_{h \rightarrow 0} h[2h^2 + 8h^2 + \dots + 2(n-1)2h^2] + \lim_{h \rightarrow 0} h[9h + 18h \\ &+ \dots + 9(n-1)h] \\ &= \lim_{h \rightarrow 0} 7nh + \lim_{h \rightarrow 0} 2h^3[1^2 + 2^2 + 3^2 + \dots + (n-1)^2] + \lim_{h \rightarrow 0} 9h^2[1 + 2 + \dots + (n-1)] \\ &= \lim_{h \rightarrow 0} 7(2) + \lim_{h \rightarrow 0} \frac{2h^3 \cdot n(n-1)(2n-1)}{6} + \lim_{h \rightarrow 0} \frac{9h^2 \cdot n(n-1)}{2} \\ [\because \sum n &= \frac{n(n+1)}{2}, \sum n^2 = \frac{n(n+1)(2n+1)}{6}] \\ &= 14 + \lim_{h \rightarrow 0} \frac{nh(nh-h)(2nh-h)}{3} + \lim_{h \rightarrow 0} \frac{9}{2} \cdot nh(nh-h) \\ &= 14 + \frac{2(2-0)(4-0)}{3} + \frac{9}{2} \cdot 2(2-0) \\ &= 14 + \frac{16}{3} + 18 \\ &= \frac{42+16+54}{3} \\ &= \frac{112}{3} \text{ sq units.} \end{aligned}$$