1. The curve y = $a x^3 + bx^2 + c x$ is inclined at $45°$ to the X – axis at (0, 0) but it touches X – axis at (1, 0) , then the values of a, b, c, are given by

   a. a = 1, b = – 2, c = 1
   b. a = 1, b = 1, c = -2
   c. a = -2, b = 1, c = 1
   d. a = -1, b = 2, c = 1.

2. The function f (x) = x^3 - 3x has a

   a. local minima at x = 1
   b. local maxima at x = 1
   c. point of inflexion at 0
   d. none of these

3. Minimum value of the function f(x) = x^2 + x + 1 is

   a. none of these
   b. 3
   c. $\frac{3}{4}$
   d. 1

4. The maximum value of $\frac{\log x}{x}$ in $0 < x < \infty$ is

   a. 0
   b. none of these
   c. -e
   d. $\frac{1}{e}$

5. If the radius of a sphere is measures as 7m with an error of 0.02m then find the approximate error in calculating its volume.

   a. None of these

b. $1.96\pi \quad m^2$

c. $2.16\pi \quad m^2$

d. $3.92\pi \quad m^3$

6. Maximum slope of the curve y = $-x^3 + 3x^2 + 9x - 27$ is _____.

7. If x is real, the minimum value of $x^2 - 8x + 17$ is _____.

8. The function f(x) = $\frac{2x^2-1}{x^4}$, x > 0, decreases in the interval _____.

9. Find the maximum value of the function $f(x) = cosx + cos\left(\sqrt{2}\ x\right)$.

10. Find the interval in which the function f(x) = $cot^{-1}x + x$ is increasing.

11. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

12. The volume of a cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

13. A balloon, which always remains spherical has a variables radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

14. Show that the tangents to the curve y = $7x^3 = 11y = 7x^3 + 11$ at the points where x = 2 and x = - 2 are parallel.

15. Find point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are

    i. parallel to x –axis
    ii. parallel to y – axis.

16. Find the value(s) of x for which y = $[x(x - 2)]^2$ is an increasing function.

17. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$.

18. A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

Solution

1. (a) a = 1, b = – 2, c = 1

   Explanation: $y = ax^3 + bx^2 + cx$

   $\Rightarrow \frac{dy}{dx} = 3ax^2 + 2bx + c.$

   At (0 , 0) , slope of tangent $= \tan 45° = 1. \Rightarrow$ c = 1. At (1 ,0) , slope of tangent = 0.

   $\Rightarrow$ 3a+2b+c=0. From this,we get ,3a+2b=-1........(1)

   Also, when x = 1 , y = 0 , therefore , a + b + c = 0 . From this,we get, a+b=-1.......(2)

   From(1) and (2),we get,

   a=1 ,b= -2 and c =1

2. (a) local minima at x = 1

   Explanation: Given, f(x) = x^3 - 3x

   f'(x) = 3x^2 - 3

   For point of inflexion we have f'(x) = 0

   $f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 = 3(x-1)(x+1) \Rightarrow x = \pm 1$

   Hence, f(x) has a point of inflexion at x = 0.

   When , x is slightly less than 1, f'(x) = (+)(-)(+) i.e, negative

   When x is slightly greater than 1, f'(x)= (+)(+)(+) i.e, positive

   Hence, f'(x) changes its sign from negative to positive as x increases through 1 and hence x = 1 is a point of local minimum.

3. (c) $\frac{3}{4}$

   Explanation: Given, f(x) = x^2 + x +1

   $\Rightarrow$ f'(x) = 2x + 1

   For minimum value of f(x) we have f'(x) = 0

   $\Rightarrow$ 2x + 1 = 0 $\Rightarrow$ x = $\frac{-1}{2}$

   Now, f''(x) = 2 >0, hence the minimum of f(x) exist at x = $\frac{-1}{2}$ and minimum value

   $= f\left(\frac{-1}{2}\right) = \frac{3}{4}$

4. (d) $\frac{1}{e}$

   Explanation: Consider f(x) = $\frac{\log x}{x}$

Then, f'(x) $= \dfrac{x. \frac{1}{x} - logx.1}{x^2} = \dfrac{1 - logx}{x^2}$

For maximum or minimum values of x we have f'(x) = 0

f'(x) = 0 $\Rightarrow$ (1 - logx) = 0

$\Rightarrow$ logx = 1 $\Rightarrow$ x = e.

Now, f'(x) $= \dfrac{x^2. \frac{-1}{x} - (1-logx)2x}{x^4} = \left[\dfrac{-3 + 2logx}{x^3}\right]$

f''(x) at $\quad x = e = \dfrac{-3}{e^3} < 0$

Therefore f(x) is maximum at x = e and the max. value $= \dfrac{loge}{e} = \dfrac{1}{e}$

5. (d) $3.92\pi m^3$

   Explanation: Given, radius of the sphere is 7 m

   Error in the measurement of radius = $\triangle r$ = 0.02 m

   We have volume of sphere = $V = \frac{4}{3}\pi r^3$

   Now, $dV = \left(\dfrac{dV}{dx}\right)\triangle r = \frac{4}{3}\pi.3r^2\triangle r = 4\pi r^2.\triangle r$

   $= 4\pi \times 49 \times 0.02 = 3.92\pi \ m^3$

6. 12

7. 1

8. $(1, \infty)$

9. we have, f(x) = cos x + $cos\left(\sqrt{2}\ x\right)$

   using the inequality $|a + b| \leq |a| + |b|$

   |f(x)| = $\left|cosx + cos\left(\sqrt{2}\ x\right)\right| \leq |cos\ x| + \left|cos\left(\sqrt{2}\ x\right)\right| \leq 1 + 1 = 2, \forall\ x\ inR$

   $\sin ce\ -1 \leq \cos x \leq 1 \Rightarrow |\cos x| \leq 1$ Which is true for any given angle.

   Hence, maximum value of f(x) = 2

10. Since, f(x)= $\cot^{-1}$x+x

    On differentiating w.r.t x,we get

    f'(x)=$-\dfrac{1}{1+x^2} + 1 = \dfrac{x^2}{1+x^2} \geq 0$, since for -ve values of x , the expression becomes

    positive since we have x$^{2.}$

    Also when x = 0 , the value is 0. And the positive values of x gives values greater than zero. Since the derivative of f(x) is non negative, f(x) is increasing function for all x $\in (-\infty, \infty)$.

11. Given: $f\left(x\right) = e^{2x}$

    $\therefore f'\left(x\right) = e^{2x}\dfrac{d}{dx}\left(2x\right) = e^{2x}\left(2\right) = 2e^{2x} > 0$ i.e., positive for all $x \in R$

    Therefore, f(x) is strictly increasing on R.

12. Let the side of a cube be x unit.

∴ Volume of cube (V) = x^3

On differentiating both side w.r.t. t, we get

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} = k$ [constant]

$\Rightarrow \frac{dx}{dt} = \frac{k}{3x^2}$ ...(i)

Also, surface area of cube, S = $6x^2$

On differentiating w.r.t. t, we get

$\frac{dS}{dt} = 12x . \frac{dx}{dt}$

$\Rightarrow \frac{dS}{dt} = 12x . \frac{k}{3x^2}$ [using Eq. (i)]

$\Rightarrow \frac{dS}{dt} = \frac{12k}{3x} = 4\left(\frac{k}{x}\right)$

$\Rightarrow \frac{dS}{dt} \alpha \frac{1}{x}$

Hence, the surface area of the cube varies inversely as the length of the side.

13. Since, $V = \frac{4}{3}\pi x^3$

$\therefore \frac{dV}{dx} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{4}{3}\pi . 3r^2 = 4\pi r^2$

At x = 10 cm

$\Rightarrow \frac{dV}{dx} = 4\pi (10)^2 = 400\pi$

Therefore, the volume is increasing at the rate of $400\pi cm^3 / sec$.

14. Given: Equation of the curve y = $7x^3$ + 11

$\therefore$ Slope of tangent at $(x, y) = \frac{dy}{dx} = 21x^2$

At the point x = 2, Slope of the tangent = $21(2)^2$ = 21 × 4 = 84

At the point x = - 2, Slope of the tangent = $21(-2)^2$ = 21 × 4 = 84

Since, the slopes of the two tangents are equal.

Therefore, tangents at x = 2 and x = - 2 are parallel.

15. $\frac{x^2}{4} + \frac{y^2}{25} = 1$...(i)

Differentiate both sides w.r.t. to x

$\frac{2x}{4} + \frac{2y}{25}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-25}{4} . \frac{x}{y}$

For tangent || to x – axis the slope of tangent is zero

$\frac{0}{1} = \frac{-25x}{4y}$

x = 0

Put x = 0 in equation (1)

$y = \pm 5$

Points are (0, 5) and (0, -5)

now the tangent is || is to y – axis

y = 0

Put y = 0 in equation (1)

$x = \pm 2$

So points on the curve are (2,0) and (-2,0)

16. Given function is y = [x(x - 2)]² = [x² - 2x]².

Therefore, on differentiating both sides w.r.t x, we get,

$$\frac{dy}{dx} = 2\left(x^2 - 2x\right)\frac{d}{dx}\left(x^2 - 2x\right)$$

= 2(x² - 2x) (2x - 2)

= 4x(x - 2) (x - 1)

Therefore, on putting $\frac{dy}{dx}$ = 0, we get,

4x(x - 2)(x - 1) = 0 ⟹ x = 0, 1 and 2.

Now, we find interval in which f(x) is strictly increasing or strictly decreasing.

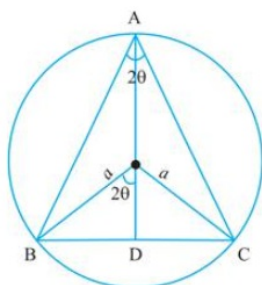| Interval	f'(x) = 12x(x + 1)(x - 2)	Sign of f'(x)
(-∞,0)	(-)(-)(-)	-ve
(0, 1)	(+)(-)(-)	+ve
(1, 2)	(+)(-)(+)	-ve
(2,∞)	(+)(+)(+)	+ve

Therefore, y is strictly increasing in (0, 1) and (2, ∞).

Also, y is a polynomial function, so it continuous at x = 0, 1 and 2.

Hence, y is increasing in [0, 1] ∪ [2, ∞).

17. Let ABC be an isosceles triangle inscribed in the circle with radius a such that AB = AC.

$AD = AO + OD = a + a\cos 2\theta$ and $BC = 2BD = 2a\sin 2\theta$ (see fig.)

Therefore, area of the triangle ABC i.e. $\triangle = \frac{1}{2} BC \cdot AD$

$= \frac{1}{2} 2a \sin 2\theta \cdot (a + a \cos 2\theta)$

$= a^2 \sin 2\theta (1 + \cos 2\theta)$

$\Rightarrow \triangle = a^2 \sin 2\theta + \frac{1}{2} a^2 \sin 4\theta$

Therefore, $\frac{d\triangle}{d\theta} = 2a^2 \cos 2\theta + 2a^2 \cos 4\theta$

$= 2a^2 (\cos 2\theta + \cos 4\theta)$

$\frac{d\triangle}{d\theta} = 0 \Rightarrow \cos 2\theta = -\cos 4\theta = \cos(\pi - 4\theta)$

Therefore, $2\theta = \pi - 4\theta \Rightarrow \theta = \frac{\pi}{6}$

$\frac{d^2\triangle}{d\theta^2} = 2a^2 (-2 \sin 2\theta - 4 \sin 4\theta) < 0 \left(at\ \theta = \frac{\pi}{6} \right)$

Therefore, Area of triangle is maximum when $\theta = \frac{\pi}{6}$.

18. Let P be a point on the hypotenuse AC of right-angled $\triangle$ABC, Such that $PL \perp AB$ and PL=a and $PM \perp BC$ and PM=b.

Let $\angle APL = \angle ACB = \theta$[say]

Then, $AP = a \sec\theta, PC = b\ cosec\theta$

Let l be the length of the hypotenuse, then

l = AP+ PC

$\Rightarrow l = a \sec\theta + b\ cosec\theta, 0 < \theta < \frac{\pi}{2}$
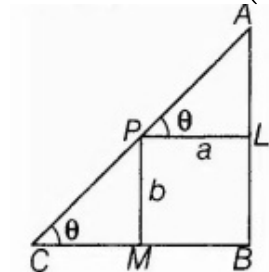
On differentiating both sides w.r.t. θ, we get,

$\frac{dl}{d\theta} = a \sec\theta \tan\theta - b\ cosec\theta \cot\theta$ ....(i)

For maxima or minima, put $\frac{dl}{d\theta} = 0$

$\Rightarrow a \sec\theta \tan\theta = b\ cosec\theta \cot\theta$

$\Rightarrow \quad \frac{a \sin\theta}{\cos^2\theta} = \frac{b \cos\theta}{\sin^2\theta}$

$\Rightarrow \quad \tan\theta = \left(\frac{b}{a} \right)^{1/3}$

Again, on differentiating both sides of Eq.(i) w.r.t. θ we get

$\frac{d^2l}{d\theta^2} = a \left(\sec\theta \times \sec^2\theta + \tan\theta \times \sec\theta \tan\theta \right) - b \left[cosec\theta \left(- cosec^2\theta \right) + \cot\theta (- cosec\theta \cot\theta) \right]$

$= a \sec\theta \left(\sec^2\theta + \tan^2\theta \right) + b\ cosec\theta \left(cosec^2\theta + \cot^2\theta \right)$

For $0 < \theta < \frac{\pi}{2}$, all trigonometric ratios are positive

Also, a > 0 and b > 0

$\because \frac{d^2 l}{d\theta^2}$ is positive.

Thus, l is least when $\tan\theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$
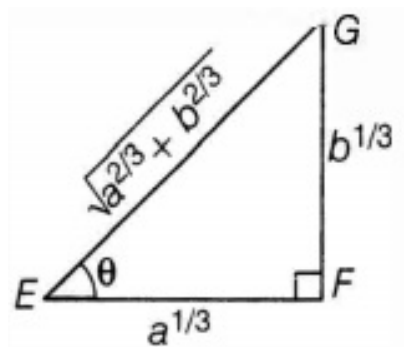
$\because$ Least value of,

$$l = a\sec\theta + b\,cosec\theta$$

$$= a\frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} + b\frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}$$

$$= \sqrt{a^{2/3}+b^{2/3}}\left(a^{2/3}+b^{2/3}\right) = \left(a^{2/3}+b^{2/3}\right)^{3/2}$$

$$\left[\therefore \text{in } \Delta EFG, \tan\theta = \frac{b^{1/3}}{a^{1/3}}, \sec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}} \text{ and } \cos ec\theta = \frac{\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}}\right]$$