CBSE Test Paper 01
Chapter 6 Application of Derivatives

1. The instantaneous rate of change at t = 1 for the function f (t) =te^{-t} + 9 is
   a. 2
   b. 9
   c. -1
   d. -0
2. The function f (x) = x^2, for all real x, is
   a. Neither decreasing nor increasing
   b. Increasing
   c. Decreasing
   d. None of these
3. The slope of the tangent to the curve x = a sint, y = a $\left\{\cos t+\log(\tan\frac{t}{2})\right\}$ at the point ‘t’ is
   a. $\tan\frac{t}{2}$
   b. none of these
   c. tan t
   d. cot t
4. The function f (x) = x^2 - 2x is strict decreasing in the interval
   a. none of these
   b. R
   c. $[1,\infty)$
   d. $(-\infty,\ 1)$
5. The equation of the tangent to the curve y^2 = 4ax at the point (at^2, 2at) is
   a. ty = x + at^2

b. none of these

c. tx + y =at^3

d. ty = x - at^2

6. The maximum value of $\left(\frac{1}{x}\right)^x$ is ________.
7. The minimum value of f if f(x) = sin x in [$\frac{-\pi}{2}$, $\frac{\pi}{2}$] is ________.
8. The equation of normal to the curve y = tan x at (0, 0) is ________.

9. Find the approximate value of f(3.02) where f(x) = 3x^2 + 5x + 3.

10. If the line ax+by+c=0 is a normal to the curve xy=1,then show that either a>0,b<0 or a<0,b>0

11. Find the interval in which the function f(x) = x^{2}e^{-x} is increasing.

12. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

13. Find the approximate value of $(1.999)^5$.

14. Show that the function f(x) = 4x^3 - 18x^2 + 27x - 7 is always increasing on R.

15. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$.

16. A particle moves along the curve 6y = x^3 + 2. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x – coordinate.

17. Find the equation of tangent to the curve $y = \frac{x-7}{x^2-5x+6}$ at the point, where it cuts the X-axis.

18. Show that semi – vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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Solution

1. (d) 0, **Explanation:** $f'(t) = te^{-t}(-1) + e^{-t} \Rightarrow f'(1) = -e^{-1} + e^{-1} = 0$
2. (a) Neither decreasing nor increasing, **Explanation:** f(x) = x^2
   $\Rightarrow$ f'(x) = 2x for all x in R.
   Since f '(x) = 2x > 0 for x >0, and f ' (x) = 2x< 0 for x < 0, therefore on R, f is neither increasing nor decreasing. Infact , f is strict increasing on [0,∞) and strict decreasing on (- $\infty, 0$].
3. (d) cot t, **Explanation:** Given, $x = asint, y = a\left\{\cos t + \log(\tan\frac{t}{2})\right\}$
   $dxdt = a\cos t, \frac{dy}{dt} = a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}.sec^2\frac{t}{2}.\frac{1}{2}\right] = a\left[-\sin t + \frac{1}{2sin\frac{t}{2}.cos\frac{t}{2}}\right]$
   $= a\left[-\sin t + \frac{1}{sint}\right] = a\frac{cos^2t}{sint}$
   Slope of the tangent$= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\frac{cos^2t}{sint}}{a\cos t} = \cot t$
4. (d) $(-\infty, 1)$, **Explanation:** f ' (x) = 2x – 2 = 2 (x - 1) <0 if x < 1 i.e. x $x \in (-\infty, 1)$.
   Hence f is strict decreasing in$(-\infty, 1)$
5. (a) ty = x +at^2, **Explanation:** $y^2 = 4ax$
   $\Rightarrow 2y\frac{dy}{dx} = 4a$
   $\Rightarrow \frac{dy}{dx} = \frac{2a}{y}$
   $\Rightarrow \frac{dy}{dx}$ at $(at^2, 2at)$ is $\frac{2a}{2at} = \frac{1}{t}$
   $\Rightarrow$ Slope of tangent $= m = \frac{1}{t}$
   Hence, equation of tangent is $y - y_1 = m(x - x_1)$
   $\Rightarrow y - 2at = \frac{1}{t}(x - at^2)$
   $\Rightarrow yt - 2at^2 = x - at^2$
   $\Rightarrow yt = x + at^2$
6. $e^{\frac{1}{e}}$
7. -1
8. x + y = 0
9. $x = 3, \Delta x = 0.02$
   $f(x + \Delta x) = f(x) + f'(x)\Delta x$

$f(x+\Delta x)=(3x^2+5x+3)+(6x+5)\times 0.02$

Put $x=3, \Delta x=0.02$

f(3.02)={3(9)+5(3)+3}+{6(3)+5}×0.02 =45+0.46

f(3.02) = 45.46

10. we have, xy =1

$\Rightarrow y=\frac{1}{x}$

$\therefore \ \frac{dy}{dx}=-\frac{1}{x^2}$

The slope of the normal = x^2

If ax+by+c=0 is normal to the curve xy=1,then

$x^2=-\frac{a}{b}$ $[\because slope\ of\ normal\ =-\frac{coeff.\ of\ x}{coeff.of\ y}]$

$\therefore\ -\frac{a}{b}>0$

$\Rightarrow\ a>0, b<0\ or\ a<0, b>0$

11. f(x) = x^2e^{-x}

Differentiating w.r.t x, we get,

f'(x) = $-x^2e^{-x}+2xe^{-x}=xe^{-x}(2-x)$

For increasing function, f'(x)≥ 0

$xe^{-x}(2-x)\geq 0$

$x(2-x)\geq 0$ [$\because\ e^{-x}$ is always positive]

$x(x-2)\leq 0$ [since - (x - 2) will change the inequality)

Here x < 0 & (x - 2) > 0 ⇒ x < 0 & x > 2 ⇒ 0 < x < 2

But when x > 0 & (x - 2) < 0 ⇒ x > 0 & x < 2

$0\leq x\leq\ 2$

12. Let r be the radius of sphere and V be its volume.

Then V = $\frac{4}{3}\pi r^3$........(i)

Given, $\frac{dV}{dt}$ = 3 cm^3/s

Differentiating (i) both sides w.r.t x,we get,

$\frac{dV}{dt}=\frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$

$\Rightarrow\quad 3=\frac{4}{3}(3\pi r^2)\frac{dr}{dt}$

$\Rightarrow\quad \frac{dr}{dt}=\frac{3}{4\pi r^2}$.......(ii)

Now, let S be the surface area of sphere, then S = $4\pi r^2$

$\Rightarrow\quad \frac{dS}{dt}=4\pi(2r)\frac{dr}{dt}$

$\Rightarrow\quad \frac{dS}{dt}=8\pi r\left(\frac{3}{4\pi r^2}\right)$[using Eq.(ii)]

$\Rightarrow \quad \left(\frac{dS}{dt}\right) = \frac{6}{r}$

when r = 2, then $\frac{dS}{dt} = \frac{6}{2}$ = 3 cm^2/s

Therefore,the rate of inrcrease of the surface area of sphere is 3 cm^2/s when it's radius is 2 cm

13. Let x = 2

    and $\Delta x = -0.001\ [\because 2 - 0.001 = 1.999]$

    let y = x^5

    On differentiating both sides w.r.t. x, we get

    $\frac{dy}{dx} = 5x^4$

    Now, $\Delta y = \frac{dy}{dx}.\Delta x = 5x^4 \times \Delta x$

    $= 5 \times 2^4 \times [-0.001]$

    $= -80 \times 0.001 = -0.080$

    $\therefore (1.999)^5 = y + \Delta y$

    $= 2^5 + (-0.080)$

    = 32 - 0.080 = 31.920

14. Here, f(x) =$4x^3 - 18x^2 + 27x - 7$

    On differentiating both sides w.r.t. x, we get

    f'(x) = $12x^2 - 36x + 27$

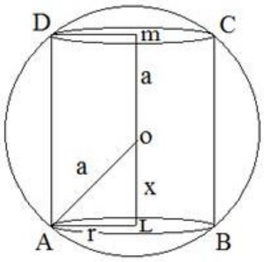
    $\Rightarrow$f'(x) = 3($4x^2$ -12 + 9)

    $\Rightarrow$ f'(x) = $3(x - 3)^2$

    $\Rightarrow$ f'(x) ≥ 0

    Since, a perfect square number cannot be negative]

    $\therefore$ Given function f(x) is an increasing function on R.

15. 

    $v = \pi r^2.2x \quad \begin{bmatrix} \because OL = x \\ LM = 2x \end{bmatrix}$

    $= \pi.\left(a^2 - x^2\right).2x$

    $V = 2\pi\left(a^2x - x^3\right)$

$\frac{dv}{dx}=2\pi\left(a^2-3x^2\right)$
$\frac{d^2v}{dx^2}=2\pi\left[0-6x\right]$
$=-12\pi x$

For maximum/minimum

$\frac{dv}{dx}=0$

$2\pi\left[a^2-3x^2\right]=0$

$a^2=3x^2\Rightarrow\sqrt{\frac{a^2}{3}}=x$

$\Rightarrow x=\frac{a}{\sqrt{3}}$

$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{a}{\sqrt{3}}}=-12\pi.\frac{a}{\sqrt{3}}$

= - tive maximum

Volume is maximum at $x=\frac{a}{\sqrt{3}}$

Height of cylinder of maximum volume is

= 2x

$=2\times\frac{a}{\sqrt{3}}$

$=\frac{2a}{\sqrt{3}}$

16. Given curve is 6y = x^3 + 2 ...(i)

so, $6\frac{dy}{dt}=3x^2\frac{dx}{dt}$

$\Rightarrow 6\times 8\frac{dx}{dt}=3x^2\frac{dx}{dt}\ \left[\because\frac{dy}{dt}=8\frac{dx}{dt}\right]$

$\Rightarrow 16=x^2$

$\Rightarrow x=\pm 4$

Put the value of x in equation (1)

When x = 4

6y = $(4)^3$ + 2

$\Rightarrow$ 6y = 64 + 2

$\Rightarrow$ 6y = 66

$\therefore y=\frac{66}{6}=11$

So, point is (4, 11)

Now, When x = - 4

6y = (- 4$\}^3$ + 2

= - 64 + 2

$\therefore y = \frac{-62}{6} = \frac{-31}{3}$

So the point is $\left(-4, \frac{-31}{3}\right)$

17. Given equation of curve is

$y = \frac{x-7}{x^2-5x+6}$ ....(i)

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{(x^2-5x+6)\cdot 1-(x-7)(2x-5)}{(x^2-5x+6)^2}\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{\left[(x^2-5x+6)-y(x^2-5x+6)(2x+5)\right]}{(x^2-5x+6)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1-(2x-5)y}{x^2-5x+6}$[dividing numerator and denominator by x^2 - 5x + 6]

Also, given that curve cuts X-axis, so its y-coordinate is zero.

Put y = 0 in Eq. (i), we get

$\frac{x-7}{x^2-5x+6} = 0$

$\Rightarrow$ x= 7

So, curve passes through the point (7, 0).

Now, slope of tangent at (7,0) is

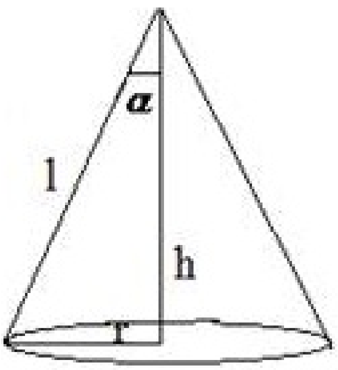
$m = \left(\frac{dy}{dx}\right)_{(2,0)} = \frac{1-0}{49-35+6} = \frac{1}{20}$

Hence, the required equation of tangent passing through the point (7, 0) having slope 1/20 is

y - 0 = $\frac{1}{20}$ (x - 7)

$\Rightarrow$ 20y = x - 7

$\therefore$ x - 20y = 7

18. 

$s = \pi r^2 + \pi r l$ (given)

$\Rightarrow l = \frac{s-\pi r^2}{\pi r}$

Let v be the volume

$v = \frac{1}{3}\pi r^2 h$

$\Rightarrow v^2 = \frac{1}{9}\pi^2 r^4 h^2 \ \left[h^2 = l^2 - r^2\right]$
$\Rightarrow v^2 = \frac{1}{9}\pi^2 r^4 \left(l^2 - r^2\right)$
$\Rightarrow v^2 = \frac{1}{9}\pi^2 r^4 \left[\left(\frac{s-\pi r^2}{\pi r}\right)^2 - r^2\right]$
$= \frac{1}{9}\pi^2 r^4 \left[\frac{\left(s-\pi r^2\right)^2}{\pi^2 r^2} - \frac{r^2}{1}\right]$
$= \frac{1}{9} r^2 \left[\left(s - \pi r^2\right)^2 - \pi^2 r^4\right]$
$= \frac{1}{9} r^2 \left[s^2 + \pi^2 r^4 - 2s\pi r^2 - \pi^2 r^4\right]$
$= \frac{1}{9} r^2 \left[s^2 - 2s\pi r^2\right]$
$z = \frac{1}{9}\left[s^2 r^2 - 2s\pi r^4\right]$
$\left[\because v^2 = z\right]$
Now $\frac{dz}{dr} = \frac{1}{9}\left[2rs^2 - 8s\pi r^3\right]$
$0 = \frac{1}{9}\left[2rs^2 - 8s\pi r^3\right]$
$8s\pi r^2 = 2rs^2$
$\implies 4\pi r^2 = s$
Now $\frac{d^2 z}{dx^2} = \frac{1}{9}\left[2s^2 - 24s\pi r^2\right]$
$\left.\frac{d^2 z}{dx^2}\right]_{r^2 = \frac{s}{4\pi}} = \frac{1}{9}\left[25^2 - 24\pi . \frac{5}{4\pi}\right]$
= + ve
Hence minimum
Now $s = 4\pi r^2$
We have $s = \pi r l + \pi r^2$
$4\pi r^2 = \pi r l + \pi r^2$
$\Rightarrow 3\pi r^2 = \pi r l$
$\Rightarrow$ 3 r = l
$\Rightarrow \frac{r}{l} = \frac{1}{3}$
$\Rightarrow \sin\alpha = \frac{1}{3}$
$\therefore \alpha = \sin^{-1}\left(\frac{1}{3}\right)$