**CBSE Test Paper 02**

**Chapter 5 Continuity and Differentiability**

1. $\underset{x \to 0}{Lt} \frac{1-\cos x}{x^2}$ is equal to
   a. 1
   b. -1
   c. $\frac{1}{2}$
   d. 0
2. Let f(x) = $\begin{cases} e^{1/x}, x < 0 \\ x, x \geqslant 0 \end{cases}, then \underset{x \to 0}{Lt} f(x).$
   a. does not exist
   b. is equal to 0
   c. is equal to non – zero real number
   d. None of these
3. Let f and g be differentiable functions such that fog= I, the identity function. If g'(a) = 2 and g(a)=b, then f '(b)=.
   a. -2
   b. None of these
   c. 2
   d. $\frac{1}{2}$
4. $\frac{d^4}{dx^4}(\sin^3 x)$ is equal to
   a. $\frac{3}{4}\cos x - \frac{3^4 \cos 3x}{4}$
   b. None of these
   c. $\frac{3 \sin x - 3^4 \sin 3x}{4}$
   d. $\frac{3}{4}\sin x - \frac{3^4 \cos 3x}{4}$
5. The differential coefficient of log (|log x|) w.r.t. log x is
   a. $\frac{1}{x|\log x|}$

b. $\frac{1}{x \log x}$

c. None of these

d. $\frac{1}{\log x}$

6. The value of c in Rolle's Theorem for the function f(x) = $e^x$ sin x, x $\in [0, \pi]$ is ________.
7. The set of points where the functions f given by f(x) = |x - 3|cos x is differentiable is ________.
8. The derivative of $\log_{10}$x w.r.t. x is ________.
9. Differentiate the following function with respect to x : $\sin(ax+b)$.
10. Differentiate the following function with respect to x: cos(log x + $e^x$), x > 0.
11. Find $\frac{dy}{dx}$ if $y = \frac{\sin(ax+b)}{\cos(cx+d)}$.
12. If $x = a\sec^3\theta$ and $y = a\tan^3\theta$ , find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.
13. Find $\frac{dy}{dx}$ , if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.
14. Examine the continuity of the function $f(x) = x^3 + 2x^2 - 1$ at x = 1.
15. Find the value of k so that the function f is continuous at the indicated point:

$$f(x) = \begin{cases} \dfrac{1-\cos kx}{x\sin x}, & if\ x \neq 0 \\ \dfrac{1}{2}, & if\ x = 0 \end{cases}$$ at x = 0.

16. If y = $\left(x+\sqrt{1+x^2}\right)^n$, then show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$.
17. If $y^x = e^{y-x}$ prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.
18. Find $\frac{dy}{dx}$ , if $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$.

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### Solution

1. c. $\frac{1}{2}$, **Explanation:** $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to0}\frac{\sin x}{2x}=\frac{1}{2}$ (Using L'hospital Rule ).

2. b. is equal to 0, **Explanation:** $\lim\limits_{x\to0^-}f(x)=\lim\limits_{x\to0^-}e^{\frac{1}{x}}=0$
$\lim\limits_{x\to0^+}f(x)=\lim\limits_{x\to0^+}x=0\therefore\lim\limits_{x\to0}f(x)=0$

3. d. $\frac{1}{2}$, **Explanation:** $(fog)'(x)=1\forall x$
$\Rightarrow(fog)'(a)=1$
$\Rightarrow f'(g(a))g'(a)=1$
$\Rightarrow f'(b)g'(a)=\frac{1}{2}$

4. c. $\frac{3\sin x-3^4\sin 3x}{4}$, **Explanation:** $\frac{d}{dx}(\sin^3 x)=3\sin^2 x\cos x$
$\frac{d^2}{dx^2}(\sin^3 x)=\frac{d}{dx}(3\sin^2 x\cos x)=6\sin x\cos^2 x-3\sin^3 x$
$\frac{d^3}{dx^3}(\sin^3 x)=\frac{d}{dx}(6\sin^2 x\cos^2 x-3\sin^3 x)$
$=6\cos^3 x-12\sin^2 x\cos x-9\sin^2 x\cos x=6\cos^3 x-21\sin^2 x\cos x$
$\frac{d^4}{dx^4}(\sin^3 x)=\frac{d}{dx}(6\cos^3 x-21\sin^2 x\cos x)$
$=-18\cos^2 x\sin x-42\sin x\cos^2 x+21\sin^3 x$
$=60\sin x\cos^2 x+21\sin^3 x=-60\sin x(1-\sin^2 x)+21\sin^3 x$
$=-60\sin x+60\sin^3 x+21\sin^3 x=-60\sin x+81\sin^3 x$
$=-60\sin x+81\left[\frac{3\sin x-\sin 3x}{4}\right]=\frac{3\sin z-3^4\sin 3z}{4}$

5. d. $\frac{1}{\log x}$, **Explanation:** Let y = log(|logx|) and z = logx, then ,we have;
$\frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\left(\frac{1}{\log x}\cdot\frac{1}{x}\right)/\left(\frac{1}{x}\right)\Rightarrow\frac{1}{\log x}$

6. $\frac{3\pi}{4}$
7. R - {3}
8. $(\log_{10}e)\frac{1}{x}$
9. Let $y=\sin(ax+b)$
$\therefore\frac{dy}{dx}=\cos(ax+b)\frac{d}{dx}(ax+b)$
$=\cos(ax+b)(a+0)$

$= a\cos(ax + b)$

10. Let y = cos(logx + e$^{x}$)

$\therefore \frac{dy}{dx} = -\sin(\log x + e^x)\frac{d}{dx}(\log x + e^x)$

$= -\sin(\log x + e^x).\left(\frac{1}{x} + e^x\right)$

11. $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

$\frac{dy}{dx} = \frac{\cos(cx+d)\frac{d}{dx}\sin(ax+b) - \sin(ax+b)\frac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$

$\frac{dy}{dx} = \frac{\cos(cx+d)\cos(ax+b).a + \sin(ax+b)\sin(cx+d).c}{\cos^2(cx+d)}$

12. We have $x = a\sec^3\theta$ and $y = a\tan^3\theta$

Differentiating w.r.t. $\theta$ , we get

$\frac{dx}{d\theta} = 3a\sec^2\theta\frac{d}{d\theta}(\sec\theta) = 3a\sec^3\theta\tan\theta$

and $\frac{dy}{d\theta} = 3a\tan^2\theta\frac{d}{d\theta}(\tan\theta) = 3a\tan^2\theta\sec^2\theta$

Thus $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \frac{\tan\theta}{\sec\theta} = \sin\theta$

Hence, $\left(\frac{dy}{dx}\right)_{at\ \theta = \frac{\pi}{3}} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

13. Given: $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

To simplify the given Inverse Trigonometric function,we put, $x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow y = 2\tan^{-1}x$

$\Rightarrow \frac{dy}{dx} = 2.\frac{1}{1+x^2} = \frac{2}{1+x^2}$

14. We have, $f(x) = x^3 + 2x^2 - 1$ at x = 1

$\therefore \lim\limits_{x\to 1^+} f(x) = \lim\limits_{h\to 0}(1+h)^3 + 2(1+h)^2 - 1 = 2$

and $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{h\to 0}(1-h)^3 + 2(1-h)^2 - 1 = 2$

$\therefore \lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^-} f(x)$ and $f(1) = 1 + 2 - 1 = 2$

So, f(x) is continuous at x = 1.

15. We have, $f(x) = \begin{cases} \dfrac{1-\cos kx}{x\sin x},\ if\ x \neq 0 \\ \dfrac{1}{2},\ if\ x = 0 \end{cases}$

At x = 0, $LHL = \lim\limits_{x\to 0^-}\frac{1-\cos kx}{x\sin x} = \lim\limits_{h\to 0}\frac{1-\cos k(0-h)}{(0-h)\sin(0-h)}$

$$= \lim_{h \to 0} \frac{1-\cos(-kh)}{-h\sin(-h)}$$

$$= \lim_{h \to 0} \frac{1-\cos kh}{h\sin\ h} [\because \cos(-\theta) = \cos\theta, \sin(-\theta) = -\sin\theta]$$

$$= \lim_{h \to 0} \frac{1-1+2\sin^2\frac{kh}{2}}{h\sin\ h} \left[\because \cos\theta = 1 - 2\sin^2\frac{\theta}{2}\right]$$

$$= \lim_{h \to 0} \frac{2\sin^2\frac{kh}{2}}{h\sin\ h}$$

$$= \lim_{h \to 0} \frac{2\sin\frac{kh}{2}}{\frac{kh}{2}} \cdot \frac{\sin\frac{kh}{2}}{\frac{kh}{2}} \cdot \frac{1}{\frac{\sin\ h}{h}} \cdot \frac{k^2h/4}{h}$$

$$= \frac{2k^2}{4} = \frac{k^2}{2} \left[\because \lim_{h \to 0} \frac{\text{sinh}}{h} = 1\right]$$

Also, $f(0) = \frac{1}{2} \Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$

16. According to the question, y = $\left(x + \sqrt{1+x^2}\right)^n$ ..........(i)

Differentiating both sides w.r.t x,

$\Rightarrow \frac{dy}{dx} = n\left(x + \sqrt{1+x^2}\right)^{n-1}\left(1 + \frac{2x}{2\sqrt{1+x^2}}\right)$ [ Using chain rule of derivative]

$$\Rightarrow \quad \frac{dy}{dx} = n\left(x + \sqrt{1+x^2}\right)^{n-1}\left(\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{n\left(x+\sqrt{1+x^2}\right)^n}{\sqrt{1+x^2}}$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{ny}{\sqrt{1+x^2}}$ [ From Equation(i)]

$\Rightarrow \sqrt{1+x^2}\frac{dy}{dx} = ny$.........(ii)

Differentiating both sides w.r.t x again,

$$\Rightarrow \sqrt{1+x^2}\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{1+x^2}} \cdot \frac{dy}{dx} = n\frac{dy}{dx}$$

$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n \cdot \sqrt{1+x^2}\frac{dy}{dx}$ [ multiplying both sides by $\sqrt{1+x^2}$]

$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2} \cdot \frac{ny}{\sqrt{1+x^2}}$ [ From Equation(ii)]

$\therefore \left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$ Hence Proved

17. We have, $y^x = e^{y-x}$

$$\Rightarrow \log y^x = \log e^{y-x}$$

$$\Rightarrow x\log y = (y-x).\log e = (y-x)[\because \log e = 1]$$

$\Rightarrow \log y = \frac{(y-x)}{x}$ ...(i)

Now, differentiating w.r.t. x, we get

$$\frac{d}{dx}\log y.\frac{dy}{dx}=\frac{d}{dx}\frac{(y-x)}{x}$$
$$\Rightarrow\frac{1}{y}\cdot\frac{dy}{dx}=\frac{x.\frac{d}{dx}(y-x)-(y-x).\frac{d}{dx}.x}{x^2}$$
$$\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{x\left(\frac{dy}{dx}-1\right)-(y-x)}{x^2}$$
$$\Rightarrow\frac{x^2}{y}\cdot\frac{dy}{dx}=x\frac{dy}{dx}-x-y+x$$
$$\Rightarrow\frac{dy}{dx}\left(\frac{x^2}{y}-x\right)=-y$$
$$\therefore\frac{dy}{dx}=\frac{-y^2}{x^2-xy}=\frac{-y^2}{x(x-y)}$$
$$=\frac{y^2}{x^2(y-x)}.x=\frac{y^2}{x^2}\cdot\frac{1}{\frac{(y-x)}{x}}$$
$$=\frac{(1+\log y)^2}{\log y}\left[\because\log y=\frac{y-x}{x}\Rightarrow\log y=\frac{y}{x}-1\Rightarrow 1+\log y=\frac{y}{x}\right]$$
Hence Proved.

18. Let $y=u+v$

Where $u=(x\cos x)^x, v=(x.\sin x)^{\frac{1}{x}}$

$u=(x\cos x)^x$

Taking log both sides

$\log u=\log(x\cos x)^x$

$\log u=x.\log(x.\cos x)$

Differentiating both sides

$$\frac{1}{u}\cdot\frac{du}{dx}=x.\frac{1}{x\cos x}(-x\sin x+\cos x.1)+\log(x\cos x).1$$
$$\frac{du}{dx}=u\left[-x\tan x+1+\log(x.\cos x)\right]$$
$v=(x.\sin x)^{\frac{1}{x}}$

Taking log both side

$\log v=\log(x.\sin x)^{\frac{1}{x}}$

$\log v=\frac{1}{x}.\log(x.\sin x)$

Differentiate

$$\frac{1}{v}\cdot\frac{dv}{dx}=\frac{1}{x}\cdot\frac{1}{x.\sin x}(x\cos x+\sin x.1)+\log(x.\sin x)\left(-\frac{1}{x^2}\right)$$
$$\frac{dv}{dx}=v\left[\frac{\cot x}{x}+\frac{1}{x^2}-\frac{\log(x\sin x)}{x^2}\right]$$
$$\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$$
$$=(x\cos x)^x\left[-x.\tan x.1+\log(x.\log x)\right]+(x.\sin x)^{\frac{1}{x}}\left[\frac{\cot x}{x}+\frac{1}{x^2}-\frac{\log(x.\sin x)}{x^2}\right]$$