1. The roots of the equation det. $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0$ are

   a. None of these

   b. 2 and 3

   c. 1, 2 and 3

   d. 1 and 3

2. If A' is the transpose of a square matrix A, then

   a. $|A| + |A'| = 0$

   b. $|A| = |A'|$

   c. $|A| \neq |A'|$

   d. None of these

3. If f(x) = $\begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then, $f(\frac{\pi}{3})$ =.

   a. 0

   b. 1

   c. -1

   d. 2

4. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are

   a. −1, −2

   b. −1, 2

   c. 1, −2

   d. 1, 2

5. If A and B are any 2×2 matrices , then det. (A+B) = 0 implies

   a. det A + det B = 0

   b. det A = 0 or det B = 0

   c. None of these

d. det A = 0 and det B = 0

6. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then x is _____.

7. Multiplying a determinant by k means multiplying the elements of only one row (or one column) by _____.

8. If elements of a row (or a column) in a determinant can be expressed as the sum of two or more elements, then the given determinant can be expressed as the _____ of two or more determinants.

9. Find adj A for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.

10. $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular or not.

11. Evaluate $2 \begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

12. Evaluate: $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

13. Find the area of Δ whose vertices are (3, 8) (-4, 2) and (5, 1).

14. Find the equation of the line joining A (1, 3) and B (0, 0) using det. Find K if D (K, 0) is a point such that area of ΔABD is 3 square unit.

15. If A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.

16. If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find matrix B such that AB = I.

17. Using properties of determinants, prove that
$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$

18. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$. find AB and use this result in solving the following system of equation.

x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1

Solution

1. c. 1 , 2 and 3

    Explanation: Expanding along C_1

    $$\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0 \Rightarrow (1 - x)(2 - x)(3 - x) = 0 \Rightarrow x = 1, 2 , 3.$$

2. b. $|A| = |A'|$

    Explanation: The determinant of a matrix A and its transpose always same. Because if we interchange the rows into column in a determinant the value of determinant remains unaltered.

3. c. -1

    Explanation: $\begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$

    Put $x = \frac{\pi}{3}$, $\begin{vmatrix} 2\cos\frac{\pi}{3} & 1 & 0 \\ 1 & 2\cos\frac{\pi}{3} & 1 \\ 0 & 1 & 2\cos\frac{\pi}{3} \end{vmatrix}$

    $\Rightarrow \begin{vmatrix} 2 \cdot \frac{1}{2} & 1 & 0 \\ 1 & 2 \cdot \frac{1}{2} & 1 \\ 0 & 1 & 2 \cdot \frac{1}{2} \end{vmatrix}$

    $\Rightarrow \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \Rightarrow 1(0) - 1(1) = -1$

4. b. $-1 , 2$

    Explanation: $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

    Apply, $R_3 \rightarrow R_3 - R_1$, $R_2 \rightarrow R_2 - R_1$,

$$\Rightarrow \begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} = 0$$

$\Rightarrow$ -6(5x^2 - 20) + 15(2x - 4) =0

$\Rightarrow$ (x - 2)(x + 1) = 0 $\Rightarrow$ x= 2 , -1.

5. c. None of these

      Explanation: If det (A+B)=0 implies that A+B a Singular matrix.

6. x = ± 3

7. k

8. sum

9. $adjA = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

$$\because \quad A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$
change sign   inter-change

10. $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix}$

= 8 - 8

= 0

Hence A is singular

11. According to the question, we have to evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

Now, $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix} = 2[35 - (20)]$

$= 2 \times 15 = 30$

12. Let $\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Expanding along first row,

$= \cos\alpha\cos\beta(\cos\alpha\cos\beta - 0) - \cos\alpha\sin\beta(-\cos\alpha\sin\beta - 0)$

$- \sin\alpha(-\sin\alpha\sin^2\beta - \sin\alpha\cos^2\beta)$

$= \cos^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta + \sin^2\alpha(\sin^2\beta + \cos^2\beta)$

$= \cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha(\sin^2\beta + \cos^2\beta)$

$= \cos^2\alpha + \sin^2\alpha$

= 1

13. $\Delta = \dfrac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \dfrac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[3(2-1) - 8(-4-5) + 1(-4+10)]$

$= \dfrac{1}{2}[3 + 72 - 14] = \dfrac{61}{2}$

14. Let P (x, y) be any point on AB. Then the equation of line AB is,

$\dfrac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$

y = 3x

Area $\Delta ABD = 3$ square unit

$\dfrac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ K & 0 & 1 \end{vmatrix} = \pm 3$

$k = \pm 2$

15. If A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then we have to find $(A')^{-1}$.

Now, A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ Therefore, we have, $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{vmatrix}$

= 1 (-1 - 8) + 2 (0 + 8) + 3 (0 - 2)

[expanding along R_1]

=-9+16-6=1$\neq$0

Therefore, A is non-singular matrix and hence its inverse exists.

Cofactors of an element of |A| are given by

$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} = (-1 - 8) = -9$

$A_{12} = (-1)^{1} + 2 \begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} = -(0 + 8) = -8$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -1 \\ -2 & 2 \end{vmatrix} = (0-2) = -2$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} = -(-2-6) = 8$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = (1+6) = 7$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2-4) = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ -1 & 4 \end{vmatrix} = (-8+3) = -5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4-0) = -4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = (-1-0) = -1$$

Thus, adj A = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}$ adj $A = \frac{1}{1} \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$

Now, $(A')^{-1} = (A^{-1})' = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}' = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

16. $|A| = 2 \neq 0$

Therefore A^{-1} exists

AB = I

A^{-1} AB = A^{-1}I

B = A^{-1}

$$adjA = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adjA)$$

$$= \frac{1}{2} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

Hence $B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

17. According to the question,we have to use properties of determinants to prove that,

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

Let LHS = $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

Therefore,on applying $C_1 \to C_1 + C_2 + C_3$ we get,

$$\Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}$$

on taking 2 common from C_1,we get,

$$\Delta = 2\begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

On applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3$ - C12,

we get

$$\Delta = 2\begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

on applying $C_1 \to C_1 + C_2 + C_3, we\ get,$

$$\Delta = 2\begin{vmatrix} a & -b & -c \\ p & -q & -r \\ x & -y & -z \end{vmatrix}$$

$\therefore \Delta = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ [taking (-1) common from both C_2 and C_3]

= RHS

18. x - y + z = 4

x - 2y - 2z = 9

2x + y + 3z = 1

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

AX = C

$$AB = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

AB = 8I

$$A^{-1} = \tfrac{1}{8} B \begin{bmatrix} \because A^{-1}AB = 8A^{-1}I \\ B = 8A^{-1} \end{bmatrix}$$

$$= \tfrac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

X = A^{-1}C

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

x = 3, y = -2, z = -1