

**CBSE Test Paper 02**  
**Chapter 2 Inverse Trigonometric Functions**

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1.  $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{3}$  is equal to
  - a. None of these
  - b.  $\frac{\pi}{2}$
  - c.  $\frac{\pi}{4}$
  - d.  $\frac{3\pi}{4}$
2.  $\sin\left\{\sin^{-1}\left(\frac{\sqrt{3}}{5}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{5}\right)\right\}$  is equal to
  - a. 1
  - b.  $\frac{\pi}{2}$
  - c. None of these
  - d. 0
3.  $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$  holds true for
  - a. all  $x \in R - \{0\}$
  - b. all  $x > 0$
  - c. all  $x > 1$
  - d. all  $x \in R$
4.  $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$  is equal to
  - a.  $\frac{3\pi}{4}$
  - b. None of these
  - c.  $\frac{5\pi}{4}$
  - d.  $-\frac{\pi}{4}$
5. If  $\sin^{-1}x = \frac{\pi}{5}$ , then  $\cos^{-1}x$  is equal to
  - a.  $\frac{5\pi}{4}$
  - b.  $\frac{7\pi}{4}$
  - c.  $\frac{3\pi}{10}$

- d.  $\frac{\pi}{10}$
6. The principle value branch of  $\cos^{-1}x$  is \_\_\_\_\_.
7. If  $\cos(\tan^{-1}x + \cot^{-1}\sqrt{3}) = 0$ , then value of x is \_\_\_\_\_.
8. The set of values of  $\sec^{-1}\left(\frac{1}{2}\right)$  is \_\_\_\_\_.
9. Find the value of  $\cos(\sec^{-1}x + \csc^{-1}x)$ . (1)
10. Find the value of  $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ . (1)
11. Using the principal values, evaluate  $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$ .
12. If  $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\csc\theta)$  then show that  $\theta = \frac{\pi}{4}$ , where n is any integer. (2)
13. Evaluate:  $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ .
14. Evaluate:  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ . (2)
15. Find the value of  $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos(\tan^{-1}\sqrt{3})$ .
16.  $\sin(\tan^{-1}x) =$ . (4)
17. Prove that  $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$ . (4)
18. If  $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$ . Prove that  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\alpha = \sin^2\alpha$ .

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**Solution**

1. c.  $\frac{\pi}{4}$

**Explanation:**  $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{\frac{2 \cdot \frac{1}{3}}{1 - (\frac{1}{3})^2}}{1 - \frac{1}{7} \cdot \frac{3}{4}}$   
 $= \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

2. a. 1

**Explanation:**  $\sin\left\{\sin^{-1}\left(\frac{\sqrt{3}}{5}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{5}\right)\right\} = \sin(\pi/2) = 1.$  [  
 $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ ]

3. b. all  $x > 0$

**Explanation:**  $\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$   
 $\tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \tan^{-1}x = \cot^{-1}x \quad \forall x \in R$   
 $\therefore \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$   
 $\Rightarrow x > 0$

Since, we know that  $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$  when  $x > 0$

4. a.  $\frac{3\pi}{4}$

**Explanation:** We know that principle value branch of  $\cos^{-1}$  is  $[0, \pi]$

and  $\frac{3\pi}{4} \notin [0, \pi]$  but  $\left(2\pi - \frac{5\pi}{4}\right) \in [0, \pi]$

$\therefore \cos^{-1}\left(\cos\frac{5\pi}{4}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{4}\right)\right) = \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$

5. c.  $\frac{3\pi}{10}$

**Explanation:**  $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$   
 $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$

6.  $[0, \pi]$

7.  $\sqrt{3}$

8.  $\phi$

9.  $\cos(\sec^{-1}x + \csc^{-1}x)$

$\cos\frac{\pi}{2} = 0$

10.  $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$$\begin{aligned}
&= \tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3}) [\because \cot^{-1}(-x) = \pi - \cot^{-1}x] \\
&= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3} \\
&= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi [\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}] \\
&= \frac{\pi}{2} - \frac{\pi}{1} = \frac{-\pi}{2}
\end{aligned}$$

11. using the principal values, we have to evaluate  $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$ .

$$\begin{aligned}
&= \tan^{-1}\left(\tan \frac{\pi}{4}\right) + \sin^{-1}\left(-\sin \frac{\pi}{6}\right) \\
&\quad \left[\because \tan \frac{\pi}{4} = 1 \text{ and } \sin \frac{\pi}{6} = \frac{1}{2}\right] \\
&= \tan^{-1}\left(\tan \frac{\pi}{4}\right) + \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] \\
&\quad [\because -\sin \theta = \sin(-\theta)] \\
&= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12} \\
&\quad [\because \tan^{-1}(\tan \theta) = \theta; \forall \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \sin^{-1}(\sin \theta) = \theta; \forall \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]]
\end{aligned}$$

12. We have,  $2\tan^{-1}(\cos \theta) = \tan^{-1}(2\cos ec\theta)$ ,

$$\begin{aligned}
&\Rightarrow \tan^{-1}\left(\frac{2\cos \theta}{1-\cos^2 \theta}\right) = \tan^{-1}(2\cos ec\theta) \\
&\quad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right] \\
&\Rightarrow \left(\frac{2\cos \theta}{\sin^2 \theta}\right) = (2\cos ec\theta) \\
&\Rightarrow (\cot \theta \cdot 2\cos ec\theta) = (2\cos ec\theta) \Rightarrow \cot \theta = 1 \\
&\Rightarrow \cot \theta = \cot \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}
\end{aligned}$$

13.  $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - [\pi - \sec^{-1}2]$

$$\begin{aligned}
&= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right) \\
&= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}
\end{aligned}$$

14. Let  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

$$\begin{aligned}
&\Rightarrow \sec y = \frac{2}{\sqrt{3}} \\
&\Rightarrow \sec y = \sec \frac{\pi}{6}
\end{aligned}$$

Since, the principal value branch of  $\sec^{-1}$  is  $[0, \pi]$ .

Therefore, Principal value of  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$  is  $\frac{\pi}{6}$ .

15. Let  $\tan^{-1}\frac{2}{3} = x$  and  $\tan^{-1}\sqrt{3} = y$

so that  $\tan x = \frac{2}{3}$  and  $\tan y = \sqrt{3}$

Therefore,

$$\begin{aligned}
&\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos(\tan^{-1}\sqrt{3}) \\
&= \sin(2x) + \cos y
\end{aligned}$$

$$\begin{aligned}
&= \frac{2 \tan x}{1 + \tan^2 x} + \frac{1}{\sqrt{1 + \tan^2 y}} \\
&= \frac{2 \cdot \frac{2}{3}}{1 + \frac{4}{9}} + \frac{1}{\sqrt{1 + (\sqrt{3})^2}} \\
&= \frac{12}{13} + \frac{1}{2} = \frac{37}{26}
\end{aligned}$$

16. Let  $\tan^{-1}x = \theta$

$$\begin{aligned}
&= \frac{x}{1} = \tan \theta \\
\Rightarrow \sin \theta &= \frac{x}{\sqrt{1+x^2}} \\
\Rightarrow \theta &= \sin^{-1} \frac{x}{\sqrt{1+x^2}} \\
\Rightarrow \tan^{-1}x &= \sin^{-1} \frac{x}{\sqrt{1+x^2}} \\
\Rightarrow \sin(\tan^{-1}x) &= \sin \left( \sin^{-1} \frac{x}{\sqrt{1+x^2}} \right) \\
&= \frac{x}{\sqrt{1+x^2}}
\end{aligned}$$

17. To prove,  $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$

$$\begin{aligned}
&\Rightarrow 2 \left[ \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) \right] = \tan^{-1}\left(\frac{4}{3}\right) \\
&\text{LHS} = 2 \left[ \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) \right] \\
&= 2 \left[ \tan^{-1} \left( \frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}} \right) \right] [\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right); xy < 1] \\
&= 2 \tan^{-1} \left( \frac{\frac{9+8}{36}}{\frac{36-2}{36}} \right) \\
&= 2 \tan^{-1} \left( \frac{17}{34} \right) \\
&= 2 \tan^{-1} \left( \frac{1}{2} \right) \\
&= \tan^{-1} \left( \frac{2 \times \left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2} \right) [\because 2 \tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right); -1 < x < 1] \\
&= \tan^{-1} \left( \frac{1}{1 - \frac{1}{4}} \right) \\
&= \tan^{-1} \left( \frac{4}{3} \right) = \text{RHS} \\
&\therefore \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)
\end{aligned}$$

**Hence proved.**

18.  $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$   
 $[\because \cos^{-1}x + \cos^{-1}y = \cos^{-1} \left( xy - \sqrt{1-x^2}\sqrt{1-y^2} \right)]$

$$\cos^{-1} \left[ \frac{x}{a} \cdot \frac{y}{b} - \sqrt{1 - \frac{x^2}{a^2}} \cdot \sqrt{1 - \frac{y^2}{b^2}} \right] = \alpha$$

$$\frac{xy}{ab} - \sqrt{1 - \frac{x^2}{a^2}} \cdot \sqrt{1 - \frac{y^2}{b^2}} = \cos \alpha$$

$$\frac{xy}{ab} - \cos \alpha = \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}}$$

Squaring both sides,

$$\left( \frac{xy}{ab} - \cos \alpha \right)^2 = \left( \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}} \right)^2$$

$$\frac{x^2 y^2}{a^2 b^2} + \cos^2 \alpha - 2 \cdot \frac{xy}{ab} \cdot \cos \alpha = \left( 1 - \frac{x^2}{a^2} \right) \left( 1 - \frac{y^2}{b^2} \right)$$

$$\frac{x^2 y^2}{a^2 b^2} + \cos^2 \alpha - 2 \cdot \frac{xy}{ab} \cdot \cos \alpha = 1 - \frac{y^2}{b^2} - \frac{x^2}{a^2} + \frac{x^2 y^2}{a^2 b^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2 \frac{xy}{ab} \cos \alpha = 1 - \cos^2 \alpha$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2 \frac{xy}{ab} \cos \alpha = \sin^2 \alpha$$