

CBSE Test Paper 01
Chapter 2 Inverse Trigonometric Functions

1. The period of the function $f(x) = \cos 4x + \tan 3x$ is
 - a. $\frac{\pi}{3}$
 - b. π
 - c. None of these
 - d. $\frac{\pi}{2}$
2. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$. Then, $x =$
 - a. $\frac{1}{\sqrt{3}}$
 - b. $\frac{1}{\sqrt{2}}$
 - c. 2
 - d. 1
3. The value of $\tan 15^\circ + \cot 15^\circ$ is
 - a. 4
 - b. Not defined
 - c. $\sqrt{3}$
 - d. $2\sqrt{3}$
4. The values of x which satisfy the trigonometric equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ are:
 - a. ± 2
 - b. $\pm \frac{1}{2}$
 - c. $\pm \frac{1}{\sqrt{2}}$
 - d. $\pm \sqrt{2}$
5. The minimum value of $\sin x - \cos x$ is
 - a. $-\sqrt{2}$

- b. -1
c. 0
d. 1
6. The principle value of $\tan^{-1}\sqrt{3}$ is _____.
7. If $y = 2 \tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ for all x , then _____ < y < _____.
8. The value of $\cos(\sin^{-1}x + \cos^{-1}x)$, $|x| \leq 1$ is _____.
9. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.
10. Write the principal value of $\cos^{-1}1$ [$\cos(680^\circ)$].
11. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$. (1)
12. Find the value of the expression $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$.
13. Solve the equation: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.
14. Find the value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$. (2)
15. Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$.
16. Solve for x , $\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}, \sqrt{6} > x > 0$.
17. Find the value of the following: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.
18. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$.

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Solution

1. b. π , **Explanation:** $f(\pi) = (\cos 4\pi + \tan 3\pi)$ gives the same value as $f(0)$.
 Therefore, the period of the function is π .
2. a. $\frac{1}{\sqrt{3}}$, **Explanation:** $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$
 Put $x = \tan\theta$
 $3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$
 $3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$
 $3.2\theta - 4.2\theta + 2.2\theta = \frac{\pi}{3} \Rightarrow 2\theta = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$
 $\therefore \tan^{-1}x = \frac{\pi}{6} \Rightarrow x = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$
3. a. 4, **Explanation:** $\tan 15^\circ + \cot 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$
 $= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{2} = \frac{8}{2} = 4$
4. c. $\pm\frac{1}{\sqrt{2}}$, **Explanation:** $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$
 $\tan^{-1}\left[\frac{\left(\frac{x-1}{x-2}\right) + \left(\frac{x+1}{x+2}\right)}{1 - \left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$
 $\tan^{-1}\left[\frac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x+1)(x-1)}\right] = \frac{\pi}{4}$
 $\left(\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right) = \tan^{-1}\left(\frac{\pi}{4}\right)$
 $\left(\frac{2x^2-4}{-3}\right) = 1$
 $\therefore 2x^2 - 4 = -3$
 $\Rightarrow 2x^2 = 1$
 $x = \pm\frac{1}{\sqrt{2}}$
5. a. $-\sqrt{2}$, **Explanation:** Since, range of sine function and cosine function is $[-1, 1]$.
 But, sine is increasing function and cosine is decreasing function. Therefore, the lowest that both together can attain is -45° .
 $\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}}\right) = -\sqrt{2}$
6. $\frac{\pi}{3}$

7. $-2\pi, 2\pi$

8. 0

9. Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \theta$

$$\Rightarrow \sin \theta = \frac{1}{\sqrt{2}}$$

We know that $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \sin \theta = \sin \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$

Therefore, principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$

10. We know that, principal value branch of $\cos^{-1} x$ is $[0, 180^\circ]$.

Since, $680^\circ \in [0, 180^\circ]$, so write 680° as $2 \times 360^\circ - 40^\circ$

$$\text{Now, } \cos^{-1}[\cos(680^\circ)] = \cos^{-1} [\cos(2 \times 360^\circ - 40^\circ)]$$

$$= \cos^{-1}(\cos 40^\circ) [\because \cos(4\pi - \theta) = \cos \theta]$$

Since, $40^\circ \in [0, 180^\circ]$

$$\therefore \cos^{-1}[\cos(680^\circ)] = 40^\circ$$

$$[\because \cos^{-1}(\cos \theta) = \theta; \forall \theta \in [0, 180^\circ]]$$

which is the required principal value.

11. LHS = $\tan^{-1}\sqrt{x}$

$$\text{Let } \tan \theta = \sqrt{x}$$

$$\tan^2 \theta = x$$

$$\text{R.H.S.} = \frac{1}{2} \cos^{-1} \left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right)$$

$$= \frac{1}{2} \cos^{-1} (\cos 2\theta) = \frac{1}{2} \times 2\theta = \theta$$

$$= \tan^{-1}\sqrt{x}$$

12. $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

$$= \tan^{-1}\left(\tan \frac{4\pi-\pi}{4}\right)$$

$$= \tan^{-1}[\tan(\pi - \frac{\pi}{4})]$$

$$= \tan^{-1}[-\tan \frac{\pi}{4}]$$

$$= \tan^{-1} \tan\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

13. $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$$\Rightarrow \tan^{-1}\left(\frac{2 \cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$$

$$\Rightarrow \frac{2 \cos x}{1-\cos^2 x} = \frac{2}{\sin x}$$

$$\Rightarrow \frac{\cos x}{\sin x} = 1$$

$$\Rightarrow \cot x = 1 \Rightarrow x = \frac{\pi}{4}$$

$$\begin{aligned}
14. \quad & \sin^{-1} \left(\sin \frac{2\pi}{3} \right) \\
&= \sin^{-1} \left(\sin \frac{3\pi - \pi}{3} \right) \\
&= \sin^{-1} \left[\sin \left(\pi - \frac{\pi}{3} \right) \right] \\
&= \sin^{-1} \sin \frac{\pi}{3} = \frac{\pi}{3}
\end{aligned}$$

15. To prove, $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

$$\begin{aligned}
\text{LHS} &= \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) \\
&= \tan^{-1}(\tan \frac{\pi}{4}) + \frac{\pi}{2} - \cot^{-1}(2) + \frac{\pi}{2} - \cot^{-1}(3) \quad [\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}] \\
&= \frac{\pi}{4} + \pi - [\cot^{-1}(2) + \cot^{-1}(3)] \quad [\because \tan^{-1}(\tan \theta) = \theta; \forall \theta \in (-\frac{\pi}{2}, \frac{\pi}{2})] \\
&= \frac{5\pi}{4} - \left[\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) \right] \quad [\because \cot^{-1}x = \tan^{-1}\frac{1}{x}, x > 0] \\
&= \frac{5\pi}{4} - \left[\tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right) \right] \quad [\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), \text{ if } xy < 1] \\
&= \frac{5\pi}{4} - \tan^{-1}\left(\frac{5/6}{5/6}\right) \\
&= \frac{5\pi}{4} - \tan^{-1}(1) = \frac{5\pi}{4} - \frac{\pi}{4} = \frac{4\pi}{4} = \pi = \text{RHS (Hence Proved)}
\end{aligned}$$

16. Here, we have to find the value of x. Now, we are given that

$$\begin{aligned}
\tan^{-1} \frac{x}{2} + \tan^{-1} \frac{x}{3} &= \frac{\pi}{4}, \sqrt{6} > x > 0 \\
\Rightarrow \tan^{-1} \left(\frac{\frac{x}{2} + \frac{x}{3}}{1 - \frac{x^2}{6}} \right) &= \frac{\pi}{4} \quad [\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right); xy < 1]
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \frac{\frac{3x+2x}{6}}{\frac{6-x^2}{6}} &= \tan \frac{\pi}{4} \quad \{ \text{taking tan on both sides} \} \\
\Rightarrow \frac{5x}{6-x^2} &= 1 \quad [\because \tan \frac{\pi}{4} = 1]
\end{aligned}$$

$$\Rightarrow 5x = 6 - x^2$$

$$\Rightarrow x^2 + 5x - 6 = 0$$

$$\Rightarrow x^2 + 6x - x - 6 = 0$$

$$\Rightarrow x(x+6) - 1(x+6) = 0$$

$$\Rightarrow (x-1)(x+6) = 0$$

$$\therefore x = 1 \text{ or } -6$$

But it is given that, $\sqrt{6} > x > 0 \Rightarrow x > 0$

$\therefore x = -6$ is rejected.

Hence, $x = 1$ is the only solution of the given equation.

17. $\tan^{-1} [2 \cos(2 \sin^{-1} \frac{1}{2})]$

$$\begin{aligned}
&= \tan^{-1} \left[2 \cos \left(2 \sin^{-1} \sin \frac{\pi}{6} \right) \right] \\
&= \tan^{-1} \left[2 \cos \left(2 \times \frac{\pi}{6} \right) \right] \\
&= \tan^{-1} \left[2 \cos \frac{\pi}{3} \right] \\
&= \tan^{-1} \left[2 \times \frac{1}{2} \right] = \tan^{-1} 1 \\
&= \tan^{-1} \tan \frac{\pi}{4} = \frac{\pi}{4}
\end{aligned}$$

18. Let $\theta = \sin^{-1}(\frac{12}{13})$

$$\Rightarrow \sin \theta = \frac{12}{13}$$

$$\Rightarrow \sqrt{1 - \cos^2 \theta} = \frac{12}{13}$$

$$\Rightarrow 1 - \cos^2 \theta = \frac{(12)^2}{(13)^2}$$

$$\Rightarrow \cos^2 \theta = \frac{(5)^2}{(13)^2}$$

$$\Rightarrow \cos \theta = \frac{5}{13}$$

$$\text{Since, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{12}{5} \right)$$

$$\text{Thus, } \theta = \sin^{-1} \left(\frac{12}{13} \right) = \tan^{-1} \left(\frac{12}{5} \right)$$

$$\text{Similarly, } \cos^{-1} \left(\frac{5}{13} \right) = \tan^{-1} \left(\frac{12}{5} \right)$$

$$\text{We have, LHS} = \sin^{-1} \left(\frac{12}{13} \right) + \cos^{-1} \left(\frac{4}{5} \right) + \tan^{-1} \left(\frac{63}{16} \right)$$

$$= \tan^{-1} \left(\frac{12}{5} \right) + \tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{63}{16} \right)$$

$$= \left[\tan^{-1} \left(\frac{12}{5} \right) + \tan^{-1} \left(\frac{3}{4} \right) \right] + \tan^{-1} \left(\frac{63}{16} \right)$$

$$\{\text{since } \frac{12}{5} \times \frac{3}{4} = \frac{9}{5} > 1, \text{ therefore, } \tan^{-1} A + \tan^{-1} B = \pi + \tan^{-1} \frac{A+B}{1-AB}\}$$

$$= \pi + \tan^{-1} \left(\frac{\frac{12}{5} + \frac{3}{4}}{1 - \left(\frac{12}{5} \right) \left(\frac{3}{4} \right)} \right) + \tan^{-1} \left(\frac{63}{16} \right)$$

$$= \pi + \tan^{-1} \left(-\frac{63}{16} \right) + \tan^{-1} \left(\frac{63}{16} \right)$$

$$= \pi - \tan^{-1} \left(\frac{63}{16} \right) + \tan^{-1} \left(\frac{63}{16} \right) = \pi \text{ Hence Proved.}$$