## CBSE Test Paper 02

## Chapter 12 Linear Programming

1. Maximise $Z=5 x+3 y$ subject to $3 x+5 y \leq 15,5 x+2 y \leq 10, x \geq 0, y \geq 0$.
a. Maximum $\mathrm{Z}=\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$
b. Maximum $\mathrm{Z}=\frac{255}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$
c. Maximum $\mathrm{Z}=\frac{245}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$
d. Maximum $\mathrm{Z}=\frac{275}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$
2. Minimise $Z=-3 x+4 y$ subject to $x+2 y \leq 8,3 x+2 y \leq 12, x \geq 0, y \geq 0$.
a. Minimum $\mathrm{Z}=-13$ at $(4,1)$
b. Minimum $\mathrm{Z}=-14$ at $(5,0)$
c. Minimum $\mathrm{Z}=-12$ at $(4,0)$
d. Minimum $\mathrm{Z}=-15$ at $(5,1)$
3. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)

| From/To | A | B |
| :---: | :---: | :---: |
| D | 6 | 4 |
| E | 3 | 2 |
| F | 2.5 | 3 |

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?
a. From A : 12,52, 40 units; From B: 50,0,0 units to D, E and F respectively and minimum cost $=$ Rs 530
b. From A : 10,50, 40 units; From B: 50,0,0 units to D, E and F respectively and minimum cost $=$ Rs 510
c. From A : 10,53, 44 units; From B: 50,0,0 units to D, E and F respectively and minimum cost $=$ Rs 570
d. From A : 10,52, 42 units; From B: 50,0,0 units to D, E and F respectively and minimum cost $=$ Rs 550
4. In an LPP if the objective function $\mathrm{Z}=\mathrm{ax}+$ by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same
a. Upper limit value
b. Minimum value
c. Maximum value
d. Mean value
5. The feasible region for a LPP is shown in Figure. Find the minimum value of $Z=11 x+$ $7 y$.


a. 22
b. 21
c. 19
d. 20
6. If the feasible region R is $\qquad$ , then a maximum or a minimum value of the objective function may or may not exist.
7. A feasible region of a system of linear inequalities is said to be $\qquad$ if it can be enclosed within a circle.
8. The vertex of common graph of inequalities $2 x+y \geq 2$ and $x-y \leq 3$ is $\qquad$ .
9. Minimise $\mathrm{Z}=13 \mathrm{x}-15 \mathrm{y}$ subject to the constraints:
$x+y \leqslant 7,2 x-3 y+6 \geqslant 0, x \geqslant 0 y \geqslant 0$.
10. In Fig, the feasible region (shaded) for a LPP is shown. Determine the maximum and
minimum value of $Z=x+2 y$.

11. A city building code requires that the area of the windows must be at least $\frac{1}{8}$ of the area of walls and roofs of all new buildings. The daily heating cost of a new building is Rs. 3 per square metre of window area and Rs. 1 per square metre of wall and roof area. To the nearest square metre, what is the largest surface area a new building can have if its daily heating cost cannot exceed Rs. 1000 ?
12. Maximize $Z=3 x+2 y$

Subject to constraints
$x+2 y \leq 10$
$3 x+y \leq 15$
$x, y \geq 0$
13. The feasible region for an LPP is shown in fig. Evaluate $Z=4 x+y$ at each of the corner points of this region. Find the minimum value of $Z$, if it exists.

14. Solve the following problem graphically minimize or maximize $Z=3 x+9 y$ Subject to the constraints :

$$
\begin{aligned}
& x+3 y \leq 60 \\
& x+y \geq 10 \\
& x \leq y \\
& x \geq 0, y \geq 0
\end{aligned}
$$

15. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital are required to produce one unit of B. If A and B are priced at Rs. 100 and Rs. 120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as an L.P.P. and solve graphically.
Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?
16. Minimize $\mathrm{Z}=3 \mathrm{x}+5 \mathrm{y}$ such that $x+3 y \geqslant 3, x+y \geqslant 2, x, y \geqslant 0$.
17. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs 5760 to invest and has space for atmost 20 items for storage. An electronic sewing machine cost Rs 360 and a manually operated sewing machine Rs 240. He can sell an electronic sewing machine at a profit of Rs22 and a manually operated sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he can buy,
how should he invest his money in oder to maximise his profit? Make it as an LPP and solve it graphically.
18. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 h and that of second machine is 9 h per day. Each unit of product A requires 3 h on both machines and each unit of product $B$ requires 2 h on the first machine and 1 h on the second machine. Each unit of product A is sold at a profit of 7 and B at a profit of 4 . Find the production level per day for maximum profit graphically.


## CBSE Test Paper 02

## Chapter 12 Linear Programming

## Solution

1. a. Maximum $\mathrm{Z}=\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$

Explanation: Objective function is $Z=5 x+3 y$
The given constraints are : $3 x+5 y \leq 15,5 x+2 y \leq 10, x \geq 0, y \geq 0$.
The corner points obtained by drawing the lines $3 x+5 y=15$ and $5 x+2 y=10$ are (0,0),(0,3), (2,0) and (20/19,45/19)

| Corner points | $\mathbf{Z}=\mathbf{5 x}+\mathbf{3} \mathbf{y}$ |
| :---: | :---: |
| $\mathrm{O}(0,0)$ | 0 |
| $\mathrm{~B}(2,0)$ | 10 |
| $\mathrm{C}(0,3)$ | 9 |
| $\mathrm{D}(20 / 19,45 / 19)$ | $235 / 19 .(\mathrm{Max})$. |

Here, $Z=235 / 19$ is maximum at (20/19, 45/19).
2. c. Minimum $Z=-12$ at $(4,0)$

Explanation: Objective function is $Z=-3 x+4 y$
The given constraints are : $x+2 y \leq 8,3 x+2 y \leq 12, x \geq 0, y \geq 0$.
The corner points obtained by constructing the line $x+2 y=8$ and $3 x+2 y=12$ are $(0,0),(0,2),(3,0)$ and (20/19, 45/19)

| Corner points | $\mathbf{Z}=\mathbf{5 x}+\mathbf{3} \mathbf{y}$ |
| :---: | :---: |
| $\mathrm{O}(0,0)$ | 0 |
| $\mathrm{~B}(2,0)$ | 10 |
| $\mathrm{C}(0,3)$ | 9 |
| $\mathrm{D}(20 / 19,45 / 19)$ | $235 / 19$. (Max.) |

Here, $\mathrm{Z}=-12$ is minimum at $\mathrm{C}(4,0)$
3. b. From A : 10,50, 40 units; From B: 50,0,0 units to D, E and F respectively and
minimum cost $=$ Rs 510
Explanation: Let the number of units of grain transported from godown A to D $=x$ And the number of units of grain transported from godown A to $E=y$
Therefore, the number of units of grain transported from godown A to F = 100 $(x+y)$ Therefore, the number of units of grain transported from godown $B$ to $D=$ $60-x$ The number of units of grain transported from godown B to $E=50-y$ The number of units of grain transported from godown $B$ to $F=x+y-60$. Here, the objective function is: Minimise $Z=2.5 x+1.5 y+410$., subject to constraints: $60-$ $x \geq 0,50-y \geq 0,100-(x+y) \geq 0,(x+y)-60 \geq 0, x, y \geq 0$.,

| Corner points | $\mathbf{Z}=\mathbf{2 . 5 x} \mathbf{+ 1 . 5} \mathbf{y}+\mathbf{4 1 0}$ |
| :---: | :---: |
| $\mathrm{C}(60,0)$ | 560 |
| B $(60,40)$ | 620 |
| D(50,50) | 610 |
| A(10,50) | $510 .(M i n)$. |

Here $Z=510$ is minimum.i.e. From A : 10,50,40 units; From B: 50,0,0 units to D, E and $F$ respectively and minimum cost $=$ Rs 510.
4. c. Maximum value

Explanation: In an LPP if the objective function $\mathrm{Z}=\mathrm{ax}+$ by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same maximum value . If the problem has multiple optimal solutions at the corner points, then both the points will have the same (maximum or minimum)value.
5. b. 21

## Explanation:

| Corner points | $\mathbf{Z}=\mathbf{1 1 x}+7 \mathbf{y}$ |
| :---: | :---: |
| $(0,5)$ | 35 |
| $(0,3)$ | 21 |
| $(3,2)$ | 47 |

Hence the minimum value is 21
6. unbounded
7. Bounded
8. $\left(\frac{5}{3},-\frac{4}{3}\right)$
9. Minimise $\mathrm{Z}=13 \mathrm{x}-15 \mathrm{y}$ subject to the constraints:
$x+y \leqslant 7,2 x-3 y+6 \geqslant 0, x \geqslant 0, y \geqslant 0$.

replace $(0,7)$ by $(7,0)$ in horizontal line
Shaded region shown as OABC is bounded and coordinates of its corner points are ( 0 , 0 ), ( 7,0 ), ( 3,4 ) and ( 0,2 ), respectively.

| Corner Points | Corresponding value of Z |
| :---: | :---: |
| $(0,0)$ | 0 |
| $(7,0)$ | 91 |
| $(3,4)$ | -21 |
| $(0,2)$ | -30 (Minimum) |

Hence, the minimum value of Z is -30 at $(0,2)$.
10. From the shaded bounded region, it is clear that the coordinates of corner points are $\left(\frac{3}{13}, \frac{24}{13}\right),\left(\frac{18}{7}, \frac{2}{7}\right),\left(\frac{7}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{15}{4}\right)$
Also, we have to determine maximum and minimum value of $Z=x+2 y$.

| Corner Points | Corresponding value of Z |
| :--- | :--- |
| $\left(\frac{3}{13}, \frac{24}{13}\right)$ | $\frac{3}{13}+\frac{48}{13}=\frac{51}{13}=3 \frac{12}{13}$ |
| $\left(\frac{18}{7}, \frac{2}{7}\right)$ | $\frac{18}{7}+\frac{4}{7}=\frac{22}{7}=3 \frac{1}{7}$ (Minimum) |
|  |  |


| $\left(\frac{7}{2}, \frac{3}{4}\right)$ | $\frac{7}{2}+\frac{6}{4}=\frac{20}{4}=5$ |
| :--- | :--- |
| $\left(\frac{3}{2}, \frac{15}{4}\right)$ | $\frac{3}{2}+\frac{30}{4}=\frac{36}{4}=9$ (Maximum) |

Hence, the maximum and minimum value of are 9 and $3 \frac{1}{7}$ respectively.
11. Let the number of square metres of area of window $=x$ the number of square metres of area of walls and roofs $=y$
Total surface area $=x+y$
Now, the problem can be formulated as an L.P.P. as follows :
Maximize, $f(x, y)=x+y$
Subject to constraints
$x \geq \frac{1}{8} y$
$3 x+y \leq 1000$

$$
\text { s.t. } x \geq 0, y \geq 0
$$

We draw the lines $8 x-y=0$ and $3 x+y=1000$ and shaded the feasible region.
Corner points $O(0,0) A\left(333 \frac{1}{3}, 0\right)$ and $B\left(\frac{1000}{11}, \frac{8000}{11}\right)$. Evaluating the value of $f(x, y)$ at each corner point we have
$f(0,0)=0$
$f\left(333 \frac{1}{3}, 0\right)=333 \frac{1}{3}$
$f\left(\frac{1000}{11}, \frac{8000}{11}\right)=\frac{9000}{11}=818$ sqm. (Maximum)
Here, at $(x, y)=\left(\frac{1000}{11}, \frac{8000}{11}\right), f(x, y)$ has the maximum value.
Hence, the largest possible surface area a new building can have is nearly 818 sq . m.

12. Linear constraints
$x+2 y \leq 10$
$3 x+y \leq 15$
$x, y \geq 0$
and objective function is $\max (Z)=3 x+2 y$.
Reducing the above inequations into equations and finding their point of intersections, i.e.,
$x+2 y=10 \ldots$ (i)
$3 x+y=15$... (ii)
$x=0, y=0 \ldots$ (iii)

| Equations | Point of intersection |
| :--- | :--- |
| (i) and (ii) | $x=4$ and $y=3 \Rightarrow(4,3)$ |
| (i) and (iii) | at $x=0 \Rightarrow y=5 \Rightarrow(0,5)$ |
|  | at $\mathrm{y}=0 \Rightarrow \mathrm{x}=10 \Rightarrow(10,0)$ |
| (ii) and (iii) | at $\mathrm{x}=0 \Rightarrow \mathrm{y}=15 \Rightarrow(0,15)$ |
|  | at $\mathrm{y}=0 \Rightarrow \mathrm{x}=5 \Rightarrow(5,0)$ |

Now for feasible region, using origin testing method for each constraint
$x+2 y \leq 10$, let $\mathrm{x}=0, \mathrm{y}=0$
$\Rightarrow 0 \leq 10$ i.e., true
$\Rightarrow$ The shaded region will be toward the origin.
Non negative restrictions $x \geq 0, y \geq 0$ indicates that the feasible region will exist in first quadrant.
Now, corner points are $A(0,5), B(4,3) C(5,0)$ and $D(0,0)$.
For optimal solution substituting the value of all corner points in $Z=3 x+2 y$,

| Corner points | z |
| :--- | :--- |
| $A(0,5)$ | 10 |
| $B(4,3)$ | 18 maximum |
| $C(5,0)$ | 15 |
| $D(0,0)$ | 0 |

Hence, the maximum value of $Z$ exist when $x=4$ and $y=3$

13. Consider $x+y=3$

When $\mathrm{x}=0$, then $\mathrm{y}=3$ and when $\mathrm{y}=0$, then $\mathrm{x}=3$
So $A(0,3)$ and $B(3,0)$ are the points on the line $x+y=3$
Consider $x+2 y=4$
When $\mathrm{x}=0$, then $\mathrm{y}=2$ and when $\mathrm{y}=0$, then $\mathrm{x}=4$.
So C $(0,2)$ and $D(4,0)$ are the points on the line $x+2 y=4$
The two lines $x+y=3$ and $x+2 y=4$, intersect each other at $M(2,1)$.
So the feasible region is unbounded. Therefore, minimum value may or may not occur. If it occurs, it will be on the corner point.
The comer points are $(4,0),(2,1)$ and $(0,3) Z=4 x+y$
At $(4,0), Z=4(4)+0=16$
$\operatorname{At}(2,1), Z=4(2)+1=9$
At $(0,3), Z=4(0)+3=3$ (minimum)
If we draw the graph of $4 x+y<3$, we see that open half plane determined by
$4 x+y<3$ and feasible region do not have a point in common other than ( 0,3 ).
Hence, 3 is the minimum value of $Z$ at $(0,3)$.
14. The linear inequations or constraints
$x+3 y \leq 60$
$x+y \geq 10$
$x \leq y$
$x \geq 0, y \geq 0$ and objective functions is max or $\min (Z)=3 x+9 y$
Reducing the above inequations into equations and finding their point of intersection i.e.,

$$
\begin{aligned}
& x+3 y=60 \ldots \text { (i) } \\
& x+y=10 \ldots \text { (ii) } \\
& x=y \ldots \text { (iii) } \\
& x=0, y=0 \ldots \text { (iv) }
\end{aligned}
$$

| Equations | Point of Intersection |
| :--- | :--- |
| (i) and (ii) | $x=-15, y=25$ |
|  | Point is $\Rightarrow(-15,25)$ |
| (i) and (iii) | $\mathrm{x}=15 \Rightarrow \mathrm{y}=15$ |
|  | Point is $\Rightarrow(15,15)$ |
| (ii) and (iii) | $x=5, y=5$ |
|  | Point is $(5,5)$ |
| (i) and (iv) | when $\mathrm{x}=0 \Rightarrow \mathrm{y}=20$, |
|  | Point is $(0,20)$ |
|  | when $\mathrm{y}=0 \Rightarrow \mathrm{x}=60$, |
|  | Point is $(60,0)$ |
| (ii) and (iv) | when $\mathrm{x}=0 \Rightarrow \mathrm{y}=10$, |
|  | Point is $(0,10)$ |
|  | when $\mathrm{y}=0 \Rightarrow \mathrm{x}=10$, |
|  | Point is $(10,0)$ |
|  |  |

Now for feasible region,
For $x+3 y \leq 60$, putting $\mathrm{x}=0$ and $\mathrm{y}=0$, we have
$0+0 \geq 10$ i.e., Not true
$\Rightarrow$ The shaded region will be away from origin.
Also, we have $x \geq 0$, and $y \geq 0$ indicates that the shaded part will exist in first quadrant only. Here feasible region will be ABCDA, having corner points as
$A(0,20), B(15,15), C(5,5)$ and $D(0,10)$.
For optimal point substituting the value of all corner points in objective function
$Z=3 x+9 y$

| Corner points | z |  |
| :--- | :--- | :--- |
| $A(0,20)$ | 180 | Maximum |
| $B(15,15)$ | 180 |  |
| $C(5,5)$ | 60 | Maximum |
| $D(0,10)$ | 90 |  |

So that the minimum value of Z is 60 at $\mathrm{C}(5,5)$ of the feasible region and the maximum value at $\mathrm{A}(0,20)$ and $\mathrm{B}(15,15)$ is $\mathrm{Z}=180$.

15. Let $x$ and $y$ be the number of goods of type $A$ and of type $B$ respectively.
$\therefore$ No. of units of labour $=2 x+3 y$
As 30 units of labour are available
$\therefore 2 x+3 y \leq 30$
Similarly, constraint for capital is
$3 x+y \leq 17$
and non-zero constraints are
$x \geq 0, y \geq 0$
Objective function
$Z=100 x+120 y$
Consider
$2 x+3 y=30$
When $\mathrm{x}=0$, then $\mathrm{y}=10$
When $\mathrm{x}=15$, then $\mathrm{y}=0$
$\therefore 2 x+3 y=30$
passes through $\mathrm{A}(0,10)$ and $\mathrm{B}(15,0)$
Consider
$3 x+y=17$
When $x=0$, then $y=17$
When $y=0$, then $x=\frac{17}{3}$
$\therefore 3 x+y=17$ passes through $C(0,17)$ and $\mathrm{D}\left(\frac{17}{3}, 0\right)$.
Further above two equations intersect at $E(3,8)$, vertices of the feasible region are $A(0,10)$,
$\mathrm{O}(0,0), \mathrm{D}\left(\frac{17}{3}, 0\right)$ and $\mathrm{E}(3,8)$.
At $A(0,10), Z=100(0)+120(10)=$ Rs. 1200
At $O(0,0), Z=100(0)+120(0)=R s .0$
At $\mathrm{D}\left(\frac{17}{3}, 0\right), \mathrm{Z}=100\left(\frac{17}{3}\right)+120(0)=$ Rs. 566.67
At $E(3,8), Z=100(3)+120(8)=R s .1260$
Thus, maximum value of $Z=R s .1260$ at $\mathrm{x}=3$ and $\mathrm{y}=8$.
Yes, the view of manufacturer that men and women workers are equally efficient is correct and so they should be paid at the same rate.

16. For plotting the graphs of $x+3 y=3$ and $x+y=2$, we have the following tables:

| x | 0 | 3 |
| :---: | :---: | :---: |
| y | 1 | 0 |
| x | 1 | 0 |
| y | 1 | 2 |

The feasible portion represented by the inequalities
$x+3 y \geqslant 3, x+y \geqslant 2$ and $x, y \geqslant 0$ is ABC which is shaded


In the figure. The coordinates of point B are $\left(\frac{3}{2}, \frac{1}{2}\right)$
Which can be obtained by solving $x+3 y=3$ and $x+y=2$.
At A(0, 2)
$Z=3 \times 0+5 \times 2=10$
At $B\left(\frac{3}{2}, \frac{1}{2}\right)$
$Z=3 \times \frac{3}{2}+5 \times \frac{1}{2}=\frac{9}{2}+\frac{5}{2}=\frac{14}{2}=7$
At C $(3,0)$
$Z=3 \times 3+5 \times 0=9$
Hence, Z is minimum is 7 when $x=\frac{3}{2}$ and $y=\frac{1}{2}$.
17. Let the dealer purchased $x$ number of electronic sewing machines and $y$ number of manually operated sewing machines.
Now, we can construct the following table using the given data.

| Type of sewing machine | Number | Investment <br> (in Rs) | Profit <br> (in Rs) |
| :--- | :---: | :---: | :---: |
| Electronic | x | Rs.360x | Rs.22x |
| Manually | y | Rs.249Y | Rs.18y |
| Total | $\mathrm{x}+\mathrm{y}$ | Rs.( $360 \mathrm{x}+240 \mathrm{y}$ ) | Rs.( $22 \mathrm{x}+18 \mathrm{y}$ ) |
| Availability | 20 | 5760 |  |

Then, given LPP is to maximise the profit in the transaction. Let $Z$ be the objective function which represents the total profit of the transaction. Then the equation of Z is given by $Z=22 x+18 y$, which is the profit function and is to be maximised,
Subject to constraints, $x+y \leq 20$ ( number constraints)
$360 x+240 y \leq 5760$ or $3 x+2 y \leq 48$ (investment constraints)
$x \geq 0, y \geq 0$ (non-negative constraints, which will restrict the feasible region to the
first quadrant only.)
Let us consider the inequalities as equations, we get
$x+y=20$
$3 x+2 y=48$
and $x=0, y=0$
Table of values for line $\mathrm{x}+\mathrm{y}=20$ is given as follows.

| x | 0 | 20 |
| :---: | :---: | :---: |
| y | 20 | 0 |

So, the line $\mathrm{x}+\mathrm{y}=20$ passes through the points with coordinates $(0,20)$ and $(20,0)$.
On replacing $\mathrm{O}(0,0)$ in the inequality $x+y \leq 20$, we get
$0+0 \leq 20 \Rightarrow 0 \leq 20$ [which is true]
So, the half plane for the inequality of the line $x+y=20$ is towards the origin, which means that the point which is the origin lies in the feasible region of the inequality of the line $\mathrm{x}+\mathrm{y}=20$.
Table of values for line $3 x+2 y=48$ is given as follows.

| x | 0 | 16 |
| :---: | :---: | :---: |
| y | 24 | 0 |

So, it passes through the points $(0,24)$ and $(16,0)$. On putting $(0,0)$ in the inequality
$3 x+2 y \leq 48$ we get
$0 \leq 48$ [which is true]
So, the half plane of the inequality of the line $3 x+2 y=48$ is towards the origin, which means that the origin $O(0,0)$ lies in the feasible region of the inequality of the line $3 x+$ $2 \mathrm{y}=48$.
On solving Eqs. (ii) and (iii), we get
$\mathrm{x}=8$ and $\mathrm{y}=12$ So, the point of intersection of the lines $\mathrm{x}+\mathrm{y}=20$ and $3 \mathrm{x}+2 \mathrm{y}=48$ is $\mathrm{B}(8,12)$. Also, $x \geq 0$ and $y \geq 0$, so the feasible region lies in the first quadrant.
Now, we draw all the lines on a graph paper and we get the feasible region OABCO, which is bounded.


The coordinates of the comer points of the feasible region are $0(0,0), A(16,0), B(8,12)$ and $C(0,20)$.
The values of $Z$ at comer points are as follows

| Comer Points | $\mathrm{Z}=\mathbf{2 2 x}+\mathbf{1 8 y}$ |
| :---: | :---: |
| $0(0,0)$ | $\mathrm{Z}=22(0)+\mathbf{1 8}(0)=0$ |
| $\mathrm{~A}(16,0)$ | $z=22 \times 16+0=352$ |
| $\mathrm{~B}(8,12)$ | $Z=22 \times 8+18 \times 12=392$ |
| (maximum) |  |

From the table, maximum value of $Z=R s 392$ at point $B(8,12)$.
Hence, dealer should purchase 8 electronic sewing machines and 12 manually operated sewing machines to get maximum profit of Rs. 392.
18. Let the manufacturer produces $x$ units per day of product $A$ and $y$ units per day of product B respectively. Using the given information, construct the following table to make the required in equations and the objective functions.

| Products | Produce(in <br> units) | Machine I(in <br> hours) | Machine II (in <br> hours) | Profit (in Rs) |
| :---: | :---: | :---: | :---: | :---: |
| A | x units | 3 x hours | 3 x hours | Rs. 7x |
| B | y units | 2 y hours | 1 y hours | Rs. 4 y |
| Total | $(\mathrm{x}+\mathrm{y})$ units | $(3 \mathrm{x}+2 \mathrm{y})$ hours | $(3 \mathrm{x}+\mathrm{y})$ hours | Rs. (7x + 4y) |
| Availability |  | maximum 12 | maximum 9 hours | to be |



To get the maximum profit,we need to take the objective function , as $7 \mathrm{x}+4 \mathrm{y}$, which the equation for the profit function of the given data.
i.e. Let $Z=7 x+4 y$ be the objective function to be maximised subject to the following constraints,
$3 x+2 y \leq 12$ ( time constraint for machine I)
$3 \mathrm{x}+\mathrm{y} \leq 9$ ( time constraint or machine II)
and $\mathrm{x} \geq 0, \mathrm{y} \geq 0$ (non negative constraints which represents the first quadrant)
Now, consider the equations of the given in equations, and make a table to sketch the lines in the graph.
$3 x+2 y=12 \ldots . . .$. (i)
$3 x+y=9$
Table for line $3 x+2 y=12$ or $y=\frac{12-3 x}{2}$ is

| $\mathbf{x}$ | 0 | 4 |
| :---: | :---: | :---: |
| $\mathbf{y}$ | 6 | 0 |

From the above table, we get the information that the line (i) passes though the points $(0,6)$ and $(4,0)$.To get the feasible region of the inequality (i),
On replacing the point $\mathrm{O}(0,0)$ in the inequality $3 \mathrm{x}+2 \mathrm{y} \leq 12$, we get
$0+0 \leq 12$
$\Rightarrow 0 \leq 12$ [true]
So, the half plane includes the origin and represents the region below the line.
Table for line $3 x+y=9$ or $y=9-3 x$ is given as follows.

| $\mathbf{x}$ | 0 | 3 |
| :---: | :---: | :---: |
| $\mathbf{y}$ | 9 | 0 |

From the table we can get the points $(0,9)$ and $(3,0)$ through which the line (ii) passes. To get the feasible region of the given line (ii) ,
On replacing the point $O(0,0)$ in the inequality $3 x+y \leq 9$, we get $0+0 \leq 9$, which is true.

So, the half plane includes the origin and below the line. Also, $x \geq 0$ and $y \geq 0$, is region representing only the Ist quadrant. Hence the feasible region has the corner points OABCO, which is the bounded feasible region.
Now, to find the point of intersection of the given lines (i) and (ii) , we subtract Eq.
(ii) from Eq. (i). we get
$(3 x+2 y)-(3 x+y)=12-9,3 x+2 y-3 x-y=12$
$\Rightarrow y=3$. Replacing the value of $y=3$ in the given equation, we get,
$\therefore 3 \mathrm{x}=12-2 \mathrm{y}$
$=12-(2 \times 3)=6$, when $3 x=6$, we get $x=3$.
$\Rightarrow \mathrm{x}=2$
Thus, the point of intersection of the two given lines is at $\mathrm{B}(2,3)$. Hence, all the triple regions meet in the bounded region represented by the corner points OABCO.


Here, we see that OABCO is the required bounded feasible region, whose comer points are $\mathrm{O}(0,0), \mathrm{A}(3,0), \mathrm{B}(2,3)$ and $\mathrm{C}(0,6)$ which is the bounded area in the first quadrant only.
The values of Z at these comer points are as follows

| Corner points | $\mathrm{Z}=7 \mathbf{x}+\mathbf{4 y}$ |
| :---: | :---: |
| $\mathrm{O}(0,0)$ | $\mathrm{Z}=0+0=0$ |
| $\mathrm{~A}(3,0)$ | $\mathrm{Z}=(7 \times 3)+0=21$ |
| $\mathrm{~B}(2,3)$ | $\mathrm{Z}=(7 \times 2)+(4 \times 3)=14+12=26$ |
| $\mathrm{C}(0,6)$ | $\mathrm{Z}=(7 \times 0)+(4 \times 6)=0+24=24$ |

Hence, to obtain the maximum profit of Rs.26, the manufacturer has to produce 2 units per day of product $A$ and 3 units per day of product $B$.

