CBSE Test Paper 02
Chapter 10 Vector Algebra

1. Find $|\vec{a}|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=8$ and $|\vec{a}|=8\left|\vec{b}\right|$.
   a. $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$
   b. $\frac{19\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{5}}{3\sqrt{7}}$
   c. $\frac{17\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{3}}{3\sqrt{7}}$
   d. $\frac{21\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{6}}{3\sqrt{7}}$
2. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.
   a. $\frac{5}{2}\hat{i}-\frac{3\sqrt{3}}{2}\hat{j}$
   b. $\frac{-5}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$
   c. $\frac{-5}{2}\hat{i}-\frac{3\sqrt{3}}{2}\hat{j}$
   d. $\frac{5}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$
3. Direction angles are
   a. The angles denoted by $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$
   b. The angles that show the tip of the vector
   c. The angles α, β, γ made by the position vector $\vec{r}$ with the positive directions of x, y and z-axes respectively
   d. The angles α, β, γ made by the perpendicular to position vector $\vec{r}$with the negative directions of x, y and z-axes respectively
4. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}\ with\ \hat{i},\ \frac{\pi}{4}\ with\ \hat{j}\ and\ an\ acute\ angle\ \theta\ with\ \hat{k}$, then the components of $\vec{a}$ are
   a. $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{3}$
   b. $\frac{1}{3}, \frac{1}{\sqrt{2}}, \frac{1}{2}$
   c. $\frac{1}{3}, \frac{1}{\sqrt{3}}, \frac{1}{2}$
   d. $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$
5. Find $|\vec{x}|$, if for a unit vector $\widehat{a}$, $(2\vec{x}-3\vec{a}).(2\vec{x}+3\vec{a})=91$.

a. 5
b. $\sqrt{17}$
c. $\sqrt{15}$
d. $\sqrt{19}$

6. The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is ________.
7. The vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if $\overline{a}$ and $\overline{b}$ are ________ vectors.
8. The vectors $\vec{a}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{k}$ are the adhacent sides of a parallelogram. The acute angle between its diagonals is ________.
9. Find the direction ratios and the direction cosines of the vector $\vec{r}=\hat{i}+\hat{j}+\hat{k}$.
10. Is the measure of 10 Newton a scalar or vector ?
11. Find $|\vec{x}|$. if for a unit Vector $\hat{a}$ $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=15$.
12. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3,\left|\vec{b}\right|=4,|\vec{c}|=5$ and each one of them being $\perp$ to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$.
13. Find the angle between vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}|=\sqrt{3},\left|\vec{b}\right|=2.\ \vec{a}.\vec{b}=\sqrt{6}$.
14. Find the sine of the angle between the vectors $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$.
15. Show that $\vec{a}=\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right),\vec{b}=\frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right),\ \vec{c}=\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$ are mutually $\perp$unit vectors.
16. If $\vec{a}=\hat{i}+2\hat{j}+\hat{k},\vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{i}-4\hat{j}-5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a}-\vec{b})$and $(\vec{c}-\vec{b})$.
17. If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors.
18. The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence, find the unit vector along $\vec{b}+\vec{c}$.

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Solution

1. a. $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$

Explanation: $(\overrightarrow{a} + \overrightarrow{b}).(\overrightarrow{a} - \overrightarrow{b}) = 8$
$\Rightarrow |\overrightarrow{a}|^2 - |\overrightarrow{b}|^2 = 8$
$\Rightarrow 64|\overrightarrow{b}|^2 - |\overrightarrow{b}|^2 = 8 \Rightarrow 63|\overrightarrow{b}|^2 = 8 \Rightarrow |\overrightarrow{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$
$\Rightarrow |\overrightarrow{a}| = 8|\overrightarrow{b}| \Rightarrow |\overrightarrow{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}$

2. b. $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Explanation: Let $\overrightarrow{r} = -x\hat{i} + y\hat{j}, x = \frac{5}{2}, y = \frac{3\sqrt{3}}{2} \therefore \overrightarrow{r} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

3. c. The angles α, β, γ made by the position vector $\vec{r}$ with the positive directions of x, y and z-axes respectively

Explanation: the angles α, β, γ are called direction angles, which the position vector $\overrightarrow{r}$ makes with the positive x-axis ,y-axis and z-axis respectively

4. d. $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

Explanation: Let, $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then,
$\Rightarrow a_1^2 + a_2^2 + a_3^2 = 1$.....(1)
$\therefore \vec{a}.\hat{i} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}).\hat{i} \Rightarrow |\vec{a}||\hat{i}|cos\frac{\pi}{3} = a_1 \Rightarrow a_1 = \frac{1}{2}$
$\therefore \vec{a}.\hat{j} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}).\hat{j} \Rightarrow |\vec{a}||\hat{j}|cos\frac{\pi}{4} = a_2 \Rightarrow a_2 = \frac{1}{\sqrt{2}}$
$\therefore \vec{a}.\hat{k} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}).\hat{k} \Rightarrow |\vec{a}||\hat{k}|cos\frac{\pi}{4} = a_3 \Rightarrow a_3 = \cos\theta$
Putting these values in (1) , we get :
$\frac{1}{4} + \frac{1}{2} + cos^2\theta = 1$
$\Rightarrow \frac{3}{4} = 1 - cos^2\theta \Rightarrow sin^2\theta = \frac{3}{4} \Rightarrow sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^o$
$\therefore a_3 = cos60^o = \frac{1}{2}$
$\Rightarrow \vec{a} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}$

5. a. 5

Explanation: It is given that:

$$\left(2\overrightarrow{x}-3\overrightarrow{a}\right).\left(2\overrightarrow{x}+3\overrightarrow{a}\right)=91$$
$$\Rightarrow 4\left|\overrightarrow{x}\right|^{2}-9\left|\overrightarrow{a}\right|^{2}$$
$$=91\Rightarrow 4\left|\overrightarrow{x}\right|^{2}-9.1=91$$
$$\Rightarrow 4\left|\overrightarrow{x}\right|^{2}=100\Rightarrow\left|\overrightarrow{x}\right|=5$$

6. $\sqrt{3}$
7. equal
8. $\frac{\pi}{4}$

9. D.R of $\vec{r}$ are 1,1,1
   $|\vec{r}|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$
   D.C of $\vec{r}$ are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

10. Vector because Newton is a unit of force and force has both magnitude and direction.

11. Here,we have to find $|\vec{x}|$. Given ,$\hat{a}$ is a unit vector. Then, $|\hat{a}|=1$.
    Now, we are given that dot product is equal to$(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$
    $\Rightarrow\quad \vec{x}\cdot\vec{x}-\vec{a}\cdot\vec{x}+\vec{x}\cdot\vec{a}-\vec{a}\cdot\vec{a}=15$
    $\Rightarrow\quad \vec{x}\cdot\vec{x}-\vec{a}\cdot\vec{x}+\vec{a}\cdot\vec{x}-\vec{a}\cdot\vec{a}=15$
    [∵ scalar product is commutative, i.e. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$]
    $\Rightarrow\quad |\vec{x}|^2-|\vec{a}|^2=15\quad\left[\because\vec{z}\cdot\vec{z}=|\vec{z}|^2\right]$
    $\Rightarrow\quad |\vec{x}|^2-1=15\quad$ [given, $|\hat{a}|=1$]
    $\Rightarrow\quad |\vec{x}|^2=16$
    $\therefore\quad |\vec{x}|=4$
    [∵ length cannot be negative]

12. $\vec{a}.\left(\vec{b}+\vec{c}\right)=0,\vec{b}.(\vec{c}+\vec{a})=0,\vec{c}.\left(\vec{a}+\vec{b}\right)=0$ (given)
    $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$
    $=\vec{a}.\vec{a}+\vec{a}\left(\vec{b}+\vec{c}\right)+\vec{b}.\vec{b}+\vec{b}(\vec{a}+\vec{c})+\vec{c}.\vec{c}+\left(\vec{a}+\vec{b}\right)$
    $=|\vec{a}|^2+\left|\vec{b}\right|^2+|\vec{c}|^2$
    $=9+16+25$
    = 50

$\left|\vec{a}+\vec{b}+\vec{c}\right|=\sqrt{50}$
$=5\sqrt{2}$

13. $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$
$=\frac{\sqrt{6}}{(\sqrt{3}).(2)}=\frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}.2}=\frac{\sqrt{2}}{\sqrt{2}.\sqrt{2}}$
$\Rightarrow\cos\theta=\frac{1}{\sqrt{2}}$
$\Rightarrow\theta=\frac{\pi}{4}$

14. Here, a_1 = 3, a_2 = 1, a_3 = 2 and b_1 = 2, b_2 = -2, b_3 = 4

We know that,
$\cos\theta=\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$
$=\frac{3\times2+1\times(-2)+2\times4}{\sqrt{3^2+1^2+2^2}\sqrt{2^2+(-2)^2+4^2}}$
$=\frac{6-2+8}{\sqrt{14}\sqrt{24}}=\frac{12}{2\sqrt{14}\sqrt{6}}=\frac{6}{\sqrt{84}}=\frac{6}{2\sqrt{21}}=\frac{3}{\sqrt{21}}$
$\therefore\sin\theta=\sqrt{1-\cos^2\theta}$
$=\sqrt{1-\frac{9}{21}}=\sqrt{\frac{12}{21}}=\frac{2\sqrt{3}}{\sqrt{3}\sqrt{7}}=\frac{2}{\sqrt{7}}$

15. $|\vec{a}|=\frac{1}{7}\sqrt{36+4+9}=\frac{1}{7}\sqrt{49}=1$
$\left|\vec{b}\right|=\frac{1}{7}\sqrt{36+4+9}=\frac{1}{7}\sqrt{49}=1$
$|\vec{c}|=\frac{1}{7}\sqrt{9+36+4}=\frac{1}{7}\sqrt{49}=1$
Hence they are unit vectors
$\vec{a}.\vec{b}=\frac{1}{49}(2\hat{i}+3\hat{j}+6\hat{k})(6\hat{i}+2\hat{j}-3\hat{k})$
$=\frac{1}{49}(12+6-18)=0$
$\vec{b}.\vec{c}=\frac{1}{49}(6\hat{i}+2\hat{j}-3\hat{k})(3\hat{i}-6\hat{j}+2\hat{k})$
$=\frac{1}{49}(18-12-6)=0$
$\vec{c}.\vec{a}=\frac{1}{49}(3\vec{i}-6\vec{j}+2\vec{k})(2\vec{i}+3\vec{j}+6\vec{k})$
$=\frac{1}{49}(6-18+12)=0$
$\vec{a}\perp\vec{b},\ \vec{b}\perp\vec{c}$ and $\vec{c}\perp\vec{a}$
So they are $\perp$ to each other.

16. According to the question,

$\vec{a} = \hat{i} + 2\hat{j} + \hat{k},$

$\vec{b} = 2\hat{i} + \hat{j}$ and

$\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$

Now, $\vec{a} - \vec{b} = (i + 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j}) = -\hat{i} + \hat{j} + \hat{k}$

Now, $\vec{c} - \vec{b} = (3\hat{i} - 4\hat{j} - 5\hat{k}) - (2\hat{i} + \hat{j}) = \hat{i} - 5\hat{j} - 5\hat{k}$

Now, a vector perpendicular to $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$ is given by

$$\overrightarrow{(a - \vec{b})} \times (\vec{c} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix}$$

$= \hat{i}(-5 + 5) - \hat{j}(5 - 1) + \hat{k}(5 - 1)$

$= \hat{i}(0) - \hat{j}(4) + \hat{k}(4)$

$= -4\hat{j} + 4\hat{k}$

Unit vector along $(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})$ is given by

$\frac{-4\hat{j}+4\hat{k}}{|-4\hat{j}+4\hat{k}|}$

$= \frac{-4\hat{j}+4\hat{k}}{\sqrt{(-4)^2+4^2}}$

$= \frac{-4\hat{j}+4\hat{k}}{\sqrt{32}}$

$= \frac{-4\hat{j}+4\hat{k}}{4\sqrt{2}}$

$= -\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$

17. According to the question,

Given vectors are , $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

Now, $\vec{a} + \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) + (5\hat{i} - \hat{j} + \lambda\hat{k}) = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}$

Now, $\vec{a} - \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) - (5\hat{i} - \hat{j} + \lambda\hat{k}) = -4\hat{i} + (7 - \lambda)\hat{k}$

Since, $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular vectors, then dot product of these vectors will be zero , i.e. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$\Rightarrow [6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}] \cdot [-4\hat{i} + (7 - \lambda)\hat{k}] = 0$

$\Rightarrow \quad -24 + (7 + \lambda)(7 - \lambda) = 0$

$\Rightarrow \quad 49 - \lambda^2 = 24$

$\Rightarrow \quad \lambda^2 = 25$

$\Rightarrow \quad \lambda = \pm 5$

$\therefore \quad \lambda = \pm 5$

18. According to the question, $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$.

Now, $\vec{b} + \vec{c} = 2\hat{i} + 4\hat{j} - 5\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore \quad |\vec{b} + \vec{c}| = \sqrt{(2+\lambda)^2 + (6)^2 + (-2)^2}$

$= \sqrt{4 + \lambda^2 + 4\lambda + 36 + 4}$

$= \sqrt{\lambda^2 + 4\lambda + 44}$

The unit vector along $\vec{b} + \vec{c}$

$= \frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$ ...(i)

According to the question, the scalar product of $(\hat{i} + \hat{j} + \hat{k})$ with unit vector $\vec{b} + \vec{c}$ is 1.

$\therefore \quad (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = 1$

$\Rightarrow \quad (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \quad \frac{1(2+\lambda)+1(6)+1(-2)}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \quad \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}} = 1$

$\Rightarrow \quad \lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 44}$

$\Rightarrow \quad (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$ [squaring both sides]

$\Rightarrow \quad \lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44$

$\Rightarrow \quad 8\lambda = 8$

$\Rightarrow \quad \lambda = 1$

The value of λ is 1.

Substituting the value of λ in Eq. (i), we get unit vector along $\vec{b} + \vec{c}$

$= \frac{(2+1)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(1)^2+4(1)+44}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{1+4+44}}$

$= \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{49}} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$