CBSE Test Paper 01

Chapter 10 Vector Algebra

1. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively, having $\vec{a}\,.\,\vec{b} = \sqrt{6}$.
   a. $\frac{\pi}{5}$
   b. $\frac{\pi}{3}$
   c. $\frac{\pi}{2}$
   d. $\frac{\pi}{4}$

2. Find the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.
   a. $\cos^{-1}\left(\frac{4}{7}\right)$
   b. $\cos^{-1}\left(\frac{6}{7}\right)$
   c. $\cos^{-1}\left(\frac{5}{9}\right)$
   d. $\cos^{-1}\left(\frac{5}{7}\right)$

3. Vector has
   a. direction
   b. None of these
   c. magnitude
   d. magnitude as well as direction

4. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k},\ \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.
   a. $-\hat{i} + 4\hat{j} - \hat{k}$
   b. $-4\hat{j} - \hat{k}$
   c. $-\hat{i} - 4\hat{j} - \hat{k}$
   d. $\hat{i} - 4\hat{j} - \hat{k}$

5. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

a. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$
b. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$
c. $\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$
d. $-\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

6. The values of k which $|k\vec{a}| < |\vec{a}|$ and $k\vec{a} + \frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are ________.
7. If $\vec{r}.\vec{a} = 0, \vec{r}.\vec{b} = 0$, and $\vec{r}.\vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}(\vec{b} \times \vec{c})$ is ________.
8. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, $\vec{a}.\vec{b}$ = $2\sqrt{3}$ is ________.
9. Find $\vec{a} \times \vec{b}$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.
10. Find the projection of $\vec{a}$ on $\vec{b}$, if $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.
11. $\vec{a}$ Is unit vector and $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 8$, Then find $|\vec{x}|$.
12. Find the position vector of the mid-point of the vector joining the points P (2, 3, 4) and Q(4,1, - 2)
13. Find sine of the angle between the vectors. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$.
14. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$
15. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$.Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.
16. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ .Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.
17. A girl walks 4 km towards west, then she walks 3 km in a direction 30^0 east of north and stops. Determine the girl's displacement from her initial point of departure.
18. Find a vector $\vec{d}$ which is $\perp$ to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$ Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

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Solution

1. d. $\frac{\pi}{4}$, **Explanation:** $|\overrightarrow{a}| = \sqrt{3}, |\overrightarrow{b}| = 2, \overrightarrow{a}.\overrightarrow{b} = \sqrt{6}$
$\Rightarrow \overrightarrow{a}.\overrightarrow{b} = |\overrightarrow{a}|.|\overrightarrow{b}|\cos\theta \Rightarrow \sqrt{6}$
$= 2\sqrt{3}\cos\theta$
$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

2. d. $\cos^{-1}\left(\frac{5}{7}\right)$, **Explanation:** $\overrightarrow{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{b} = 3\hat{i} - 2\hat{j} + \hat{k}$
$\Rightarrow |\overrightarrow{a}| = \sqrt{14}, |\overrightarrow{b}| = \sqrt{14}, \overrightarrow{a}.\overrightarrow{b} = 10$
$\Rightarrow \frac{\overrightarrow{a}.\overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|} = \cos\theta \Rightarrow \frac{10}{14} = \cos\theta$
$\Rightarrow \cos\theta = \frac{5}{7} \Rightarrow \theta = \cos^{-1}\frac{5}{7}$

3. d. magnitude as well as direction, **Explanation:** A vector has both magnitude as well as direction.

4. b. $-4\hat{j} - \hat{k}$, **Explanation:** We have: vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,
$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and

5. a. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$, **Explanation:** Let $\overrightarrow{a} = \hat{i} + 2\hat{j} + 3\hat{k}$,

Then, $\widehat{a} = \frac{\overrightarrow{a}}{|\overrightarrow{a}|} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{1^2+2^2+3^2}} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$

Therefore , the D.C.'s of vector a are :
$\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

6. k $\in$]-1, 1 [k $\neq -\frac{1}{2}$

7. 0

8. $\frac{\pi}{3}$

9. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$

$= \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$

$= -17\hat{i} + 13\hat{j} + 7\hat{k}$

10. We are given that, $\vec{a} \cdot \vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore$ The projection of $\vec{a}$ on $\vec{b}$ is given as = $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{8}{\sqrt{2^2+6^2+3^2}}$

$= \frac{8}{\sqrt{4+36+9}}$

$= \frac{8}{\sqrt{49}} = \frac{8}{7}$

11. $|\vec{a}| = 1$

$(\vec{x} - \vec{a}) . (\vec{x} + \vec{a}) = 8$

$|\vec{x}|^2 - |\vec{a}|^2 = 8$

$|\vec{x}|^2 - 1 = 8$

$|\vec{x}|^2 = 9$

$|\vec{x}| = 3$

12. Given: Point P (2, 3, 4) and Q(4,1, - 2)

$\therefore$ Position vector of point P is $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

And Position vector of point Q is $\vec{b} = 4\hat{i} + \hat{j} - 2\hat{k}$

And Position vector of mid-point R of PQ is $\frac{\vec{a}+\vec{b}}{2} = \frac{2\hat{i}+3\hat{j}+4\hat{k}+4\hat{i}+\hat{j}-2\hat{k}}{2}$

$= \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$

13. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$

$= -11\hat{i} - \hat{j} + 7\hat{k}$

$\left|\vec{a} \times \vec{b}\right| = \sqrt{(-11)^2 + (-1)^2 + (7)^2}$

$= \sqrt{171} = 3\sqrt{19}$

$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{|\vec{a}|\left|\vec{b}\right|} = \frac{3\sqrt{19}}{\sqrt{14}.\sqrt{14}} = \frac{3}{14}\sqrt{19}$

14. Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$

Projection of vector $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \frac{(1)(7)+(3)(-1)+7(8)}{\sqrt{(7)^2+(-1)^2+(8)^2}}$

$= \frac{7-3+56}{\sqrt{49+61+64}} = \frac{60}{\sqrt{114}}$

15. According to the question ,

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$,

$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

Now , $2\vec{a} - \vec{b} + \vec{3c}$

$= 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= \hat{i} - 2\hat{j} + 2\hat{k}$

$\Rightarrow \quad 2\vec{a} - \vec{b} + 3\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$

Now, a unit vector in the direction of vector is $2\vec{a} - \vec{b} + 3\vec{c} = \frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|}$

$= \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{9}}$

$= \frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

$= \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

Vector of magnitude 6 units parallel to the vector is ,

$= 6\left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

$= 2\hat{i} - 4\hat{j} + 4\hat{k}$

16. Given: Vectors $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

We know that the cross-product of two vectors, $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$

Hence, vector $\vec{d}$ which is also perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$ where $\lambda = 1$ or some other scalar.

Therefore, $\vec{d} = \lambda\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$

$= \lambda \left[\hat{i}\,(28+4) - \hat{j}\,(7-6) + \hat{k}\,(-2-12) \right]$

$\Rightarrow \vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$...(i)

Now given $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c}.\vec{d} = 15$

$\vec{c}.\vec{d} = 15$

$= 2\,(32\lambda) + (-1)\,(-\lambda) + 4\,(-14\lambda) = 15$

$\Rightarrow 64\lambda + \lambda - 56\lambda = 15$

$\Rightarrow 9\lambda = 15$

$\Rightarrow \lambda = \frac{15}{9}$

$\Rightarrow \lambda = \frac{5}{3}$

Putting $\lambda = \frac{5}{3}$ in eq. (i), we get

$\vec{d} = \frac{5}{3}\left[32\hat{i} - \hat{j} - 14\hat{k}\right]$

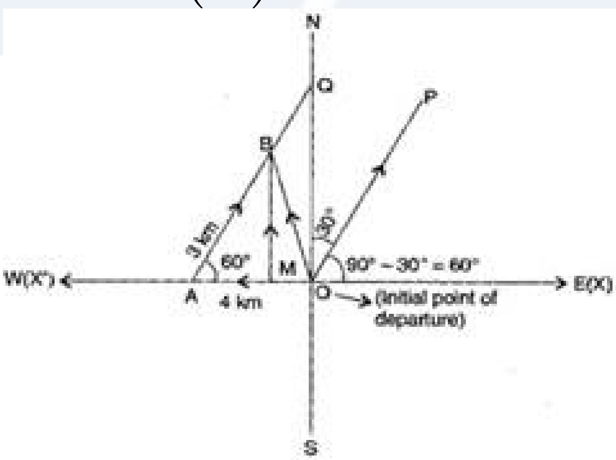
$\Rightarrow \vec{d} = \frac{1}{3}\left[160\hat{i} - 5\hat{j} - 70\hat{k}\right]$

17. Let the initial point of departure is origin (0, 0) and the girl walks a distance OA = 4 km towards west.

Through the point A, draw a line AQ parallel to a line OP, which is 30^0 East of North, i.e., in East-North quadrant making an angle of 30^0 with North.

Again, let the girl walks a distance AB = 3 km along this direction $\overrightarrow{OQ}$

$\therefore \overrightarrow{OA} = 4\left(-\vec{i}\right) = -4\hat{i}$ ...(i) [$\because$ Vector $\overrightarrow{OA}$ is along OX']



Now, draw BM perpendicular to x - axis.

In ΔAMB by Triangle Law of Addition of vectors,

$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB} = (AM)\,\hat{i} + (MB)\,\hat{i}$

Dividing and multiplying by AB in R.H.S.,

$\overrightarrow{AB} = AB\frac{AM}{AB}\hat{i} + AB\frac{MB}{AB}\hat{j} = 3\cos 60^o\hat{i} + 3\sin 60^o\hat{j}$

$\Rightarrow AB = 3\frac{1}{2}\hat{i} + 3\frac{\sqrt{3}}{2}\hat{i} = \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}j$ ...(ii)

$\therefore$ Girl's displacement from her initial point O of departure to final point B,

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = -4\hat{i} + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}\right) = \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$\Rightarrow \overrightarrow{OB} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

18. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

Let $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

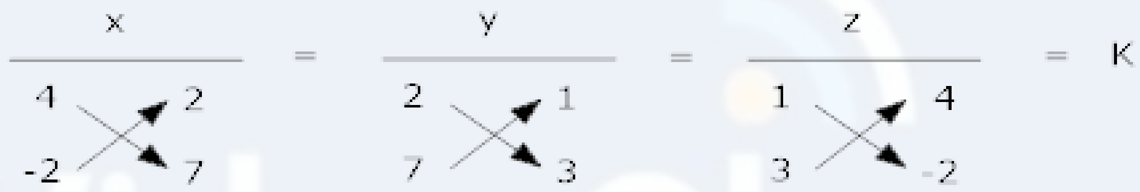
ATQ, $\vec{d}.\vec{a} = 0, \vec{d}.\vec{b} = 0$ and $\vec{c}.\vec{d} = 15$, then,

x + 4y + 2z = 0 ...(1)

3x - 2y + 7z = 0 ...(2)

2x - y + 4z = 15 ...(3)

On solving equation (1) and (2)



$\frac{x}{28+4} = \frac{y}{6-7} = \frac{z}{-2-12} = k$

x = 32k, y = -k, z = -14k

Put x, y, z in equation (3)

2(32k) - (-k) + 4(-14k) = 15

64k + k - 56k = 15

9k = 15

$k = \frac{15}{9}$

$k = \frac{5}{3}$

$x = 32 \times \frac{5}{3} = \frac{160}{3}$

$y = -\frac{5}{3}$

$z = -14 \times \frac{5}{3} = -\frac{70}{3}$

$\vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}$