CBSE Test Paper 01
Chapter 1 Relations and Functions

1. In Z , the set of integers, inverse of – 7 , w.r.t. ‘ * ‘ defined by a * b = a + b + 7 for all $a, b \in Z$,is
   a. -7
   b. -14
   c. 14
   d. 7
2. If A = { 1, 2, 3}, then the relation R = {(1, 2), (2, 3), (1, 3)} in A is ___.
   a. transitive only
   b. reflexive only
   c. symmetric only
   d. symmetric and transitive only
3. A relation R on a set A is called an empty relation if
   a. no element of A is related to any element of A
   b. every element of A is related to one element of A
   c. one element of A is related to all the elements of A
   d. every element of A is related to any element of A
4. Let f and g be two functions from R to R defined as $f(x)=\left\{\begin{array}{c}0, x \text { is rational } \\ 1, x \text { is irrational }\end{array}\right\}$, $g(x)=\left\{\begin{array}{c}-1, x \text { is rational } \\ 0, x \text { is irrational }\end{array}\right\}$ then, (gof)(e) + (fog) (π) =.
   a. 1
   b. 2
   c. -1
   d. 0
5. Let R be the relation on N defined as xRy if x + 2 y = 8. The domain of R is
   a. {2, 4, 6, 8}
   b. {2, 4, 8}
   c. {1, 2, 3, 4}
   d. {2, 4, 6}
6. If n(A) = p and n(B) = q, then the number of relations from set A to set B = ________.

7. A function is called an onto function, if its range is equal to ________.

8. A binary operation * on a set X is said to be ________, if a * b = b * a, where a, b $\in$ X.

9. Find gof f(x) = |x|, g(x) = |5x + 1|.

10. Show that function f: N $\rightarrow$ N, given by f(x) = 2x, is one – one.

11. Let S = {1, 2, 3} Determine whether the function f: S $\rightarrow$ S defined as below have inverse.
    f = {(1, 1), (2, 2), (3, 3)}

12. Let f : {1, 3, 4} $\rightarrow$ {1, 2, 5} and g : {1, 2, 5} $\rightarrow$ {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

13. Consider f: {1, 2, 3} $\rightarrow$ {a, b, c} given by f(1) = a, f(2) = b and f(3) = c find f^{-1} and show that $(f^{-1})^{-1}$ = f.

14. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g(x) = ax + b, then what value should be assigned to a and b.

15. Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$ a + d = b + c on the set N$\times$N is an equivalence relation.

16. Let the function f : R $\rightarrow$ R be defined by f(x) = cosx, $\forall$x $\in$ R. Show that f is neither one-one nor onto.

17. Let L be the set of all lines in plane and R be the relation in L define if R = {(l_1, L_2): L_1 is$\perp$ to L_2 } .Show that R is symmetric but neither reflexive nor transitive.

18. If the function f : R $\rightarrow$ R is given by f (x) = x^2 + 2 and g: R $\rightarrow$ R is given by $g(x) = \frac{x}{x-1}; x \neq 1$ then find fog and gof and hence find fog(2) and gof(- 3).

Solution

1. a. -7

   Explanation: If ‘ e ‘ is the identity ,then a*e = a $\Rightarrow$a + e + 7 = a $\Rightarrow$ e = - 7 . Also,inverse of e is e itself. Hence , inverse of -7 is -7.

2. a. transitive only

   Explanation: A relation R on a non-empty set A is said to be transitive if xRy and y Rz $\Rightarrow$ xRz, for all x $\in$ R. Here, (1, 2) and (2, 3) belongs to R implies that (1, 3) belongs to R.

3. a. no element of A is related to any element of A

   Explanation: For any set A ,an empty relation may be defined on A as: there is no element exists in the relation set which satisfies the relation for a given set A i.e.
   let A={1,2,3,4,5} and R={(a,b): a,b $\in$ A and a+b= 10},so we get R={ } which is an empty relation.

4. c. -1

   Explanation: (gof)(e) + (fog)(π) =g(f(e)) + f(g(π) = g(1) + f(0) = - 1 + 0 = -1.

5. d. {2, 4, 6}

   Explanation: As xRy if x + 2 y = 8 , therefore, domain of the relation R is given by x = 8 – 2y ∈ N. When y = 1, $\Rightarrow$ x = 6, when y = 2, $\Rightarrow$ x =4 , when y =3 , $\Rightarrow$ x = 2. Therefore, domain is { 2, 4, 6 }.

6. 2^{pq}

7. codomain

8. commutative

9. gof (x) = g [f(x)]
   = g [|x|]
   =| 5 |x| + 1 |

10. For, $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$

So, The function f is one – one

11. Since different elements have different images. So, f is one - one. Also, every element of codomain has pre-image so, f is onto

Now f is one – one and onto, so that f is invertible with inverse f^{-1} = {(1, 1) (2, 2) (3, 3)}

12. f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}

Now, f(1) = 2, f(3) = 5, f(4) = 1 and g(1) = 3, g(2) = 3, g(5) =1

gof(x)=g(f(x))

gof(1)=g(f(1))=g(2)=3

g[f(3)] = g(5) = 1 and g[f(4)] = g(1) = 3

Hence, gof = {(1, 3), (3, 1), (4, 3)}

13. f = {(1, a) (2, b) (3, c)}

f^{-1} = { (a, 1) (b, 2) (c, 3)}

$(f^{-1})^{-1}$ = {(1, a) (2, b) (3, c)}

Hence $(f^{-1})^{-1}$ = f.

14. Yes, g is a function since every element in domain has a unique image in range.

Now, Let g(x) = ax + b Then Given,

g(1) = a + b = 1 &

g(2) = 2a +b = 3

Subtracting g(1) from g(2) Gives

(2a + b) - (a + b) = a = 2 & Substituting It into g(1)

We have b = -1

15. we have,a + b = b + a for all (a, b) $\in N \times N$, which implies (a,b) R (a,b).Thus, R is reflexive.

Let (a,b),(c,d) $\in N \times N$ be such that

(a, b) R (c, d)

$\Rightarrow$ a + d= b + c

$\Rightarrow$ d + a = c + b

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

Hence R is symmetric.

Let (a,b),(c,d),(e,f) $\in N \times N$ such that (a,b) R (c,d) and (c,d) R (e,f).Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c....... (1)

(c, d) R (e, f) $\Rightarrow$ c + f = d + e ....(2)

Adding (1) and (2)

(a + d) + (c+f) = (b + c) + (d + e)

a + f = b+ e

(a, b) R (e, f)

Hence, R is transitive

So, R is an equivalence relation.

16. Given function, f(x) = cosx, $\forall$x $\in$ R

Now, $f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$\Rightarrow f\left(\frac{-\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

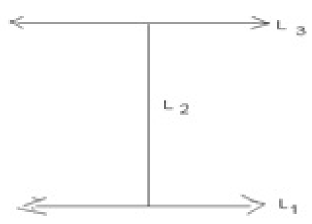
$\Rightarrow f\left(\frac{\pi}{2}\right) = f\left(\frac{-\pi}{2}\right)$

But $\frac{\pi}{2} \neq \frac{-\pi}{2} = 0$

So, f (x) is not one-one

Now, f(x) = cosx, $\forall$x $\in$ R is not onto as there is no pre-image for any real number. Which does not belong to the intervals [-1, 1], the range of cos x.

17. R is not reflexive, as a line L_1 cannot be $\perp$ to itself i.e (L_1, L_1)$\notin$ R



Now (L_1, L_2)$\in$R

$\Rightarrow L_1 \perp L_2$

$\Rightarrow L_2 \perp L_1$

$\Rightarrow$(L_2, L_1)$\in$R

R is symmetric

Now (L_1, L_2) and (L_2, L_3) $\in$ R i.e $L_1 \perp L_2$ and $L_2 \perp L_3$

Then L_1 can never be $\perp$ to L_3 in fact $L_1 \mid\mid L_3$

i.e $(L_1, L_2) \in R$, $(L_2, L_3) \in R$.

But $(L_1, L_3) \notin R$

R is not transitive.

18. We are given that, f : R $\rightarrow$ R and g : R $\rightarrow$ R defined as f(x) = x^2 + 2 and g(x) = $\frac{x}{x-1}; x \neq 1.$

First we see whether fog and gof exist for the given functions.

Since, range f $\subseteq$ domain g and range g $\subseteq$ domain f

Hence,fog and gof exist for the given functions.

Now, for any x $\in$ R - {1}, we have (fog)(x) = f[g(x)]

$= f\left[\frac{x}{x-1}\right] = \left(\frac{x}{x-1}\right)^2 + 2$

$= \frac{x^2+2(x-1)^2}{(x-1)^2} = \frac{x^2+2\left(x^2+1-2x\right)}{(x-1)^2}$

$= \frac{3x^2+2-4x}{(x-1)^2}$

$\therefore$ fog: R $\rightarrow$ R is defined by

(fog)(x)= $\frac{3x^2-4x+2}{(x-1)^2}, x \neq 1$ ......(i)

For any x $\in$ R we have

gof(x) = g[f(x)]

= g(x^2 + 2) = $\frac{x^2+2}{(x^2+2)-1} = \frac{x^2+2}{x^2+1}$

$\therefore$ gof: R $\rightarrow$ R is defined by

(gof)(x) = $\frac{x^2+2}{x^2+1}$ ....(ii)

On putting x = 2 in Eq. (i), we get

fog(2) = $\frac{3\times(2)^2-4(2)+2}{(2-1)^2} = \frac{3\times4-8+2}{(1)^2}$

= 12 - 8 + 2 = 6

On putting x = - 3 in Eq. (ii), we get

gof(-3) = $\frac{(-3)^2+2}{(-3)^2+1} = \frac{9+2}{9+1} = \frac{11}{10}$