CBSE Test Paper 02

Chapter 9 Mechanical Properties of Solids

1. Columns are loaded in **1**
   a. compression
   b. tension
   c. shear
   d. hydraulic stress
2. When water freezes, it expands by about 9.00$\%$. What would be the pressure increase inside your automobile's engine block if the water in it froze? (The bulk modulus of ice is 2.00 $\times$ 10^9 N/m^2 .) **1**
   a. 2.4×10^8 N/m^2
   b. 1.1×10^8 N/m^2
   c. 4.16×10^8 N/m^2
   d. 1.8×10^8 N/ m^2
3. If the elastic limit of copper is 1.50 $\times$ 10^8 N/m^2, determine the minimum diameter a copper wire can have under a load of 10.0 kg if its elastic limit is not to be exceeded. **1**
   a. 0.812 mm
   b. 0.912 mm
   c. 0.712 mm
   d. 1.012 mm
4. A steel rod 2.0 m long has a cross-sectional area of 0.30 cm^2 . It is hung by one end from a support, and a 550-kg milling machine is hung from its other end. Determine the resulting strain **1**
   a. 8.0×10^{-4}
   b. 7.0×10^{-4}
   c. 9.0×10^{-4}
   d. 10.0×10^{-4}
5. A copper cube measures 6.00 cm on each side. The bottom face is held in place by very strong glue to a flat horizontal surface, while a horizontal force F is applied to the upper face parallel to one of the edges. How large must F be to cause the cube to

deform by 0.250 mm? (shear modulus of copper = 4.4×10^{10} Pa) **1**

a. 6.4×10^5 N

b. 6.5×10^5 N

c. 6.3×10^5 N

d. 6.6×10^5 N

6. Show graphically the stress-strain relationship for a brittle material. **1**

7. A wire of length 1 m is stretched so as to make its length 1.01 m on applying an external force. What is the value of strain? **1**

8. What causes variation in velocity of a particle? **1**

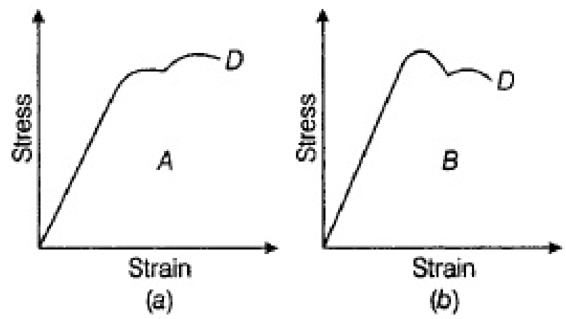
9. What is an elastomer? What are their special feature? **2**

10. Two wires made of the same material are subjected to forces in the ratio 1: 4. Their lengths are in the ratio 2: 1 and diameters in the ratio 1: 3. What is the ratio of their extensions? **2**

11. A wire of length L, area of cross-section A and young's modulus Y is stretched by an amount x. What is the work done? **2**

12. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large. **3**

13. The stress-strain graphs for materials A and B are shown in fig,(a) and Fig. (b)



The graphs are drawn to the same scale. **3**

i. Which of the materials has greater Young's modulus?

ii. Which of the two is the stronger material?

14. The elastic limit of a steel cable is 3.0×10^8 N/m^2 and the cross-section area is 4 cm^2. Find the maximum upward acceleration that can be given to a 900 kg elevator supported by the cable if the stress is not to exceed one-third of the elastic limit. **3**

15. A steel rod of length 2 l, cross sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform.) **5**

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Answer

1. a. compression

   Explanation: Stress on the columns compress it.

2. d. 1.8×10^8 N/ m^2

   Explanation: $given\ \frac{\Delta V}{V} = 9\% = \frac{9}{100}$

   bulk modulus of water = 2.00×10^9 N/m^2

   $B = \frac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow \Delta P = B \times \frac{\Delta V}{V}$

   $\Delta P = 2.00 \times 10^9 \times \frac{9}{100}$

   Delta P=1.8×10^8 N/m^2

3. b. 0.912 mm

   Explanation: given mass to be hung m=10.0 kg

   elastic limit i.e. pessure P = $\frac{force}{area} \rightarrow (1)$

   force F = mg and area $= \pi r^2$ $(r \rightarrow radius\ of\ wire)\, using\ 1$

   $1.50 \times 10^8 = \frac{10.0 \times 9.8}{\pi r^2}$

   r^2 = 20.80×10^{-8}

   r = 4.56×10^{-4} m

   diameter d = 2r = $2 \times 4.56 \times 10^{-4}$

   d=0.912 mm

4. c. 9.0×10^{-4}

   Explanation: young's modulus y = $\frac{stress}{strain} \Rightarrow strain = \frac{stress}{y}$

   $stress = \frac{restoring\ force}{area}$

   in given case restoring force will be tension, which will be equal to the weight' mg' of machine.

   using standard value of young modulus for steel y = 2×1011 N/m^2

   $strain = \frac{mg}{A \times y} \Rightarrow \frac{550 \times 9.8 \times 10^4}{0.30 \times 2 \times 10^{11}} = 8.9 \times 10^{-4} \approx 9.0 \times 10^{-4}$

5. d. 6.6×10^5 N

Explanation: shear modulus(η) is given as η = $\frac{shear\ stress}{shear\ strain} = \frac{F/A}{x/h} \rightarrow (1)$

given for copper η = 4.4×10^{10} pa

x = 0.250mm h = 6.0 cm

side of cube a = 6.0 cm

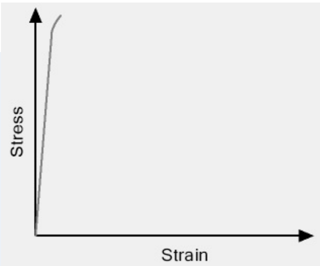
area A = a $\times$ a = 6.0 $\times$ 6.0 = 36.0 cm^2

from equation 1

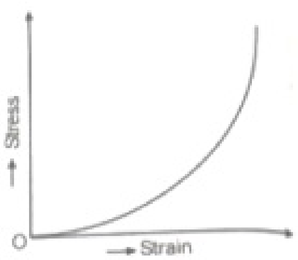
$F = \frac{\eta \times A \times x}{h} = \frac{4.4 \times 10^{10} \times 36 \times 10^{-4} \times 0.250 \times 10^{-3}}{6.0 \times 10^{-2}}$

F = 6.6×10^5 N

6. A typical stress–strain curve for a brittle material will be linear as shown:



7. Strain $= \frac{\Delta L}{L} = \frac{(1.01-1)}{1} = \frac{0.01}{1} = 0.01$
8. Either a force or an impulse can cause variation in velocity of particle.
9. Elastomers are those substances which can be stretched to cause large strains. Substances like tissue of aorta, rubber etc., are elastomers.
The stress-strain curve for an elastomer is as shown in Fig. Although elastic region is very large but the material does not obey Hooke's law over most of the region.



10. According to Hook's, Law.
Modulus of elasticity, $E = \frac{F}{\pi r^2} \times \frac{l}{\Delta l}$
$\Delta l = \frac{Fl}{\pi r^2 E}$
Δl $\propto \frac{Fl}{r^2}$ [$\because$ E is the same for two wires]
$\therefore \frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{F_2} \times \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2}$

= $\frac{1}{4} \times \frac{2}{1} \times \left(\frac{3}{1}\right)^2 = \frac{9}{2}$

$\therefore \triangle l_1 : \triangle l_2 = 9 : 2$

11. given $\Rightarrow$ change in length $\triangle L = x$

youngs's modulus is given by Y = $\frac{\text{stress}}{\text{streching}} = \frac{P \times L}{A \times \triangle L}$

Restoring force $F = \frac{YAx}{L}$

Work done in streching the wire by amount, dx

dW = F.dx

Total work done in streching the wire from 0 + x

$$W = \int dW = \int_0^x F dx = \int_0^x \frac{YAx}{L} dx = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^x$$

$$W = \frac{YAx^2}{2L}$$

12. Initial volume, V_1 = 100.0 litre = 100.0×10^{-3} m^3(As, 1 litre = 10^{-3} m^3)

Final volume, V_2 = 100.5 litre = 100.5×10^{-3} m^3

Increase in volume, $\Delta V = V_2 - V_1 = 0.5 \times 10^{-3}$ m^3

Increase in pressure, Δp = 100.0 atm = $100 \times 1.013 \times 10^5$ Pa(Since, 1 atm = 1.013×10^5 Pa)

Bulk modulus, k = $\frac{\Delta p}{\frac{\Delta V}{V}} = \frac{\Delta p \times V}{\Delta V}$

$= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$

$= 2.026 \times 10^9\ Pa$

We know that bulk modulus of air $= 1.0 \times 10^5$Pa

$\therefore \frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1.0 \times 10^5} = 2.026 \times 10^4$

We know that compressibility is inverse of bulk modulus. Now the air is much more compressible than water. That means the the compressibility of air is much more greater than water. Hence, bulk modulus of water is much more greater than air. That's why, the ratio of the bulk modulus of water to the bulk modulus of air is very high.

13. i. The slope of the graph in Fig. (a) is greater than the slope of the graph in Fig. (b), Therefore, material A has greater Young's modulus.
    ii. Material A can withstand more load without breaking. Therefore, it is stronger than material B. For material A, the break even point (D) is higher.

14. Given, mass of elevator $M = 900kg$, elastic limit of steel cable $= 3.0 \times 10^8 N/m^2$

and cross section area $A = 4cm^2 = 4 \times 10^{-4}m^2$

Suppose elevator is going upward with an acceleration 'a' so that tension in cable

$T = M(g + a)$

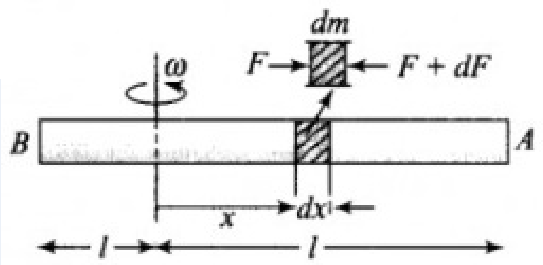
$\therefore$ Stress $\sigma = \frac{T}{A} = \frac{M(g+a)}{A} = \frac{900(9.8+a)}{4\times10^{-4}} \mathrm{N/m^2}$

As stress should not exceed one-third of elastic limit of steel cable, hence in limiting case,

$\sigma = \frac{3.0\times10^8}{3} = \frac{900(9.8+a)}{4\times10^{-4}}$

$\Rightarrow a = \frac{10^8\times4\times10^{-4}}{900} - 9.8 = 44.4 - 9.8 = 34.6ms^{-2}$

15. Consider an element of width dx at a distance x from the given axis of rotation as shown in the figure:



As rod is uniform, so mass per unit length, $\mu = \frac{M}{2l}$

Mass of small element dm = $\left(\frac{M}{2l}\right) dx$

Centripetal force acting on this element,

dF = dm.xω^2

$\Rightarrow \quad dF = \left(\frac{M}{2l}\right) dx \cdot x\omega^2$

Let tension in the rod be F at a distance x from the axis of rotation. F is due to centripetal force acting on all the elements from x to l, i.e.

$F = \frac{M\omega^2}{2l} \int_x^l x dx = \frac{M\omega^2}{4l}\left(l^2 - x^2\right)$

If d(r) is the extension in the element of length dx at position x, then,

$d(r) = \frac{Fdx}{YA} \left[\because Y = \frac{F/A}{d(r)/dx}\right]$ where Y is Young's Modulus.

Hence, extension in the half on the rod (from axis to point A) is given by

$\Delta r = \int_0^l d(r) = \int_0^l \frac{Fdx}{YA}$

$= \frac{M\omega^2}{4YAl}\left[l^2(x) - \frac{x^3}{3}\right]_0^l = \frac{M\omega^2}{4YAl}\left[l^3 - \frac{l^3}{3}\right] = \frac{M\omega^2 l^2}{6YA} = \frac{\mu\omega^2 l^3}{3YA}$ (putting M = 2μl)

Hence, total extension in whole rod of length 2l = 2$\triangle$r = $\frac{2}{3YA}\mu\omega^2 l^3$