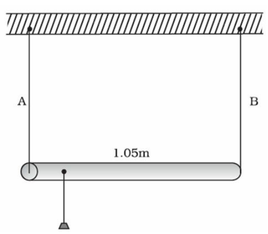
CBSE Test Paper 01

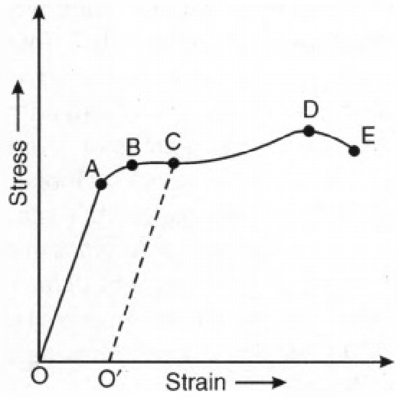
Chapter 9 Mechanical Properties of Solids

1. A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0 $\times 10^5$ N/ m^2 (normal atmospheric pressure). The sphere is lowered into the ocean to a depth at which the pressure is 2.0 $\times 10^7$ N/ m^2 . The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged? modulus of brass as 61 GPa **1**
   a. -1.7 $\times 10^{-4}$ m^3
   b. -1.4 $\times 10^{-4}$ m^3
   c. -1.5 $\times 10^{-4}$ m^3
   d. -1.6 $\times 10^{-4}$ m^3
2. For a rope of yield strength S_y loaded in tension with weight Mg the minimum area A of the rope should be **1**
   a. A $\geq$ Mg S_y
   b. A $\geq$ Mg/ S_y
   c. A $\geq$ Mg/3 S_y
   d. A $\geq$ Mg/2 S_y
3. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminum (wire B) of equal lengths as shown in Figure. The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2 , respectively. At what point along the rod should a mass m be suspended in order to produce equal strains in both steel and aluminum wires. Take Young's modulus of steel as 200 GPa, for aluminum 70 GPa **1**



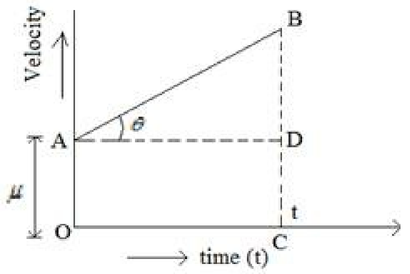
   a. 0.42 m from steel wire
   b. 0.40 m from steel wire
   c. 0.43 m from steel wire
   d. 0.44 m from steel wire

4. Material is said to be brittle if **1**
   a. material cross section is significantly reduced at failure
   b. fracture occurs soon after the elastic limit is passed
   c. a large amount of plastic deformation takes place between the elastic limit and the fracture point
   d. material elongates a lot before finally breaking
5. A piece of copper having a rectangular cross-section of 15.2 mm $\times$ 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain? Take Young's modulus of copper as 11 $\times$ 10^{10} Pa **1**
   a. 0.06×10^{-2}
   b. 0.11×10^{-2}
   c. 0.04×10^{-2}
   d. 0.14×10^{-2}
6. Define Poisson's ratio? What is its unit? **1**
7. What are ductile and brittle materials? **1**
8. What is the Young’s modulus for a perfect rigid body? **1**
9. Why do we prefer a spring made of steel and not of copper? **2**
10. When a load of a wire is increased from 3 kg wt to 5 kg wt, the length of that wire changes from 0.61 mm to 1.02 mm. calculate the change in the elastic potential energy of the wire. **2**
11. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each is hung at the two ends, then what will be the elongation of the wire in mm? **2**
12. The stress-strain graph for a metal wire is given in the figure. Up to the point B, the wire returns to its original state O along the curve BAO, when it is gradually unloaded. Point E corresponds to the fracture point of the wire. **3**



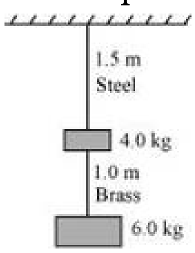
i. Up to which point of curve, is Hooke's law obeyed? This point is also called 'Proportionality limit'.
ii. Which point on the curve corresponds to elastic limit and yield point of the wire?
iii. Indicate the elastic and plastic regions of the stress-strain curve.
iv. What change happens when the wire is loaded up to a stress corresponding to point C on curve, and then unloaded gradually?

13. Establish $s = ut + \frac{1}{2}at^2$ from velocity-time graph for a uniform accelerated motion? **3**



14. A man carrying mass M = 125 kg makes a flying tackle at Vj = 4 m/s on a stationary quarterback of mass m = 85 kg and his helmet makes solid contact with quarterback's femur. **3**
    i. What is the final speed of two athletes immediately after contact and also determine the average force exerted on the quarterback's femur, when the last collision occur at 0.100 s?
    ii. If the area of cross-section of quarterback's femur is 5×10^4 m^2, then estimate the shear stress exerted on the femur in the collision.

15. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires. **5**



Answer

1. a. (d) -1.6 $\times\ 10^{-4}$ m^3

   Explanation: given initial pressure p_1 = $1.0 \times 10^5\ N/m^2$

   final pressure p_2 = $2.0 \times 10^7\ N/m^2$

   volume of sphere V = 0.50 m^3

   bulk modulus B of brass = 61 Gpa = 61 $\times$ 10^9pa

   bulk modulus is given by B = $-\frac{\Delta p}{\frac{\Delta V}{V}} DeltaV = -\frac{\Delta p \times V}{B} = -\frac{(p_2 - p_1) \times V}{B}$

   $= -\frac{(2.0 \times 10^7 - 1.0 \times 10^5) \times 0.50}{61 \times 10^9}$

   $DeltaV = -1.6 \times 10^{-4}\ m^3$

2. b. A $\geq$ Mg/ S_y

   Explanation: We know that Yield strength times the Area give the weight of the body.

   As Yield strength = weight of body/area

   For safety purposes

   Yield strength $\geq$ weight of body/area

   S_y $\geq$ Mg/A

   A $\geq$ Mg/ S_y

3. c. 0.43 m from steel wire

   Explanation: $\frac{F_1}{A_{steel} \times y_{steel}} = \frac{F_2}{A_{al} \times y_{aluminium}}$ let L be the length of each wire.

   cross sectional area of steel A_{steel} = 2mm^2

   young modulus of steel y_{steel} = 200 Gpa= 2 $\times$ 10^{11} pa

   cross sectional area of aluminium A_{al} = 1mm^2

   young modulus of aluminium y_{al} = 70 Gpa =7 $\times$ 10^{10} pa

   Let after placing the mass m wight on lower ends of wire be F_1 and F_2 then stress on wires A and B will be $\frac{f_1}{A_{steel}}$ and $\frac{f_2}{A_{al}}$

Now given condition is strain should be equal thus

Young modulus y = $\frac{stress}{strain}$ $\Rightarrow$ strain = $\frac{stress}{y}$

Strain$_{st}$ = $\frac{\text{stress}_{st}}{y_{steel}} = \frac{F_1}{A_{steel} \times y_{steel}}$ and strain$_{al}$ = $\frac{stress_{al}}{y_{al}}$

for equal strains $\Rightarrow$ strain$_{st}$ = strain$_{al}$

$$\frac{F_1}{A_{steel} \times y_{steel}} = \frac{F_2}{A_{al} \times y_{aluminium}} \Rightarrow \frac{F_1}{F_2} = \frac{A_{\text{stel}} \times y_{\text{ steel}}}{A_{\text{al}} \times y_{al}} \rightarrow (1)$$

if mass m is placed at a distance x and y from two wires then

$F_1 x = F_2 y$

$\frac{F_1}{F_2}$ = $\frac{y}{x}$ $\rightarrow$(2)

From equation 1 and 2

$\frac{y}{x}$ = $\frac{A_{\text{steel}} \times y_{\text{steel}}}{A_{\text{al}} \times y_{al}}$ $\Rightarrow$ x = $\frac{A_{al} \times y_{al}}{A_{steel} \times y_{steel}} y \rightarrow$ (3)

also given x + y = 1.05 (total length of rod)

y = 1.05 - x $\Rightarrow$ (4)

thus from 3 and 4

x = $\frac{A_{al} \times y_{al}}{A_{steel} \times y_{steel}}(1.05 - x) \Rightarrow x A_{steel} \times y_{steel} = A_{al} y_{al} 1.05 - A_{al} y_{al} x$

$$x\left(A_{steel} y_{steel} + A_{al} y_{al}\right) = A_{al} \times y_{al} \times 1.05$$

$$\Rightarrow x = \frac{2 \times 10^{-6} \times 7 \times 10^{10} \times 7 \times 10^{10} \times 1.05}{\left(2 \times 2 \times 10^{11} + 1 \times 7 \times 10^{10}\right) \times 10^{-6}}$$

x = 0.43 m

Thus mass should be placed 0.43 m from steel wire.

4. b. fracture occurs soon after the elastic limit is passed

Explanation: If the ultimate strength and fracture points are close, it means very small plastic range beyond elastic limit, the material is said to be brittle.

5. d. 0.14×10^{-2}

Explanation: given for copper

cross section = 15.2mm $\times$ 19.1mm

thus cross sectional area (A) = 15.2 $\times$ 19.1 mm^2 = 2.9 $\times$ 10^{-4}m^2

restoring force tension (T) = 44500 N

young modulus of copper (y) = 11 $\times$ 10^{10} pa

also y = $\frac{stress}{strain} \Rightarrow strain = \frac{stress}{y}$

$$strain = \frac{44500}{2.9 \times 10^{-4} \times 11 \times 10^{10}} strain = 0.1394 \times 10^{-2} \approx 0.14 \times 10^{-2}$$

6. Poisson's ratio is the ratio of transverse contraction strain to longitudinal extension strain in the direction of stretching force . It has no units.

7. **Ductile materials:** It is the ability of a material to withstand tensile force when it is applied upon it as it undergoes plastic deformation .For eg:- copper, Iron
Brittle materials: A material is brittle if, when subjected to stress, it breaks without significant plastic deformation. For eg:- Cast Iron, Glass.

8. $Y = \frac{FL}{A\Delta L}$ a rigid body cannot be re-shaped by applying any deforming force.

   $\therefore \Delta L = 0$

   $Y = \frac{Fl}{A(0)}$ = infinity i.e. for a perfect rigid body. Hence, Young's modulus is infinity.

9. We prefer to have a spring made of steel because Young's modulus of copper is less than that of the steel. As a result of the same shearing strain the stress i.e., the restoring force developed in the spring will be more and the spring will have more strength. In other words, we can say that in copper it does not rebound back to its original shape whereas steel comes back to its original shape.

10. Here, $F_1 = 3kgf = 3 \times 9.8N = 29.4N$
    $\Delta l_1 = 0.61mm = 6.1 \times 10^{-4}m$
    $F_2 = 5kgf = 5 \times 9.8 = 49Nl_2 = 1.02mm = 1.02 \times 10^{-3}m$
    $\therefore$ $U_1 = \frac{1}{2} F_1 . \Delta l_1 = \frac{29.4 \times 6.1 \times 10^{-4}}{2} = 8.96 \times 10^{-3}$ J
    and $U_2 = \frac{1}{2} F_2 . \Delta l_2 = \frac{49 \times 1.02 \times 10^{-3}}{2}$
    $= 24.99 \times 10^{-3}$ J
    $\therefore$ Change in elastic potential energy of the wire, $\Delta U = U_2 - U_1$
    $= 24.99 \times 10^{-3} - 8.96 \times 10^{-3} = 16.03 \times 10^{-3}J$

11. According to Hooke's law,
    Modulus of elasticity, E = $\frac{W}{A} \times \frac{L}{l}$
    Where, L = original length of the wire
    A = cross-sectional area of the wire
    $\therefore$ Elongation $\Delta l = \frac{WL}{E}$ ...(i)
    On either side of the wire, tension is W and length is l/2

Δl= $\frac{WL/2}{AE} = \frac{WL}{2AE} = \frac{l}{2}$ [from Eq.(i)]

$\therefore$ Total elongation in the wire = $\frac{l}{2} + \frac{l}{2}$ = l

12. a. Hooke's law is followed up to point A.
    b. Point B corresponds to elastic limit and yield point of given wire.
    c. OB represents the elastic region while BD represents the plastic region.
    d. When the given wire is loaded up to a stress corresponding to point C and then unloaded, it does not regain its original configuration even on complete unloading. The material has some strain left (OO'), which is called a permanent set.

13. From the velocity-time graph, OA = u = initial velocity of the particle, acceleration, a = slope of the graph = tan θ = $\frac{DB}{AD}$, time taken = OC = AD = t, final velocity, v = BC

    Displacement of the particle in time (t)

    S = area under $v-t$ graph

    S = area trapezium OABC

    S = area of rectangle AODC + area of triangle ADB

    $\Rightarrow S=(OA\times OC)+\frac{1}{2}(AD\times BD)$

    $\Rightarrow S=ut+\frac{1}{2}(AD)\times\left(\frac{AD\times DB}{AD}\right)$

    $\Rightarrow S=ut+\frac{1}{2}(AD)^2\times\left(\frac{DB}{AD}\right)$

    $\Rightarrow S=ut+\frac{1}{2}(t)^2\times\left(\frac{DB}{AD}\right)$

    $\Rightarrow S=ut+\frac{1}{2}(t)^2\times(a)$

    $\left[\because a=\tan\theta=\frac{BD}{AD}\right]$

    $\therefore S=ut+\frac{1}{2}at^2$

14. i. Here, $M=125kg, v_1=4m/s, m=85kg$

       Applying law of conservation of linear momentum,

       $\rho_{\text{initial}}=\rho_{\text{final}}$ i.e. $Mv_i=(M+m)v_f$

       The value of final speed

       $v_f=\frac{Mv_i}{M+m}$ = $\frac{125\times4}{(125+85)}=2.38\text{m/s}$

    ii. Average force exerted to the quarterback’s femur

       $\Rightarrow F_{\text{av}}\times\Delta t=M\left(v_f-v_i\right)$

i.e. $F_{\text{av}} = \frac{M(v_f - v_i)}{\Delta t} = \frac{125(4-2.38)}{0.1}$

= $\frac{125 \times 1.62}{01} = 2.03 \times 10^3 \text{N}$

Shearing stress = $\frac{F}{A}$ = $\frac{2.03 \times 10^3}{5 \times 10^{-4}} = 4.06 \times 10^6 Pa$

15. Young's modulus for steel:

$$Y_1 = \frac{\left(\frac{F_1}{A_1}\right)}{\left(\frac{\Delta L_1}{L_1}\right)}$$

Where,

ΔL_1 = Change in the length of the steel wire

A_1 = Area of cross-section of the steel wire = πr_1^2

Young's modulus of steel, $Y_1 = 2.0 \times 10^{11}$ Pa

$$\therefore \ \Delta L_1 = \frac{F_1 \times L_1}{A_1 \times Y_1} = \frac{F_1 \times L_1}{\pi r_1^2 \times Y_1}$$

$$= \frac{98 \times 1.5}{\pi \left(0.125 \times 10^{-2}\right)^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4} \text{m}$$

$$Total\ force\ on\ the\ brass\ wire:\ F_2 = m_2 g$$

$$F_2 = 6 \times 9.8 = 58.8\ N$$

Young's modulus for brass:

$$Y_2 = \frac{\left(\frac{F_2}{A_2}\right)}{\left(\frac{\Delta L_2}{L_2}\right)}$$

Where,

ΔL_2 = Change in length

A_2 = Area of cross-section of the brass wire

$$\therefore \Delta L_2 = \frac{F_2 \times L_2}{A_2 \times Y_2} = \frac{F_2 \times L_2}{\pi r_2^2 \times Y_2}$$

$$= \frac{58.8 \times 1.0}{\pi \left(0.125 \times 10^{-2}\right)^2 \times \left(0.91 \times 10^{11}\right)} = 1.3 \times 10^{-4} m$$

Elongation of the steel wire = 1.49×10^{-4}m

Elongation of the brass wire = $1.3 \times 10^{-4} m$