CBSE Test Paper 02

Chapter 8 Gravitation

1. A geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero).Mass of the earth = 6.0$\times 10^{24}$kg, radius = 6400 km. **1**
   a. -8.7 $\times 10^6$ J/kg
   b. -9.4 $\times 10^6$ J/kg
   c. -9.9 $\times 10^6$ J/kg
   d. -8.4 $\times 10^6$ J/kg
2. A particle of mass 3m is located 1.00 m from a particle of mass m. Where should you put a third mass M so that the net gravitational force on M due to the two masses is exactly zero? **1**
   a. 0.63 m from the 3m
   b. 0.69 m from the m
   c. 0.534 m from the m
   d. 0.234 m from the 3m
3. The effective value of acceleration due to gravity is $\frac{g}{4}$ at the depth(in respect of R=radius of earth): **1**
   a. $\frac{3R}{4}$
   b. R
   c. $\frac{R}{4}$
   d. $\frac{R}{2}$
4. For a satellite to be in a circular orbit 780 km above the surface of the earth, what orbital speed must it be given? **1**
   a. 7260 m/s
   b. 7160 m/s
   c. 7360 m/s
   d. 7460 m/s
5. The value of the gravitational constant G is **1**
   a. G = 6.67$\times 10^{-11}$ N m^2/kg

b. $G = 6.67 \times 10^{-11}$ N m/kg^2
c. $G = 6.67 \times 10^{-11}$ N m^2/kg^2
d. $G = 6.67 \times 10^{-11}$ m^2/kg^2

6. Work done in moving a particle round a closed path under the action of gravitation force is zero? Why? **1**

7. What is a parking orbit? **1**

8. The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside is zero, so can a body be shielded from gravitational influence? **1**

9. Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle. **2**

10. Three particles A, B and C, each of mass m, are placed in a line with AB = BC = d. Find the gravitational force on a fourth particle P of same mass, placed at a distance d from the particle B on the perpendicular bisector of the line AC. **2**

11. Assuming earth to be a uniform sphere find an expression for density of earth in terms of g and G? **2**

12. Obtain an expression showing variation of acceleration due to gravity with height. **3**

13. What do you mean by gravitational potential at a point? Give its unit and dimensional formula. Obtain an expression for gravitational potential at a distance r from the centre of Earth of mass M and radius R, where $r \geq R$. **3**

14. A rocket is fired vertically from the surface of the mars with a speed of 2 km/s. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of the mars before returning to it? Mass of the mars = 6.4 $\times 10^{24}$ kg; radius of the mars = 3395 km; G = 6.67 $\times$ 10^{-11} Nm^2/kg^2. **3**

15. Two stars each of 1 solar mass (=2 $\times$ 10^{30} kg) are approaching each other for a head-on collision. When they are at a distance of 10^9 km, their speeds are negligible. What is the speed with which they will collide? The radius of each star is 10^4 km. Assume the stars to remain undistored until they collide (use the known value of G). **5**

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Answer

1. b. -9.4 $\times 10^6$ J/kg

   Explanation: Mass of the earth, M = 6.0×10^{24}kg.

   Radius of the earth, R = 6400 km = $6.4 \times 10^6 m$.

   Height of a geostationary satellite from the surface of the earth,

   h = 36000 km = 3.6$\times 10^7 m$

   Gravitational potential energy due to earth's gravity at height h,

   $= -\frac{GM}{(R+h)}$

   $= -\frac{6.67\times10^{-11}\times6\times10^{24}}{3.6\times10^7+0.64\times10^7}$

   $= -\frac{6.67\times6\times10^6}{4.24} = -9.4 \times 10^6 \text{ J/kg}$

2. a. 0.63 m from the 3m

   Explanation: Let x be the distance of third particle from the mass 3m. then (1 - x) is the distance will be the distance from mass m.

   If we require that net force on another mass M be zero, then we must have the following:

   $\frac{GM(3m)}{x^2} = \frac{GM(m)}{(1-x)^2}$

   $\Rightarrow \frac{3}{x^2} = \frac{1}{(1-x)^2}$

   $\Rightarrow x^2 = 3 \times (1-x)^2$

   $\Rightarrow x = \sqrt{3 \times (1-x)^2} = \sqrt{3}(1-x)$

   $\Rightarrow x = \frac{\sqrt{3}}{1+\sqrt{3}} = 0.63$

   $\therefore$ Particle is 0.63 m away from 3m.

3. a. $\frac{3R}{4}$

   Explanation: we Know : $g' = g\left(1 - \frac{d}{R}\right)$

   $\Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right)$

   $\Rightarrow d = \frac{3R}{4}$

4. d. 7460 m/s

Explanation: Mass of the Earth, M_e = 6.0×10^{24}kg

Radius of the Earth, Re = 6.4×10^6m

Universal gravitational constan t, G = $6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

Height of the satellite, h = 780 km $= 780 \times 10^3\ m = 0.78 \times 10^6 m$

Orbital velocity of the satellite, v = $\sqrt{\frac{GM_e}{R_e+h}}$

$= \sqrt{\frac{6.67\times10^{-11}\times6.0\times10^{24}}{6.4\times10^6+0.78\times10^6}}$

$= \sqrt{\frac{40\times10^{13}}{7.18\times10^6}}$

$= \sqrt{5.57 \times 10^7}$

$= 10^3 \times \sqrt{5.57 \times 10}$

$= 7.46 \times 10^3$ = 7460 m/sec

5. c. G = 6.67×10^{-11}N m^2/kg^2

Explanation: The value of G was experimentally determined by Lord Henry Cavendish using a torsion balance.Cavendish brought two large lead spheres near the smaller spheres attached to the rod. Since all masses attract, the large spheres exerted a gravitational force upon the smaller spheres and twisted the rod a measurable amount. Once the torsional force balanced the gravitational force, the rod and spheres came to rest and Cavendish was able to determine the gravitational force of attraction between the masses.Cavendish expressed his result in terms of the density of the Earth.

$$G = g\frac{R_{earth}^2}{M_{earth}} = \frac{3g}{4\pi R_{earth}\rho_{earth}}$$

After converting to SI units, Cavendish's value for the Earth's density, 5.448 g cm $^{-3}$, gives

G = 6.74×10^{-11} m^3 kg^{-1} s^{-2}

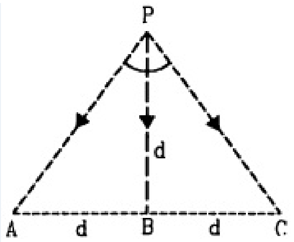
Today, the currently accepted value is 6.67259 x 10^{-11} N m^2/kg^2.

6. Gravitational force is a conservative force which means that work done by it, is independent of a path followed.

7. Parking orbit is that orbit in which the period of revolution of a satellite is equal to the period of rotation of the earth about its axis. It is a temporary orbit used during the launch of a satellite.

8. Because Gravity is still very much present, but the sum of all the gravity vectors does integrate to zero. It's called the shell theorem.

9. The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius. Consider the motion of one of the 2 particles. The force on the particle is
$F = \frac{Gm^2}{4R^2}$.
If the speed is v, its acceleration is $\frac{v^2}{R}$.
According to the Newton's law
$\frac{Gm^2}{4R^2} = \frac{mv^2}{R}$
$\Rightarrow v = \sqrt{\frac{Gm}{4R}}$

10. 

The force at P due to A is F_A = $\frac{Gm^2}{(AP)^2} = \frac{Gm^2}{2d^2}$ along PA.
The force at P due to C is $F_C = \frac{Gm^2}{(CP)^2} = \frac{Gm^2}{2d^2}$ along PC.
The force at P due to B is $F_B = \frac{Gm^2}{d^2}$ along PB.
The resultant of $F_A, F_B\ and\ F_C$ will be along PB.
Clearly $\angle APB = \angle BPC = 45^o$.
Component at F_A along $PB = F_A cos 45^\circ = \frac{Gm^2}{2\sqrt{2}d^2}$
Component at Fc along $PB = F_C\ cos 45^o = \frac{Gm^2}{2\sqrt{2}d^2}$
Component at F_B along $PB = \frac{Gm^2}{d^2}$
The resultant of the three forces =
$\frac{Gm^2}{d^2}\left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + 1\right) = \frac{Gm^2}{d^2}\left(1 + \frac{1}{\sqrt{2}}\right)$ along PB.

11. According to relation between g and G
$g = \frac{GM}{R^2}$............(1)
If earth is uniform sphere of mean density ρ
M=Vρ

$M = \cdot \left(\frac{4}{3}\pi R^3 \rho\right)$................(2)

put eq. 2 in 1

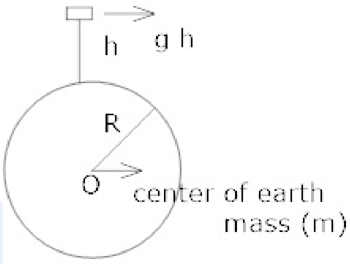
$g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$

$g = \frac{4}{3}\pi G R \rho$

$\Rightarrow \rho = \frac{3g}{4\pi G R}$

12. Now we know on the surface of earth Acceleration due to gravity is

$g = \frac{GM}{R^2}$ ....(1)



If g' is the acceleration due to gravity at a height ‘h’

Here (R + h) is the distance between the object and the center of earth.

$g' = \frac{GM}{(R+h)^2}$ ....(2)

Divide (2) by (1)

$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$

$\frac{gh}{g} = \frac{R^2}{(R+h)^2}$

$g' = \text{g}\left(\frac{R^2}{(R+h)^2}\right)$

If h <<< R then the above relation

$g' = g\frac{R^2}{R^2(1+h/R)^2}$

$g' = g\frac{1}{(1+h/R)^2}$

g' = g (1 + $\frac{h}{R}$)$^{-2}$

Expanding Binomially and neglecting higher power

So as altitude h increases, the value of acceleration due to gravity falls.

$g' = g\left(\frac{1-2h}{R}\right)$

13. The amount of work done in moving a unit test mass from infinity into the gravitational influence of source mass is known as gravitational potential. Unit of

gravitational potential is J-kg^{-1}.

Important Points:

- The gravitational potential at a point is always negative, V is maximum at infinity.
- The SI unit of gravitational potential is J/Kg.
- The dimensional formula is $M^0L^2T^{-2}$.

Let us calculate gravitational potential due to Earth at a point A near its surface, situated at a distance r from centre of Earth, where r $\geq$ R. Consider a unit mass situated at point P at a distance x from centre of Earth. Gravitational intensity i.e., gravitational force on the unit mass at P due to Earth is given by

$F = \frac{GM}{x^2}$ (towards O)

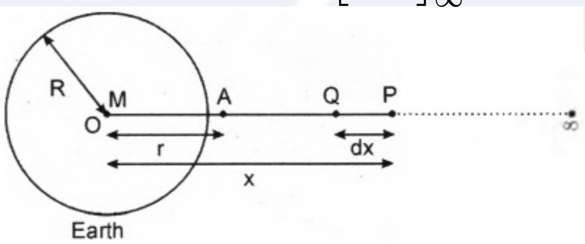
If the unit mass is shifted from point P to Q through a small distance dx, then work done by the force

$dW = Fdx = \frac{GM}{x^2}dx$

By definition gravitational potential at point A is the amount of work done to bring a unit mass from ∞ to point A, hence

$V = W = \int_{\infty}^{A} dW$

$= \int_{x=\infty}^{x=r} \frac{GM}{x^2}dx = -\left[\frac{GM}{x}\right]_{\infty}^{r} = -\frac{GM}{r}$



14. Mass of the mars, $M = 6.4 \times 10^{23}kg$

Radius of the mars, $R = 3395km = 3.395 \times 10^6m$

Velocity of the rocket, $v = 2km/s = 2 \times 10^3m/s$

Let h be the maximum height attained by the rocket. Change in potential energy of the rocket.

PE = final Potential energy - initial potential energy

$= -G\frac{Mm}{(R+h)} + G\frac{Mm}{R}$

$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = GMm\frac{h}{R(R+h)}$

Here, 20% of the kinetic energy of the rocket is lost due to Marian atmosphere.

KE of the rocket which is converted into its potential energy = $\frac{80}{100} \times \frac{1}{2} = 0.4mv^2$

Applying the law of conservation of energy,

$\Rightarrow GMm\,\frac{h}{R(R+h)} = 0.4mv^2$

$\Rightarrow GM\,\frac{h}{R^2+Rh}$ =$0.4v^2$ or $h = \frac{R^2}{\left(\frac{GM}{0.4v^2}\right)-R}$

$\Rightarrow$ h = $\frac{11.526\times10^{12}}{26.68\times10^{6}-3.395\times10^{6}}$ m

$\Rightarrow h = 495 \times 10^3 m = 495 km$

15. Mass of each star, $M = 2 \times 10^{30} kg$

Radius of each star, $r = 10^7 m$

Initial potential energy of the stars when they are 10^{12}m apart = - $\frac{GM\times M}{10^{12}}$ = - $\frac{GM^2}{10^{12}}$

[distance between two stars $= 10^{12} m$]

When the stars are just going to collide, the distance between their centres = twice the radius of each star $= 2r = 2 \times 10^7 m$

Final potential energy of the stars when they are about to collide $= -G\,\frac{M\times M}{2\times10^7}$ = - $\frac{GM^2}{2\times10^7}$

Change in potential energy of stars

= - $\frac{GM^2}{10^{12}}$ - $\left(-\frac{GM^2}{2\times10^7}\right)$ = $\frac{GM^2}{2\times10^7}$ - $\frac{GM^2}{10^{12}}$

$\approx \frac{GM^2}{2\times10^7}\ \left[\text{as } \frac{GM^2}{10^{12}} << \frac{GM^2}{2\times10^7}\right]$

Suppose v be the speed of each star just before colliding,

Final KE of the stars $= 2 \times \frac{1}{2} Mv^2 = Mv^2$

Initial KE of the stars$= 0$

(when the stars are initially $10^{12} m$ apart, their speeds are negligible).

Change in KE of the stars $= Mv^2$

Using the law of conservation of energy, from Eqs.(i) and (ii),

$\Rightarrow \frac{GM^2}{2\times10^7} = Mv^2$

$v = \sqrt{\frac{GM}{2\times10^7}}$

$v = \sqrt{\frac{6.67\times10^{-11}\times\left(2\times10^{30}\right)}{2\times10^7}}$

$v = 2.6 \times 10^6 m/s$