CBSE Test Paper 01

Chapter 8 Gravitation

1. The direction of the universal gravitational force between particles of masses m_1 and m_2 is: **1**
   a. towards m_1
   b. towards m_2 on m_1 and towards m_1 on m_2.
   c. towards the center of the earth
   d. towards m_2
2. The space shuttle releases a 470-kg communications satellite while in an orbit that is 280 km above the surface of the Earth. A rocket engine on the satellite boosts it into a geosynchronous orbit, which is an orbit in which the satellite stays directly over a single location on the Earth. How much energy did the engine have to provide? **1**
   a. 1.09×10^{10} J
   b. 1.29×10^{10} J
   c. 1.39×10^{10} J
   d. 1.19×10^{10} J
3. Time period of an earth satellite very close to the surface of earth is given by **1**
   a. $T_0 = \pi\sqrt{\frac{2R_E}{g}}$
   b. $T_0 = \pi\sqrt{\frac{R_E}{2g}}$
   c. $T_0 = 2\pi\sqrt{\frac{R_E}{g}}$
   d. $T_0 = \pi\sqrt{\frac{R_E}{g}}$
4. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth? **1**
   a. 18 N
   b. 128 N
   c. 28 N
   d. 180 N
5. Two stars each of one solar mass (= 2 $\times$ 10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 104 km. assume the

stars to remain undistorted until they collide. (Use the known value of G) **1**

a. 2.6×10^6 m/s
b. 1.6×10^6 m/s
c. 2.2×10^6 m/s
d. 2.8×10^6 m/s

6. Do the friction of force and other contact forces arise due to gravitational attraction? If not, then what is the origin of these forces? **1**

7. Define the effect of the shape of the earth on the value of g. **1**

8. A thief with a box in his hand jumps from the top of a building. What will be the load experienced by him during the state of free fall? **1**

9. If an object at the altitude of the space shuttle's orbit, about 400 km about the earth's surface, then find out the free fall acceleration of that object. **2**

10. Derive an expression for work done against gravity. **2**

11. State three essential requisites of geostationary satellite. **2**

12. Find the distance of a point from the earth's centre where the resultant gravitational field due to the earth and the moon is zero. The mass of the earth is 6.0×10^{24} kg and that of the moon is 7.4×10^{22} kg. The distance between the earth and the moon is 4.0×10^6 km. **3**

13. Two uniform solid spheres of radii R and 2R are at rest with their surfaces just touching. Find the force of gravitational attraction between them if density of spheres be P? **3**

14. Obtain an expression for escape velocity from energy considerations. **3**

15. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the Sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28×10^8kg; $G = 6.67 \times 10^{-11} \mathrm{m}^2 \mathrm{kg}^{-2}$. **5**

Answer

1. b. towards m_2 on m_1 and towards m_1 on m_2.
   Explanation: since gravitational force is attractive in nature, so if there are two particles of m_1 & m_2.
   So One Force will be on m_1 which will be directed towards m $_{2.}$ i.e
   $\overrightarrow{F}_{12} = G\frac{m_1 m_2}{{r_{21}}^2}$
   And other force wiil be on m_2 which will be directed towards m $_1$ i.e
   $\overrightarrow{F}_{21} = -G\frac{m_1 m_2}{{r_{12}}^2}$
   Clearly, It can be seen that $\overrightarrow{F}_{12} = -\overrightarrow{F}_{21}$ Because thsese forces are attracted to each other and direction of each force is opposite to other one.

2. d. 1.19×10^{10} J
   Explanation: Period of rocket in geosynchronous orbit is same as that of the earth:
   That is T = 1day = 24hours = 24 $\times$ 60 $\times$ 60 sec = 8.64 $\times$ 10^4 s
   From Keplers 3rd law
   $T^2 = K_E r^3_{GS}$
   Where $K_E = \frac{4\pi^2}{GM_E}$ = 9.89 $\times$ $10^{-14} s^2/m^3$
   Therefore the geosynchronus radius is
   $r_{GS} = \sqrt[3]{\frac{T^2}{K_E}} = \sqrt[3]{\frac{(8.64\times10^4)^2}{9.89\times10^{-14}}} = 4.23 \times 10^7 m$
   Because the initial position before the boost is 280 km = 2.8 $\times$ 10^5 m
   and the radius of the Earth is 6,370 km = 6.37 $\times$ 10^6 m
   Therefore $r_i = R_E$ + 2.80 $\times$ 10^5 m = 6.65 $\times$ 10^6 m
   The total energy needed to boost the satellite at the geosynchronus radius is the difference of total energy before and after the boost

$E=-\frac{GM_Em_s}{2}\left(\frac{1}{r_{GS}}-\frac{1}{r_i}\right)$

$=-\frac{6.67\times10^{-11}\times5.98\times10^{24}\times470}{2}\left(\frac{1}{4.23\times10^{7}}-\frac{1}{6.65\times10^{6}}\right)$ = 1.19 $\times$ 10^{10}J

3. c. $T_0=2\pi\sqrt{\frac{R_E}{g}}$

Explanation: It is the time taken by satellite to go once around the earth

$\therefore \mathrm{T}=\frac{\text{Circumference of the orbit}}{\text{orbitalvelocity}}$

$\Rightarrow T=\frac{2\pi r}{\nu}=2\pi\sqrt{\frac{r}{GM}}$ $[\text{As } \nu=\sqrt{\frac{GM}{r}}]$

Since Satellite is very close to the surface of earth, So Here, we can take r=R(Radius of earth)

$\Rightarrow T=2\pi\sqrt{\frac{R^3}{GM}}$

As $[GM=gR^2]$

$\therefore\Rightarrow T=2\pi\sqrt{\frac{R^3}{gR^2}}$

$\Rightarrow T=2\pi\sqrt{\frac{R}{g}}$

4. c. 28 N

Explanation: $g_h=g\left(1+\frac{h}{R}\right)^{-2}$

Given, h = $\frac{R}{2}$

$\therefore g_h=g\left(1+\frac{\frac{\mathrm{R}}{2}}{R}\right)^{-2}$

$\Rightarrow \frac{g_h}{g}=\frac{1}{\left(1+\frac{1}{2}\right)^2}=\frac{1}{\left(\frac{3}{2}\right)^2}=\left(\frac{2}{3}\right)^2=\frac{4}{9}$

Let m = mass of the body

If W and W_h be its weight at earth's surface and at a height h above earth's surface, then

W = mg

and $\mathrm{W}_h=mg_h=m\times\frac{4}{9}g=\frac{4}{9}mg$

$\Rightarrow \mathrm{W}_h=mg_h=\frac{4}{9}\times63=28N$

5. a. 2.6 $\times10^6$ m/s

Explanation: Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of two stars when they collide.

According to the law of conservation of momentum,

$M\vec{v}_1+M\vec{v}_2=0$

$\Rightarrow \vec{v}_1 = -\vec{v}_2$

$\Rightarrow v_1 = v_2 = v$

According to law of conservation of energy,

$2\left[\frac{1}{2}mv^2\right] = GM^2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

We Have

Mass of the star, M = 2 $\times$ 10^{30} kg

Distance between the stars, r_1 = 10^9 km = 10^{12} m

Radius of star, r = 10^4km = 10^7m

The distance between two stars when they collide is,

2r = 2 $\times$ 10^7m

$\therefore$ Speed with which the stars collide is given by,

$v = \sqrt{GM\left[\frac{1}{r_2} - \frac{1}{r_1}\right]}$

$v = \sqrt{6.67 \times 10^{-11} \times 2 \times 10^{30} \times \left[\frac{1}{2\times10^7} - \frac{1}{10^{12}}\right]}$

$v = 2.6 \times 10^6 \mathrm{m/sec}$

6. No, the origin of forces of friction or other contact forces arises from electromagnetic force. The origin of electromagnetic force is due to attraction/repulsion between particles.

7. The value of g decreases from poles to the equator. Therefore, at the surface of the earth, g is maximum at the poles and minimum at the equator

8. The load experienced by him will be zero because during the state of free fall, the acceleration is equal to the acceleration due to gravity. So, the thief will be in the state of weightlessness.

9. $The\ acceleration, g = \frac{F}{m}$ = $\frac{GMm/R^2}{m}$ = $\frac{GM}{R^2}$

   If the object is at height h above the earth's surface, then

   $g = \frac{GM}{(R+h)^2}$

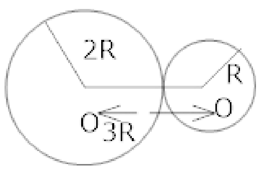
   $\Rightarrow g = \frac{6.67\times10^{-11}\cdot\times5.98\times10^{24}}{\left(6.4\times10^6+0.4\times10^6\right)^2}$ = 8.70 m/s^2

10. Potential energy of the body on the surface of the earth = $\frac{-GMm}{R}$
Potential energy at a $height\ h$ from the surface of the earth = - $\frac{GMm}{(R+h)}$
$$Work\ done = \left(-\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$$
= $\frac{GMm}{R}$ - $\frac{GMm}{R+h}$
$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$
= $\frac{GMmh}{R(R+h)}$ = $\frac{MgR^2h}{R(R+h)}$ $\left[\because g = \frac{GM}{R^2}\right]$
= $\frac{(Mgh)R}{(R+h)}$ = $\frac{Mgh}{1+\frac{h}{R}}$

11. i. The period of revolution of a satellite around the earth should be same as that of earth about its own axis (T=24 hrs)
    ii. The sense of rotation of satellite should be same as that of the earth about its own axis i.e. from west to east in anti-clockwise direction.
    iii. It should orbit in the equatorial plane and have same speed as that of earth.

12. The point must be on the line joining the centres of the earth and the moon. If the distance of the point from the earth is x, the distance from the moon is $(4.0 \times 10^5 km - x)$. The magnitude of the gravitational field due to the earth is
$$E_1 = \frac{GM_e}{x^2} = \frac{G\times 6\times 10^{24}\text{kg}}{x^2}$$
Magnitude of the gravitational field due to the moon is
$$E_2 = \frac{GM_m}{(4{\cdot}0\times 10^6\text{km}-x)^2} = \frac{G\times 7{\cdot}4\times 10^{22}\text{kg}}{(4{\cdot}0\times 10^6\text{km}-x)^2}$$
These fields are in opposite directions. $\Rightarrow E_1 = E_2$
$$\Rightarrow \frac{6\times 10^{24}\text{kg}}{x^2} = \frac{7{\cdot}4\times 10^{22}\text{kg}}{(4{\cdot}0\times 10^6\text{km}-x)^2}$$
$$\Rightarrow \frac{x}{4{\cdot}0\times 10^5\text{km}-x} = \sqrt{\frac{6\times 10^{24}}{7{\cdot}4\times 10^{22}}} = 9$$
$$\Rightarrow x = 3.6 \times 10^5 km$$

13. Two spheres of density p and radii R and 2R



Given , density of each sphere is ρ

For finding mass of each sphere , we have to find volume of each

Volume of sphere of radius R , $V_1 = \frac{4}{3}\pi R^3$

volume of sphere of radius 2R, $V_2 = \frac{4}{3}\pi(2R)^3 = \frac{32\pi R^3}{3}$

Now, mass of 1st sphere, $M_1 = V_1 \times \rho$ [mass = density $\times$ volume]

$= \frac{4\pi R^3\rho}{3}$

mass of 2nd sphere, $M_2 = V_2 \times \rho = \frac{32\pi R^3\rho}{3}$

= 2R + R = 3R

Now, Gravitational force act between M_1 and M_2 , $F = \frac{GM_1M_2}{(R+2R)^2}$

$= \frac{G\frac{4\pi R^3\rho}{32\pi R^3\rho}}{9R^2}$

$= \frac{128\pi^2R^6\rho^2}{81R^2}$

$= \frac{128\pi 2R^4\rho^2}{81}$

Hence, force between them, $F = \frac{128\pi 2R^4\rho^2}{81}$

14. Consider a body of mass m projected from the surface of Earth with a speed v_{es}, which is just sufficient to take the body up to infinite distance away.

$\therefore$ initial K.E. of body $K_1 = \frac{1}{2}mv_{es}^2$

initial P.E. of body $U_1 = -\frac{GMm}{R}$

$\therefore$ Total energy of the body at the surface of Earth $= K_1 + U_1 = \frac{1}{2}mv_{es}^2 - \frac{GMm}{R}$

When the body reaches infinity, its final velocity and hence kinetic energy is zero i.e.

$K_\infty = 0$

Also $U_\infty = -\frac{GMm}{\infty} = 0$

$\therefore$ Total energy of body at infinity = 0

According to the conservation law of mechanical energy,

$\frac{1}{2}mv_{es}^2 - \frac{GMm}{R} = 0$

$\Rightarrow \frac{1}{2}mv_{es}^2 = \frac{GMm}{R}$

$\Rightarrow v_{es} = \sqrt{\frac{2GM}{R}}$

As $g = \frac{GM}{R^2}$,

$\Rightarrow v_{es} = \sqrt{2gR}$

15. Mass of the spaceship, m_S = 1000 kg

Mass of the Sun, M = 2×10^{30} kg

Mass of Mars, m_M = 6.4×10^{23} kg

Orbital radius of Mars, R = 2.28×10^8kg = 2.28×10^{11} m

Radius of Mars, r = 3395 km = 3.395×10^6m

Universal gravitational constant, $G = 6.67 \times 10^{-11} \mathrm{N}m^2 \mathrm{kg}^{-2}$

Potential energy of the spaceship due to the gravitational attraction of the Sun

$= \frac{-GMm_2}{R}$

Potential energy of the spaceship due to the gravitational attraction of Mars

$= \frac{-GM_n m_z}{r}$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship $= \frac{-GMm_s}{R} - \frac{GM_m m_s}{r}$

$= -Gm_s \left(\frac{M}{R} + \frac{m_m}{r} \right)$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= – (Total energy of the spaceship)

$= Gm_s \left(\frac{M}{R} + \frac{m_m}{r} \right)$

$= 6.67 \times 10^{-11} \times 10^3 \times \left(\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3.395 \times 10^6} \right)$

$= 6.67 \times 10^{-8} \times 1.88 \times 10^{17}$

$= 596.97 \times 10^9$

$= 6 \times 10^{11} J$