CBSE Test Paper 01
Chapter 7 System of Particles & Rotational

1. Angular momentum of a body is defined as the product of **1**
   a. Linear velocity and angular velocity
   b. Mass and angular velocity
   c. Centripetal force and radius
   d. Moment of inertia and angular velocity
2. A dancer on ice spins faster when she folds her hands. This is due to **1**
   a. constant angular momentum and increase in kinetic energy
   b. increase in energy and decrease in angular momentum
   c. decrease in friction at the skates
   d. increase in energy and increase in angular momentum
3. A planet is revolving round the sun in an elliptical orbit. The maximum and the minimum distances of the planet from the sun are 3×10^{12} m and 2×10^{10} m respectively. The speed of the planet when it is nearest to sun is 2×10^{7} m/sec.what is the speed of the planet when it is farthest from the sun? **1**
   a. 1.5×10^{7} m/sec
   b. 2.66×10^{5} m/sec
   c. 1.33×10^{5} m/sec
   d. 3×10^{5} m/sec
4. What is the moment of inertia of a thin rod of length L and mass M about an axis passing through one end and perpendicular to its length? **1**
   a. $\frac{1}{12}\,\mathrm{ML}^2$
   b. ML^2
   c. $\frac{1}{3}\,\mathrm{ML}^2$
   d. $\frac{1}{2}\,\mathrm{ML}^2$
5. A particle moves with a constant velocity parallel to the x - axis. Its angular

momentum with respect to the origin **1**

a. goes on increasing
b. goes on decreasing
c. remains constant
d. is zero

6. What is the value of torque on the planet due to the gravitational force of sun? **1**

7. What is the position of centre of mass of a rectangular lamina? **1**

8. State the condition for translational equilibrium of a body. **1**

9. A comet revolves around the Sun in a highly elliptical orbit having a minimum distance of 7×10^{10} m and a maximum distance of 1.4×10^{13}m. If its speed while nearest to the sun is 60 kms^{-1}, find its linear speed when situated farthest from the sun. **2**

10. What is the importance of the study of centre of mass of a system in mechanics? **2**

11. An energy of 484 J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm. Calculate moment of inertia of the flywheel. **2**

12. Establish the relation $\vec{\tau}=I\vec{\alpha}$ for rotation about a fixed axis. **3**

13. (n - 1) equal point masses each of mass *m* are placed at the vertices of a regular *n*-polygon. The vacant vertex has *a* position vector $\vec{a}$ with respect to the centre of the polygon. Find the position vector of centre of mass. **3**

14. Show that the area of the triangle contained between the vectors a and b is one half of the magnitude of a $\times$ b. **3**

15. Find the centre of mass of a uniform **5**

i. half-disc,
ii. quarter-disc.

Answer

1. d. Moment of inertia and angular velocity

   Explanation: $\vec{L} = \vec{r} \times \vec{p}$

   if r is perpendicular to p (in case of circular motion)

   L = mvr

   $L = m\,(r\omega)\,r = mr^2\omega$

   L = Iω

2. a. constant angular momentum and increase in kinetic energy

   Explanation: $L = I\omega$

   if L = const.

   then $I\omega = const.$

   When a dancer is spinning, she has a certain angular momentum. When the dancer folds her arms, the distance of all the points of her body decreases with respect to axis of rotation so that her moment of inertia decreases. Thus, in order to conserve angular momentum, the speed of rotation has to increase and hence the dancer spins faster.

   $K = \frac{L^2}{2I}$

   $K \propto \frac{1}{I}$

   Hence if moment of inertia decrease then kinetic energy will increase.

3. c. 1.33 $\times\ 10^5$ m/sec

   Explanation: Perihelion is the nearest distance of planet from focus.

   aphelion is the farthest distance of planet from focus.

   $v_p = 2 \times 10^7 m/s$

   v_a = ?

   $r_p = 2 \times 10^{10} m$

   $r_a = 3 \times 10^{12} m$

   $\frac{v_p}{v_a} = \frac{r_a}{r_p}$

   $\frac{2\times10^7}{v_a} = \frac{3\times10^{12}}{2\times10^{10}}$

$v_a = 1.33 \times 10^5 m/s$

4. c. $\frac{1}{3}$ ML^2

Explanation: Using theorem of parallel axis, the axis is shifted by L/2 distance from centre of Mass where the Moment of Inertia is $ML^2/12$, thus the moment of inertia of a thin rod about an axis passing through one end and perpendicular to its length is $ML^2/12 + ML^2/4 = \frac{1}{3}$ ML^2

5. c. remains constant

Explanation: As angular momentum is $\vec{L} = \vec{p} \times \vec{r} = mvrsin\theta$, Now rsinθ=perpendicular distance from x axis which is constant, so angular momentum is remains constant.

6. Zero.

7. The centre of mass of a rectangular lamina is the point of intersection of diagonals.

8. For translations equilibrium of a body the vector sum of all the forces acting on the body must be zero.

9. Let the mass of comet be M and its angular speed be ω when situated at a distance r from the Sun, then its angular momentum

$L = I\,\omega = Mr^2\omega$

If v be the linear speed, then $L = Mr^2\omega = Mrv$

According to the conservation law of angular momentum,

$mr_1v_1 = mr_2v_2$

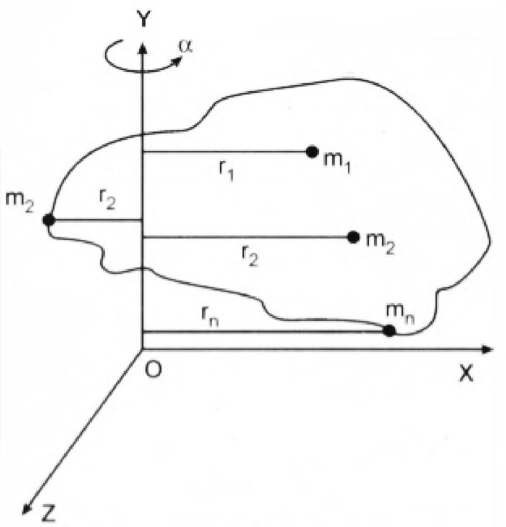
$\therefore v_2 = \frac{r_1v_1}{r_2} = \frac{7\times10^{10}\times60}{1.4\times10^{13}} = 0.3\ km/s\ \ or\ \ 300\ m/s$

10. In mechanics, the study of centre of mass of a system of particles is extremely important due to the following points :

i. Whole mass of the system may be considered to be concentrated at the centre of mass.
ii. The nature of motion executed by the system remains unaltered when the forces acting on the system are applied directly at its centre of mass.
iii. In the absence of external forces, if any, the centre of mass of the system either

remains at rest or in a state of motion having uniform velocity.

11. Energy spent, $W = 484J$
Initial speed, $\omega_1 = 60\ rpm = \frac{60}{60} \times 2\pi$ rad/s
Final speed, $\omega_2 = 360\ rpm = \frac{360}{60} \times 2\pi$ rad/s
Moment of inertia, $I = ?$
Energy spent, $W = E_2 - E_1 = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2$
$= \frac{1}{2} I \left[(12\pi)^2 - (2\pi)^2\right] = 70 I\pi^2$
$\Rightarrow I = \frac{484}{70 \times \pi^2} = 0.7 kg\ m^2$

12. 

Consider a rigid body rotating about an axis OY, as shown in the figure.
When an external torque τ be applied on the rigid body about the rotational axis, its angular velocity changes which causes an angular acceleration 'α' in the rotating body. Obviously whole rigid body has same value of angular acceleration about the axis of rotation. However, particles of masses m_1 m_2, m_3, ...etc., situated at normal distances r_1 r_2, r_3, ... from the rotational axis have different values of linear acceleration given by a_1 a_2, a_3 .... etc.

If we consider, in general, any (say ith) particle of mass m_i situated at a normal distance r_i, its linear acceleration has a magnitude equal to

$a_i = r_i \cdot \alpha$ ....(1)
It means that external force on ith particle, $F_i = m_i a_i = m_i r_i \alpha$
Therefore, the torque acting about the rotational axis due to the force F_i is given by:
$\tau_i = F_i r_i$

or $\tau_i = (m_i r_i \alpha) r_i = \left(m_i r_i^2\right) \alpha$

Therefore, the total torque on the whole rigid body about the rotational axis is given by:

$\tau = \sum_{i=1}^{n}$ = $\tau_i = \sum \left(m_i r_i^2\right) \alpha$ ....(2)

But we know that, $I = \Sigma m_i r_i^2$, therefore from equation (2) we get

$\tau = I\alpha$

Hence, torque (τ) = moment of inertia (I) $\times$ angular acceleration (α)

Since the moment of inertia is a scalar, hence, we have

$\vec{\tau} = I\vec{\alpha}$ ....(2)

which is the required relation.

13. The *center of mass* is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses. For simple rigid objects with uniform density, the center of mass is located at the centroid.If all the masses were instead placed at the centre of mass then they must give the same resultant turning moment about any point in the system as all of the individual moments added together.

$(m_1 + m_2 + \ldots.)r_{cm} = m_1 r_1 + m_2 r_2 + \ldots$

$r_{cm} = \frac{\sum m_i r_i}{\sum m_i}$

The centre of mass of a regular n-polygon lies at it's geometric centre. Let $\vec{b}$ is the position vector of the centre of mass of a regular n-polygon. (n – 1) equal point mass are placed at (n – 1) vertices of n-polygon then r_{cm} when mass m is placed at nth vertex .

$r_{cm} = \frac{(n-1)mb + ma}{(n-1)m + m}$

If mass m is placed at nth remaining vertex then

$r_{cm} = 0$

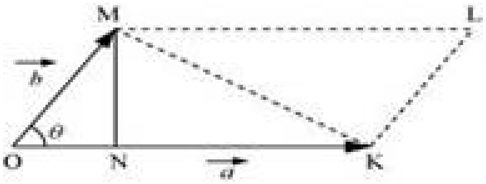
$\frac{(n-1)mb + ma}{(n-1)m + m} = 0$

$(n-1)mb + ma = 0$

$b = -\frac{ma}{(n-1)m}$

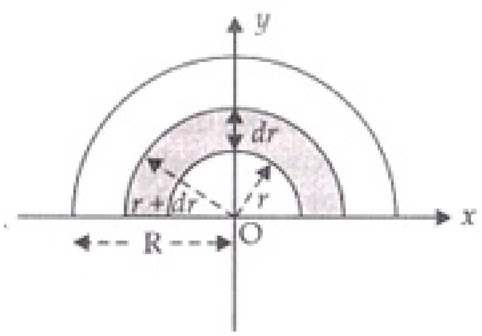
$\Rightarrow \vec{b} = -\frac{\vec{a}}{(n-1)}$

(-) sign shows that c.m. lies other side from nth vertex geometrical centre of n-polygon i.e., $\vec{b}$ is opposite to the vector $\vec{a}$-(from centre to nth vertex) as shown in above calculation.

14. To calculate the area of the parallogram using vector method ,we will proceed in the following manner.
Let there are two vectors $\overline{OK} = |\vec{a}|$ and $\overline{OM} = |\vec{b}|$, inclined at an angle θ, as shown in the following figure for which we have to find the area of corresponding parallogram OKLMO -


In $\Delta \mathrm{OMN}$, we can write the relation:
$\sin\theta = \frac{MN}{OM} = \frac{MN}{|\vec{b}|}$
$MN = |\vec{b}|\sin\theta$
$|\vec{a}\times\vec{a}| = |\vec{a}||\vec{b}|\sin\theta$
$= OK \cdot MN \times \frac{2}{2}$
= 2 × Area of ΔOMK
$\therefore$ Area of $\Delta\,\mathrm{OMK} = \frac{1}{2}|\vec{a}\times\vec{b}|$
Thus it can be proved that area of parallelogram is half the magnitude of cross product of two vectors both forming separate sides of the parallogram.OKLMO as shown in the figure.
Above problem can be Proved in such manner.

15. Let mass of half disc is M.



i. Area of element $= \frac{\pi}{2}\left[(r+dr)^2 - r^2\right]$
$= \frac{\pi}{2}\left[r^2 + dr^2 + 2rdr - r^2\right]$
$= \pi r dr$

$\therefore$ Mass of elementary Ring $dm = \frac{2M}{\pi R^2} . \pi r dr$

$\mathrm{dm} = \frac{2M}{R^2} r . dr$

Let (x, y) are the co-ordinates of c.m. of this strip $(x, y) = \left(0, \frac{2r}{\pi}\right)$

$x = x_{cm} = \frac{1}{M} \int_0^R x dm = \int_0^R 0 dm = 0$

$y_{cm} = \frac{1}{M} \int_0^R y dm = \frac{1}{M} \int_0^R \frac{2r}{\pi} \times \frac{2M}{R^2} r dr$

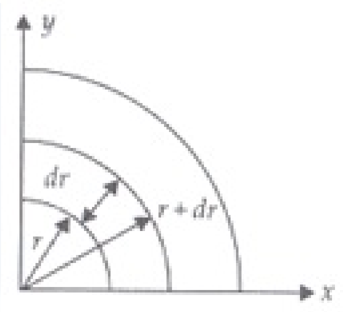
$= \frac{1}{m} \cdot \frac{4M}{\pi R^2} \int_0^R r^2 dr = \frac{4}{\pi R^2} \left[\frac{r^3}{3}\right]_0^R = \frac{4}{3\pi R^2} \cdot R^3$

$y_{cm} = \frac{4R}{3\pi}$

So centre of mass of circular half disc $= \left(0, \frac{4R}{3\pi}\right)$

ii. Mass per unit area of quarter disc

$\sigma = \frac{M}{\frac{\pi R^2}{4}} = \frac{4M}{\pi R^2}$.



Area of element = $\frac{1}{2}$

dm = $\frac{1}{2} \pi r dr \times \sigma = \frac{2Mr}{R^2} dr$

$X_{cm} = \int_0^R x dm = \frac{4R}{3\pi}$

Similarly $Y_{cm} = \frac{4R}{3\pi}$

center of mass = $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$