CBSE Test Paper 02

Chapter 4 Motion in A Plane

1. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? **1**

    a. 47.3 km/hr, 23.4 km/hr
    b. 49.3 km/hr, 21.4 km/hr
    c. 48.3 km/hr, 22.4 km/hr
    d. 46.3 km/hr, 24.4 km/hr

2. A batter hits a baseball so that it leaves the bat at speed v_0 = 37.0 m/s at an angle a = 53.1^o. Find the time when the ball reaches the highest point of its flight, and its height h at this time. **1**

    a. 3.02 s, 44.7 m
    b. 3.32 s, 41.7 m
    c. 3.12 s, 43.7 m
    d. 3.22 s, 42.7 m

3. A unit vector is a vector **1**

    a. having a magnitude of 1 and points in any chosen direction
    b. having a magnitude of 1 and points in x-direction
    c. having a magnitude of 1 and points in y-direction
    d. having a magnitude of 1 and points in z-direction

4. The basic difference between a scalar and vector is one of **1**

    a. magnitude
    b. direction
    c. origin
    d. polar angle

5. A man stands on the roof of a 15.0-m-tall building and throws a rock with a velocity of magnitude 30.0 m/s at an angle of 33.0° above the horizontal. You can ignore air resistance. Calculate the maximum height above the roof reached by the rock; **1**

   a. 12.6 m
   b. 11.7 m
   c. 13.6 m
   d. 14.2 m

6. Write an example of zero vector. **1**

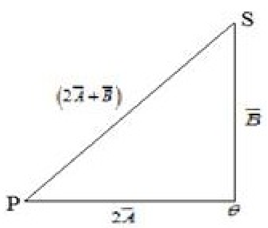
7. Two vectors of magnitude 3 units and 4 units are inclined at angle 60° w.r.t each other. Find the magnitude of their difference. **1**

8. The magnitude of vectors A, B and C are 12, 5 and 13 units respectively and A + B = C, find the angle between A and B. **1**

9. In dealing with the motion of a projectile in the air, we ignore the effect of air resistance on the motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? sketch such a trajectory and explain why you have drawn it that way. **2**

10. An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 kmh^{-1}. Compare its centripetal acceleration with the acceleration due to gravity. **2**

11. A vector $\overline{A}$ has magnitude 2 and another vector $\overline{B}$ have magnitude 3 and is perpendicular to each other. By vector diagram find the magnitude of $2\overline{A}+\overline{B}$ and show its direction in the diagram. **2**

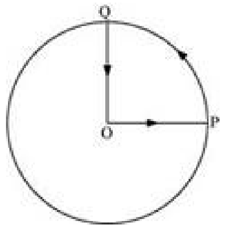


12. A projectile is projected with a certain velocity u at an angle θ with horizontal from the ground. Find expression for its trajectory. **3**

13. A projectile is fired with speed u making an angle θ with horizontal from the surface

of Earth. Prove that the projectile will hit the surface of earth with same speed and at the same angle. **3**

14. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in figure. If the round trip takes 10 min, what is the **3**



    i. net displacement,
    ii. average velocity, and
    iii. average speed of the cyclist?

15. a. What is the angle between $\overrightarrow{A}$ and $\overrightarrow{B}$ if $\overrightarrow{A}$ and $\overrightarrow{B}$ denote the adjacent sides of a parallelogram drawn form a point and the area of the parallelogram is $\frac{1}{2}AB$?
    b. State and prove triangular law of vector addition. **5**

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Answer

1. b. 49.3 km/hr, 21.4 km/hr

Explanation: (a) Total distance travelled = 23 km

Total time taken = 28 min = $\frac{28}{60}$ h

$\therefore$ Average speed of the taxi = $\frac{Total\ distance\ travelled}{Total\ time\ taken}$

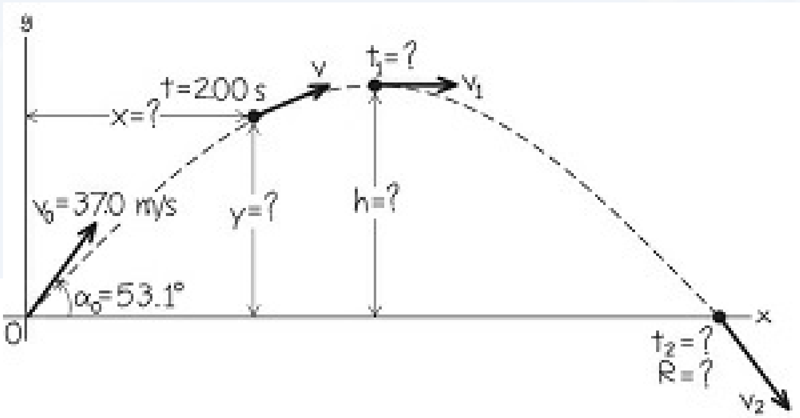
$$= \frac{\frac{23}{1}}{\frac{28}{60}} = 49.3km/h$$

(b) Distance between the hotel and the station = 10 km = Displacement of the car

$\therefore$ Average velocity = $\frac{\frac{10}{1}}{\frac{28}{60}} = 21.4\ km/h$

2. a. 3.02 s, 44.7 m

Explanation: The initial velocity of the ball has components



v_{ox} = v_o cos α_o = $37.0 \times cos53.1^o$

= 22.2 m/s

v_{oy} = vo_o sin α_o = $37.0 \times sin53.1^o$

= 29.6 m/s

At the highest point, the vertical velocity v_y is zero. Call the time when this happens t_1; then

$v_y = v_{oy} - gt_1 = 0$

$\Rightarrow t_1 = \frac{v_{oy}}{g} = \frac{29.6}{9.8}$ = 3.02 s

The height at the highest point is the value of y at time t_1:

$h = v_{oy}t_1 - \frac{1}{2}g(t_1)^2$

$= 29.6 \times 3.02 - \frac{1}{2} \times 9.8 \times (3.02)^2$

= 44.7 m

3. a. having a magnitude of 1 and points in any chosen direction

Explanation: A unit vector in a normed vector space is a vector (often a spatial vector) of length 1. A unit vector is often denoted by a lowercase letter with a circumflex, or "hat": $\hat{i}$ (pronounced "i-hat"). The term direction vector is used to describe a unit vector being used to represent spatial direction.

$\hat{i} =$ a unit vector directed along the positive x axis

$\hat{j}$ = a unit vector directed along the positive y axis

$\hat{k}$ = a unit vector directed along the positive z axis

4. b. direction

Explanation: Scalar quantity gives you an idea about how much of an object there is, but vector quantity gives you an indication of how much of an object there is and that also in which direction. So, the main difference between these two quantities is associated with the direction, i.e. scalars do not have direction but vectors do.

5. c. 13.6 m

Explanation: Let downward be the y direction.

$v_{ox} = v_o \times cos\theta = 30.0 \times cos33.0^o$

= 25.2 m/s

$v_{oy} = v_o \times sin\theta = 30.0 \times sin33.0^o$

= 16.3 m/s

At the maximum height, the velocity in the y direction v_y is zero:

Using $(v_y)^2 = (v_{oy})^2 + 2a_yh$

$=> 0 = (16.3)^2 + 2(-9.8)h$

$h = \frac{-(16.3 \times 16.3)}{-2 \times 9.8}$ = 13.6 m

6. The velocity vectors of a stationary object is a zero vector.

7. Let the vectors are A and B.

Given, |A| = 3 units, |B| = 4 units and θ= 60°

The magnitude of resultant of difference of A and B from parallelogram law of vector addition for vectors A and (-B) is given by,

$R = |\mathbf{R}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{3^2 + 4^2 - 2 \times 3 \times 4\cos 60^\circ}$

R = $\sqrt{25 - 12} = \sqrt{13}$= 3.61 units

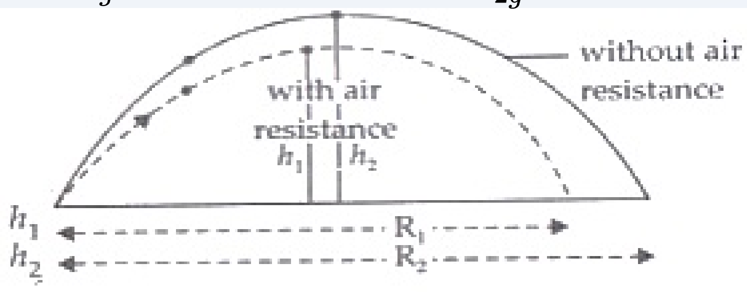
8. From the given values of vector A, B and C we know that $C^2 = A^2 + B^2$

or $13^2 = 12^2 + 5^2$, this is possible when value of cos θ = zero, that is θ = 90°

Thus, the angle between A and B is 90°.

9. When air resistance acts on projectile then its vertical and horizontal both velocity will decrease due to air resistance. Hence its maximum height becomes smaller than when there is no force of friction (resistance) of air. By formula

$R = \frac{u^2}{g}\sin 2\theta$ and $H_{\max} = \frac{u^2 \sin^2\theta}{2g}$



$\therefore h_1 < h_2$ and $R_1 < R_2$

But time of flight for both will remain same as the body in case of (with air resistance) $h_1 < h_2$ takes smaller time to rise.

10. Here, radius of the horizontal circular loop traversed by the aeroplane r = 1 km = 1000 m,

with constant speed v = 900 km h^{-1} = $\frac{900 \times (1000\text{m})}{(60 \times 60\text{s})}$ = 250 ms^{-1}

We know that Centripetal acceleration, $a = \frac{v^2}{r} = \frac{(250)^2}{1000}$

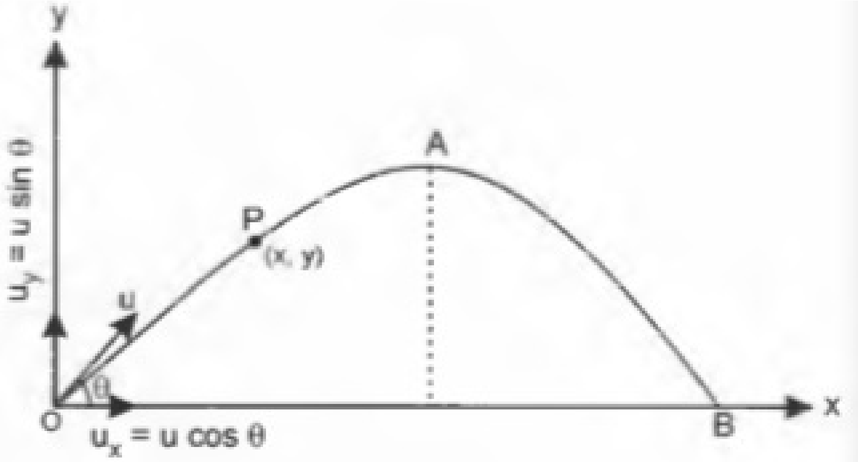
Now, $\frac{a}{g} = \frac{(250)^2}{1000} \times \frac{1}{9.8} = 6.38$

Hence a_c is 6.4 times more than the g in this case.

11. Here $(P\overline{Q}) = 2\overline{A} = 4\text{cm}$

$Q\overline{S} = \overline{B} = 3\text{cm}$

$$|P\overline{S}| = \sqrt{PQ^2 + QS^2}$$
$$|P\overline{S}| = \sqrt{4^2 + 3^2}$$
$$|P\overline{S}| = 5\text{cm}$$

12. 

Consider a projectile projected from point O with an initial velocity u inclined at an angle θ with horizontal. Here, the motion is two dimensional, where

x_0 = 0, u_x = u cosθ and a_x = 0

and y_0 = 0, u_y = +u sinθ and a_y = -g

If projectile reaches P at time t, then we have

x = u_x t + $\frac{1}{2}$ $a_x t^2$ ....(i)

or $x = u \cos\theta \cdot t$

and y = u_yt + $\frac{1}{2}$ $a_y t^2$

= u sinθ t - $\frac{1}{2}$ gt^2 ....(ii)

Substituting value of t from (i) $\left(i.e.\ t = \frac{x}{u\cos\theta}\right)$ in (ii), we get

y = u sinθ .$\frac{x}{u\cos\theta}$ - $\frac{1}{2}g \cdot \left(\frac{x}{u\cos\theta}\right)^2$

$\Rightarrow$ y = x tan θ - $\frac{gx^2}{2u^2\cos^2\theta}$

It is the equation of the trajectory of a projectile. Obviously, it is the equation of a parabola.

13. Let any projectile fire with u velocity and inclination θ angle with the horizontal and T is time of flight.

When a projectile is fired with speed u making an angle θ with horizontal, it describes a parabolic path, where instantaneous velocity v of the projectile at time has a magnitude equal to

v = $\sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

and is inclined at an angle β from horizontal such that

tanβ = $\frac{u\sin\theta - gt}{u\cos\theta}$

When the projectile hits the surface of Earth, the time t = time of flight T = $\frac{2u\sin\theta}{g}$

Therefore, v = $\sqrt{(u\cos\theta)^2 + \left(u\sin\theta - \frac{g\cdot 2u\sin\theta}{g}\right)^2}$

= $\sqrt{(u\cos\theta)^2 + (-u\sin\theta)^2}$ = u

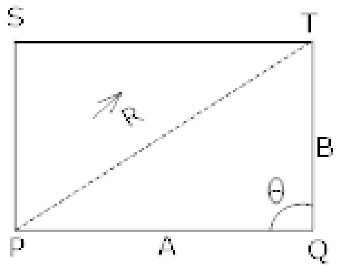
and tanβ = $\frac{u\sin\theta - g\left(\frac{2u\sin\theta}{g}\right)}{u\cos\theta}$ = -tanθ = tan(-θ)

$\Rightarrow \beta$ = - θ (it means magnitude of angle is same but direction is just opposite to each other).

So, the projectile will hit the ground with same speed u at same angle θ, which is downward.

14. i. Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point after cycling for 10 minutes.
    Hence, his net displacement is zero.
    ii. Average velocity is given by the relation: Average velocity $= \frac{\text{net displacement}}{\text{total time interval}}$
    Since the net displacement of the cyclist is zero, his average velocity will also be zero.
    iii. Average speed of the cyclist is given by the relation: Average speed $= \frac{\text{total path length}}{\text{time interval}}$
    Total path length = OP + PQ + QO
    Total path length $= 1 + \frac{1}{4}(2\pi \times 1) + 1 = 2 + \frac{1}{2}\pi$ = 3.570 km
    Time taken $= 10\text{min} = \frac{10}{60} = \frac{1}{6}h$
    $\therefore$ Average speed $= \frac{3.570}{\frac{1}{6}}$ = 21.42 km/h

15. a. Area of a parallelogram $= |\vec{A} \times \vec{B}|$
    Area of parallelogram = $AB\sin\theta$ ($\because$ Applying cross product)
    Given, area of parallelogram $= \frac{1}{2}AB$



So, $\frac{1}{2} AB = AB \sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

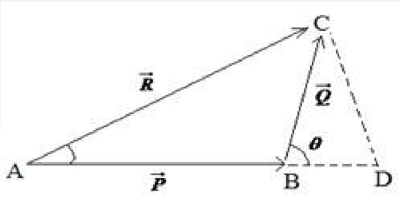
$\theta = 30^{\circ}$

b. Triangular law of vector addition states that if two vectors can be represented both in magnitude and direction by the sides of a triangle taken in order then their resultant is given by the third side of the triangle taken in opposite order.

Proof: in $\triangle$ ADC

$(AC)^2 = (AD)^2 + (DC)^2$

$(AC)^2 = (AB + BD)^2 + (DC)^2$



$(AC)^2 = (AB)^2 + (BD)^2 + 2(AB)(BD) + (DC)^2$

$(AC)^2 = \left(P^2\right) + (Q\cos\theta)2 + 2(P)(Q\cos\theta) + (Q\sin\theta)^2$

$(AC)^2 = P^2 + Q^2\left(\sin^2\theta + \cos^2\theta\right) + 2PQ\cos\theta$

$\left(\because \frac{CD}{BC} = \sin(\theta)\ ,\ \ \frac{BD}{BC} = \cos(\theta)\right)$

$(R)^2 = P^2 + Q + 2PQ\cos\theta\left(\because \sin^2\theta + \cos^2 Q\right)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$