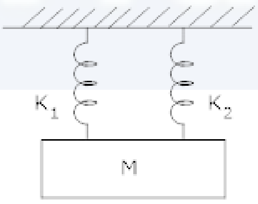
CBSE Test Paper 02

Chapter 14 Oscillations

1. The pendulum of a wall clock executes **1**
   a. a translatory motion
   b. a circular motion
   c. a rotary motion
   d. an oscillatory motion
2. The total energy of a particle, executing simple harmonic motion is **1**
   a. Independent of x
   b. $\propto \frac{x}{2}$
   c. $\propto x^2$
   d. $\propto x$
3. Frequency of the periodic motion is **1**
   a. the reciprocal of the period of oscillation
   b. the reciprocal of the wave velocity
   c. time taken to complete one oscillation
   d. twice the period of oscillation
4. Wavelength of the wave is the distance between two particles of the medium having a phase difference of **1**
   a. 3π
   b. zero
   c. 2π
   d. π
5. The ratio of maximum acceleration to maximum velocity of a particle performingS.H.M is equal to **1**
   a. Angular velocity
   b. Square of angular velocity
   c. Square of amplitude
   d. Amplitude
6. Define angular frequency. Give its S.I. unit. **1**
7. State the conditions when motion of a particle can be an SHM. **1**

8. Can the motion of an artificial satellite around the earth be taken as SHMT? **1**
9. Define the restoring force and its characteristics in case of an oscillating body. **2**
10. A spring compressed by 0.1 m develops a restoring force 10 N. A body of mass 4 kg is placed on it. Deduce **2**
    i. the force constant of the spring.
    ii. the depression of the spring under the weight of the body (take g = 10 N/kg)
    iii. the period of oscillation after the body is distributed and
    iv. frequency of oscillation
11. The kinetic energy of a particle executing S.H.M. is 16J when it is in its mean position. If the amplitude of oscillations is 25cm and the mass of the particle is 5.12kg. Calculate the time period of oscillations? **2**
12. A body of mass 0.2 kg and velocity 6 m/s (after 1 s of its motion) executes SHM. Find out the total energy and potential energy of the body if time period of the body is 8 s during the SHM. **3**
13. Determine the time period of a simple pendulum of length 'l' when mass of bob is'm' Kg? **3**
14. A mass = m suspend separately from two springs of spring constant k_1 and k_2 gives time period t_1 and t_2 respectively. If the same mass is connected to both the springs as shown in figure. Calculate the time period ‘t’ of the combined system? **3**



15. A body of mass *m* is attached to one end of a massless string which is suspended vertically from a fixed point. The mass is held in hand so that the spring is neither stretched nor compressed. Suddenly the support of the hand is removed. The lowest position attained by the mass during oscillation is 4cm below the point, where it was held in hand. **5**
    i. What is the amplitude of oscillation?
    ii. Find the frequency of oscillation?

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Answer

1. d. an oscillatory motion

   Explanation: It executes simple harmonic motion because in this motion restoring force is directly proportional to the displacement towards the mean position at any time.

2. b. $\propto \frac{x}{2}$

   Explanation: E = KE + PE

   $KE = \frac{1}{2}m\omega^2 A^2 sin^2(\omega t + \phi)$

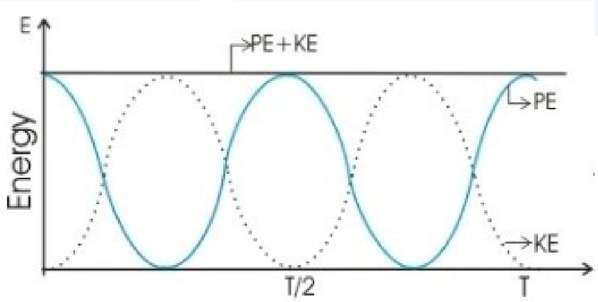
   $PE = \frac{1}{2}m\omega^2 A^2 cos^2(\omega t + \phi)$

   $E = \frac{1}{2}m\omega^2 A^2$ (as) $sin^2(\omega t + \phi) + cos^2(\omega t + \phi) = 1$

   Thus total energy of the oscillator remains constant as displacement is regained after every half cycle.

   no energy is dissipated then all the potential energy becomes kinetic and vice versa.

   Figure below shows the variation of kinetic energy and potential energy of harmonic oscillator with time where phase ϕ is set to zero for simplicity



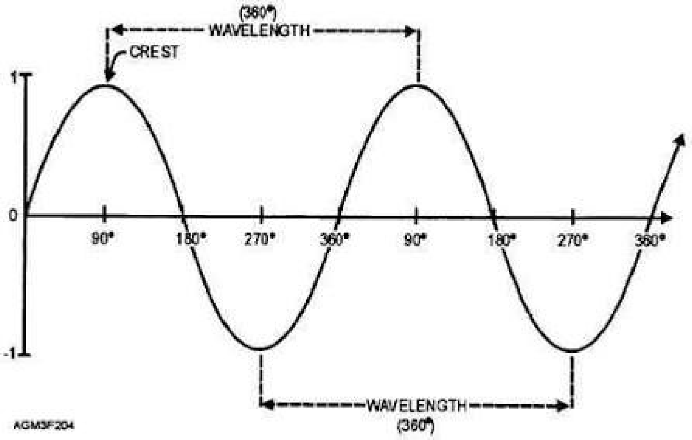
3. a. the reciprocal of the period of oscillation

   Explanation: Frequency is defined as number of oscillation completed by an oscillating body in unit time i.e. in one second during oscillatory motion.

   $\nu = \frac{1}{T}$

4. c. 2π

   Explanation: The wavelength is defined as distance between two adjacent troughs or crests. The points on two adjacent troughs or crests are in phase 2π.



5. a. Angular velocity

   Explanation: $a_{max} = w^2 A$

   $v_{max} = wA$

   thus $\frac{a_{max}}{v_{max}} = \frac{w^2 A}{wA} \Rightarrow w$

6. It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π.

   $\omega = 2\pi$ v, S.I. unit is rads s^{-1}

7. For SHM, the restoring force on the oscillating particle must be proportional to its displacement and directed towards mean position i.e. opposite in direction of the displacement vector.

8. No, it is a circular and periodic motion but not SHMT.

9. A force which takes a body back towards the mean position in oscillation is called restoring force.

   Characteristic of Restoring force : Magnitude of the restoring force at any instant is directly proportional to the displacement of the particle from its mean position but directed opposite to the displacement vector i.e. towards the mean position.

10. Here, restoring force (F) = 10 N, compression Δl = 0.1 m, mass(m) = 4 kg

    i. Spring constant, k = $\frac{F}{\Delta l} = \frac{10}{0.1}$ = 100 N/m

    ii. Depression, y = $\frac{mg}{k} = \frac{4 \times 10}{100}$ = 0.4 m

    iii. Time period, T = $2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7}\sqrt{\frac{4}{100}}$ = 1.26 s

iv. Frequency, $\nu = \frac{1}{T} = \frac{1}{1.26}$ = 0.8 Hz

11. K. E. = Kinetic energy = 16J

Now, m = Mass = 5.12kg

ω = Angular frequency

a = amplitude = 25cm or 0.25m

The Maximum value of K. E. is at mean position which is$= \frac{1}{2}mw^2a^2$

So, $16 = \frac{1}{2}m\omega^2a^2$

$16 \times 2 = m\omega^2a^2$

$32 = 5.12 \times \omega^2 \times (0.25)^2$

$32 = 5.12 \times \omega^2 \left(625 \times 10^{-4}\right)$

$\frac{32}{5.12 \times 625 \times 10^{-4}} = \omega^2$

$\frac{32}{512 \times 625 \times 10^{-2-4}} = \omega^2$

$\frac{32}{320000 \times 10} = \omega^2$

$10^{-4+6} = \omega^2$

$100 = \omega^2$

w = 10 rad/sec

Now, T = Time Period $\frac{2\pi}{w} = \frac{2\pi}{10} = \frac{\pi}{5}\text{sec}$

12. Given mass(m) = 0.2 kg, Time period(T) = 8 s and angular frequency,

$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}\text{rad/s}$

When t = 1 s, v = 6 m/s

Again we know that velocity, v(t) = ωA cosωt(A being amplitude of the wave)

$6 = \frac{\pi}{4} \times A\cos\left(\frac{\pi}{4} \times 1\right)$(putting all the above values)

$\Rightarrow 6 = \frac{\pi}{4} \times A \times \frac{1}{\sqrt{2}}$

$\Rightarrow A = \frac{4\sqrt{2}\times 6}{\pi} = \frac{24\sqrt{2}}{\pi}\text{m}$

The total energy of the body

$E = \frac{1}{2}m\omega^2A^2$

$= \frac{1}{2} \times 0.2 \times \left(\frac{\pi}{4}\right)^2 \times \left(\frac{24\sqrt{2}}{\pi}\right)^2 = \frac{230.4}{32}$

= 7.2 J

Potential energy, PE = E - KE

= 7.2 - $\frac{1}{2}$ mv^2

$= 7.2 - \frac{1}{2} \times 0.2 \times (6)^2$

= 7.2 - 3.6 = 3.6 J

Potential energy = 3.6 J

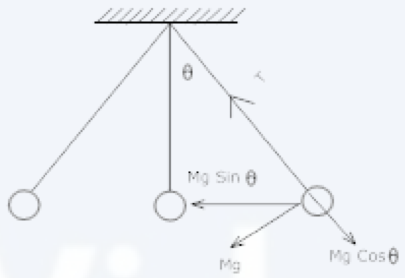
13. A **simple pendulum** is one which can be considered to be a point mass suspended from a string or rod of negligible mass. It is a resonant system with a **single**resonant frequency.It consist of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support which is free to oscillate.

The distance between point of suspension and point of oscillation is effective length of pendulum.

M = Mass of Bob

x = Displacement = OB

l = length of simple pendulum



Let the bob is displaced through a small angle θ the forces acting on it:-

i. weight = Mg acting vertically downwards.
ii. Tension = T acting upwards towards fixed point.

Divide Mg into its components → Mg Cos θ(opposite to Tension) & Mg Sin θ(Provides restoring force)

T = Mg Cos θ

F = Mg Sin θ

- ve sign shows force is diverted towards the ocean positions. If θ = Small (from 0^0 to 15^0)

$\sin\theta \cong \theta = \frac{ArcOB}{l} = \frac{x}{l}$

$F = -Mg\frac{x}{l}$

In S.H.M., vectoring fore, $\mathrm{F} = -\mathrm{mg}\ \theta$

$\mathrm{F} = -\mathrm{mg}\frac{x}{l} \rightarrow 1)$

Also, if k = spring constant

F = - k x

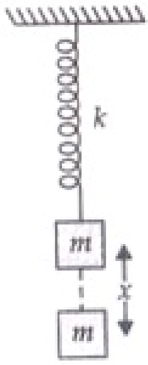
$-\mathrm{mg}\,\frac{x}{l} = -kx \left(\text{equating F} = -\mathrm{mg}\,\frac{x}{l}\right)$
$k = \frac{mg}{l}$
$T = 2\pi\sqrt{\frac{m}{k}}$
$= 2\pi\sqrt{\frac{m \times l}{mg}}$
$T = 2\pi\sqrt{\frac{l}{g}}$
i.e.

i. Time period depends on length of pendulum and 'g' of place where experiment is done(in most of the cases 'g' is assumed to be constant)
ii. T is independent of amplitude of vibration provided and it is small and also of the mass of the bob.

14. If T = Time Period of simple pendulum
m = Mass
k = Spring constant
Then, time period of simple pendulum is given as:
$\Rightarrow T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$
$\Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$
$\Rightarrow k = \frac{4\pi^2 m}{T^2}$
For first spring :
$\Rightarrow k_1 = \frac{4\pi^2 m}{t_1^2}$ let $T = t_1$
For second spring :
$\Rightarrow k_2 = \frac{4\pi^2 m}{t_2^2}$ let $T = t_2$
When springs is connected in parallel, effective spring constant, k = k = k_1 + k_2
$\Rightarrow k = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$
If t = total time period
$\Rightarrow \frac{4\pi^2 m}{t^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$
$\Rightarrow \frac{1}{t^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$
$\Rightarrow t^{-2} = t_1^{-2} + t_2^{-2}$

15. a. Let the mass reaches at its new position x unit displacement from previous.

Then P.E. of spring or mass = gravitational P.E. lost by man



$PE = mgx$

But P.E. due to spring is $\frac{1}{2}kx^2, k = \omega^2 A$

$\therefore \frac{1}{2}kx^2 = mgx$

$x = \frac{2mg}{k}$

Mean position of spring by block will be when let extension is x_0 then

$F = +kx_0$

$F = mg \therefore mg = +kx_0$ or $x_0 = \frac{mg}{k}$ ....(ii)

From (i) and (ii)

$x = 2\left(\frac{mg}{k}\right) = 2x_0$

$x = 4\text{cm} \quad \therefore 4 = 2x_0$

$x_0 = 2\text{cm}$

The amplitude of oscillator is the maximum distance from mean position i.e.,

$x - x_0 = 4 - 2 = 2\text{cm}$

b. Time Period $T = 2\pi\sqrt{\frac{m}{k}}$ which does not depend on amplitude

$\frac{2mg}{k} = x$ from (i)

$\frac{m}{k} = \frac{x}{2g} = \frac{4\times10^{-2}}{2\times9.8}$ or $\frac{k}{m} = \frac{2\times9.8}{4\times10^{-2}}$

$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\times3.144}\sqrt{\frac{2\times9.8}{4\times10^{-2}}} = \sqrt{\frac{4.9\times10^{2}}{6.28}}$

$v = \frac{10\times2.21}{6.28} = 3.52\text{Hz}$

Oscillator will not rise above the positive from where it was released because total extension in spring is 4 cm when released and amplitude is 2 cm. So it oscillates below the released position.