

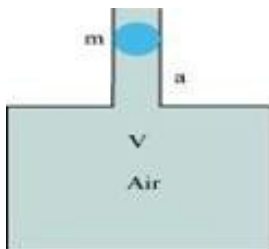
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**CBSE Test Paper 01**  
**Chapter 14 Oscillations**

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1. The total mechanical energy of a harmonic oscillator **1**
  - a. Decreases linearly with time
  - b. Increases linearly with time
  - c. Is independent of time
  - d. Increases quadratically with time
2. A spring with spring constant  $k$  when stretched through 1 cm has a potential energy  $U$ . If it is stretched by 4 cm, the potential energy will become **1**
  - a.  $16U$
  - b.  $32U$
  - c.  $4U$
  - d.  $8U$
3. The length of a second's pendulum decreases by 0.1 percent, then the clock **1**
  - a. Gains 43.2 seconds per day
  - b. Loses 37.8 seconds per day
  - c. Loses 30 seconds per day
  - d. Loses 13.5 seconds per day
4. What is constant in simple harmonic motion? **1**
  - a. Potential energy
  - b. Time period
  - c. Kinetic motion
  - d. Restoring force
5. The angular velocities of three bodies in simple harmonic motion are  $\omega_1, \omega_2, \omega_3$  with the respective amplitudes as  $A_1, A_2, A_3$ . If all the three bodies have same mass and velocity, then **1**
  - a.  $A_1^2 \omega_1 = A_2^2 \omega_2 = A_3^2 \omega_3$
  - b.  $A_1 \omega_1^2 = A_2 \omega_2^2 = A_3 \omega_3^2$
  - c.  $A_1 \omega_1 = A_2 \omega_2 = A_3 \omega_3$
  - d.  $A_1^2 \omega_1^2 = A_2^2 \omega_2^2 = A_3^2 \omega_3^2$
6. Give the name of three important characteristics of a SHM. **1**

7. What is the frequency of total energy of a particle in S.H.M.? **1**
8. Time period of a particle in S.H.M. depends on the force constant  $K$  and mass  $m$  of the particle  $\left(T = \frac{1}{2\pi} \sqrt{\frac{m}{k}}\right)$ . A simple pendulum for small angular displacement executes S.H.M approximately. Why then is the time period of a pendulum independent of the mass of the pendulum? **1**
9. A particle is executing S.H.M of amplitude 4cm and  $T = 4$ sec. find the time taken by it to move from positive extreme position to half of its amplitude? **2**
10. Show that for a particle executing S.H.M, velocity and displacement have a phase difference  $\frac{\pi}{2}$ . **2**
11. A simple harmonic motion is represented by  $x = 12 \sin (10t + 0.6)$  Find out the amplitude, angular frequency, frequency, time period and initial phase if displacement is measured in metre and time in seconds. **2**
12. Show that for a particle in linear SHM, the average kinetic energy over a period of oscillation equal to the average potential energy over the same period. **3**
13. A pipe 30.0 cm long is opened at both ends. Which harmonic mode of the pipe resonates with a 1.1 kHz source? Will resonance with the same source be observed, if one end of the pipe is closed? Take the speed of sound in air as  $330 \text{ ms}^{-1}$ . **3**
14. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body? **3**
15. An air chamber of volume  $V$  has a neck area of cross section  $a$  into which a ball of mass  $m$  just fits and can move up and down without any friction (Figure). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal. **5**



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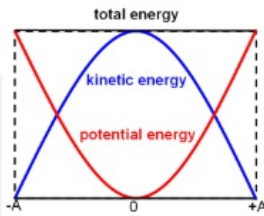
**Answer**

1. c. Is independent of time

**Explanation:** The total mechanical energy is the sum of kinetic energy(KE) and potential energy(PE), is constant.

$$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{Constant}$$

Here the figure depicts the energy of a linear harmonic oscillator vs displacement, horizontal axis represents the displacement x.



| From the graph it is clear that total energy is constant with displacement and the displacement is a function of time. So, total energy ( mechanical energy) is independent of time.

2. a. 16U

**Explanation:** Potential energy of a spring is given by

$$U = \frac{1}{2}kx^2$$

$$\text{so } U \propto x^2$$

thus for  $x = 4x$

U become 16U

3. b. Loses 37.8 seconds per day

**Explanation:** Initial time period  $T = 1 \text{ s}$

$$\text{new length } L' = L - \frac{L}{1000}$$

$$L' = \frac{999}{1000} L$$

$$\text{new time period } T' = 2\pi\sqrt{\frac{L'}{g}}$$

$$T' = 2\pi\sqrt{\frac{999L}{1000g}}$$

$$T' = 0.9995T$$

In one day there are 75600 seconds

After reducing length new time will be  $0.9995 \times 75600 = 75562.2$

Thus change is  $\Delta T = 75600 - 75562.2 = 37.8\text{s}$

Thus clock losses 37.8 s per day

4. b. Time period

**Explanation:** The time period of the SHM is given by  $T = 2\pi\sqrt{\frac{m}{k}}$  where 'm' be the mass of the body (constant), 'k' restoring force constant as T depends on 'm' and 'k' and they are constant for the system, so the corresponding Time period of the motion is Constant.

5. b.  $A_1\omega_1^2 = A_2\omega_2^2 = A_3\omega_3^2$

**Explanation:** Max acceleration is given by  $a = \omega^2 x$

At highest displacement  $x = A$

for same mass and velocity

$$A_1\omega_1^2 = A_2\omega_2^2 = A_3\omega_3^2$$

6. Three important characteristics of an SHM are as follows.

- i. A restoring force must act on the body.
- ii. Body must have an acceleration just opposite to the direction of displacement.
- iii. The motion should be an oscillatory one.

7. The frequency of total energy of a particle in S.H.M is zero because it retains constant.

8. Restoring force in case of simple pendulum is given by

$$F = \frac{mg}{l} y \Rightarrow K = mg/l$$

So force constant itself proportional to m as the value of k is substituted in the formula, m is cancelled out.

9. If Y = displacement

t = time

a = amplitude

$\omega$  = Angular frequency

Now,  $Y = a \cos \omega t$

Given  $Y = \frac{a}{2}$

So,  $\frac{a}{2} = a \cos \omega t$

$\frac{1}{2} = \cos \omega t$

Now, T = Time Period,  $\omega = \frac{2\pi}{T}$

$\frac{1}{2} = \cos \frac{2\pi}{T} \times t$  ( $T = 4 \text{ sec}$ )

$\frac{1}{2} = \cos \frac{2\pi}{4} \times t$

Let  $W_A$  is work done by spring A &  $k_A =$  Spring Constant

$W_B$  is work done by spring B &  $k_B =$  Spring Constant

$\therefore \frac{W_A}{W_B} = \frac{k_A}{k_B} = \frac{1}{3}$

$W_A : W_B = 1 : 3$

10. Consider a SHM  $x = A \sin \omega t \dots (i)$

$v = \frac{dx}{dt} = A\omega \cos \omega t = A\omega \sin(90^\circ + \omega t)$  ( $\because \sin(90^\circ + \theta) = \cos \theta$ )

$\therefore v = A\omega \sin(\omega t + \frac{\pi}{2})$

Phase of displacement from (i) is  $(\omega t)$

Phase of velocity from (ii) is  $(\omega t + \frac{\pi}{2})$

Hence, the phase difference  $= \omega t + \frac{\pi}{2} - \omega t = \frac{\pi}{2}$ .

11. Given equation of displacement,  $x = 12 \sin(10t + 0.6)$

On comparing the above equation with the general equation of displacement,  $x(t) = A \sin(\omega t + \phi)$

We have,

- i. Amplitude,  $A = 12 \text{ m}$
- ii. Angular frequency,  $\omega = 10 \text{ rad / s}$
- iii. Frequency,  $\nu = \frac{\omega}{2\pi} = \frac{10}{2\pi} = 1.59 \text{ Hz}$
- iv. Time period,  $T = \frac{2\pi}{\omega} = \frac{1}{1.59} = 0.628 \text{ s}$
- v. Initial phase,  $\omega t + \phi|_{t=0} = 10t + 0.6|_{t=0} = 0.6 \text{ rad}$

12. Suppose a particle of mass  $m$  executes SHM of time period  $T$ . The displacement of the particle at any instant  $t$  is given by the below general equation,  $y = A \sin \omega t$

Velocity of the particle,  $v = \frac{dy}{dt} = \omega A \cos \omega t$

Kinetic energy,  $E_K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \cos^2 \omega t$

and Potential energy,  $E_p = \frac{1}{2} m \omega^2 y^2 = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t$

Average KE of the particle executing SHM over a complete period of oscillation,

$$\begin{aligned}
 E_{k_{av}} &= \frac{1}{T} \int_0^T E_k dt = \frac{1}{T} \int_0^T \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t dt \\
 &= \frac{1}{2T} m\omega^2 A^2 \int_0^T \frac{(1+\cos 2\omega t)}{2} dt \\
 &= \frac{1}{4T} m\omega^2 A^2 \left[ t + \frac{\sin 2\omega t}{2\omega} \right]_0^T \\
 &= \frac{1}{4T} m\omega^2 A^2 (T) = \frac{1}{4} m\omega^2 A^2 \dots\dots\dots(i)
 \end{aligned}$$

Now average PE of the particle executing SHM over a complete period of oscillation,

$$\begin{aligned}
 E_{p_{av}} &= \frac{1}{T} \int_0^T E_p dt = \frac{1}{T} \int_0^T \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t dt \\
 &= \frac{1}{2T} m\omega^2 A^2 \int_0^T \frac{(1-\cos 2\omega t)}{2} dt \\
 &= \frac{1}{4T} m\omega^2 A^2 \left[ t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \\
 &= \frac{1}{4T} m\omega^2 A^2 (T) = \frac{1}{4} m\omega^2 A^2 \dots\dots\dots(ii)
 \end{aligned}$$

So it is clearly seen from Eqs.(i) and (ii),  $E_{k_{av}} = E_{p_{av}}$

13. Here,  $L = 30.0 \text{ cm} = 0.3 \text{ m}$

Let n-th harmonic of the open pipe resonates with 1.1 kHz source,

i.e.  $\nu_n = 1.1 \text{ kHz} = 1100 \text{ Hz}$

Again we know the formula of frequency of an open pipe for n-th harmonic,

$$\nu_n = \frac{n v}{2L}, \text{ v being speed of sound in air.}$$

$$\therefore n = \frac{2L\nu_n}{v} = \frac{2 \times 0.30 \times 1100}{330} = 2$$

i.e. 2nd harmonic resonates with open pipe.

If one end of the pipe is closed, its fundamental frequency becomes [putting  $n = 0$  in  $\nu$

$$= (2n + 1) \frac{v}{4L} ]$$

$$\nu_1 = \frac{v}{4L} = \frac{330}{4 \times 0.3} = 275 \text{ Hz}$$

As odd harmonics alone are produced in a closed organ pipe, therefore, possible frequencies are  $3\nu_1 = 3 \times 275 = 825 \text{ Hz}$ ,  $5\nu_1 = 5 \times 275 = 1375 \text{ Hz}$  and so on. As the source frequency is given 1100 Hz, therefore, no resonance can occur when the pipe is closed at one end converting the open pipe as a closed one.

14. Given

Maximum mass that the scale can read,  $M = 50 \text{ kg}$

Maximum displacement of the spring = Length of the scale,  $l = 20 \text{ cm} = 0.2 \text{ m}$

Time period,  $T = 0.6 \text{ s}$

Maximum force exerted on the spring,  $F = Mg$

Where,  $g = \text{acceleration due to gravity} = 9.8 \text{ m/s}^2$

$$\rightarrow F = 50 \times 9.8 = 490 \text{ N}$$

$$\therefore \text{Spring constant, } k = \frac{F}{l} = \frac{490}{0.2} = 2450 \text{ Nm}^{-1}$$

Since, Mass  $m$ , is suspended from the balance.

$$\rightarrow \text{Time period, } T = 2\pi \sqrt{\frac{m}{k}}$$

$$\therefore \left(\frac{T}{2\pi}\right)^2 \times k = \left(\frac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36 \text{ kg}$$

$$\therefore \text{Weight of the body} = mg = 22.36 \times 9.8 = 219.167 \text{ N}$$

Hence, the weight of the body is about 219 N.

15. Given

$$\Rightarrow \text{Volume of the air chamber} = V$$

$$\Rightarrow \text{Area of cross-section of the neck} = a$$

$$\Rightarrow \text{Mass of the ball} = m$$

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by  $x$  units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber,  $\Delta V = ax$

$$\Rightarrow \text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{ax}{V}$$

$$\Rightarrow \text{Bulk Modulus of air, } B = \frac{\text{Stress}}{\text{Strain}} = \frac{-p}{\frac{ax}{V}}$$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$\Rightarrow p = \frac{-Bax}{V}$$

$$\Rightarrow \text{The restoring force acting on the ball, } F = p \times a$$

$$= \frac{-Ba^2x}{V}$$

In simple harmonic motion, the equation for restoring force is:

$$\Rightarrow F = -kx \dots \text{(ii)}$$

Where  $k$  is the spring constant

Comparing equations (i) and (ii), we get:

$$= \frac{Ba^2}{V}$$

$$\Rightarrow \text{Time period, } T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \sqrt{\frac{Vm}{Ba^2}}$$