

**CBSE Test Paper 01**  
**Chapter 13 Kinetic Theory**

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1. In a 30.0-s interval, 500 hailstones strike a glass window with an area of  $0.600 \text{ m}^2$  at an angle of  $45.0^\circ$  to the window surface. Each hailstone has a mass of 5.00 g and a speed of 8.00 m/s. If the collisions are elastic, what are the average force and pressure on the window? **1**
  - a. 0.913 N
  - b. 0.943 N
  - c. 0.978 N
  - d. 1.043 N
  
2. Nine particles have speeds of 5.00, 8.00, 12.0, 12.0, 12.0, 14.0, 14.0, 17.0, and 20.0 m/s. What is the rms speed? **1**
  - a. 9.35 m/s
  - b. 13.3 m/s
  - c. 12.3 m/s
  - d. 14.7 m/s
  
3. The molar specific heat at constant volume,  $C_v$  for solids is **1**
  - a. R
  - b. 5R
  - c. 3R
  - d. 2R
  
4. Estimate the average thermal energy of a helium atom the temperature of 10 million Kelvin (the typical core temperature in the case of a star). **1**
  - a.  $2.1 \times 10^{-16} \text{ J}$
  - b.  $2.0 \times 10^{-16} \text{ J}$
  - c.  $1.9 \times 10^{-16} \text{ J}$
  - d.  $2.3 \times 10^{-16} \text{ J}$

5. If there are  $n$  number of molecules per unit volume and  $m$  is the mass of each,  $v_x$  is the x-component of velocity, pressure can be written as **1**
- $P = nm\overline{v_x^2}$
  - $P = n\overline{v_x^2}$
  - $P = 2nm\overline{v_x^2}$
  - $P = m\overline{v_x^2}$
6. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is  $197\text{g mole}^{-1}$ . **1**
7. What happens when an electric fan is switched on in a closed room? **1**
8. State Boyle's law in terms of pressure and density. **1**
9. At room temperature, a diatomic gas molecule has five degrees of freedom and at high temperature it has seven degrees of freedom, explain. **2**
10. A gas is contained in a closed vessel. How pressure due to the gas will be affected if force of attraction between the molecules disappear suddenly? **2**
11. Briefly explain Boyle's law on the basis of Kinetic theory of gases. **2**
12. Show that average kinetic energy of translation per molecule of a gas is directly proportional to the absolute temperature of gas. **3**
13. Show that constant - temperature bulk modulus  $K$  of an ideal gas is the pressure  $P$  of the gas. **3**
14. We have 0.5 g of hydrogen gas in a cubic chamber of size 3cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100 atm. Is one justified in assuming the ideal gas law, in the final state? (Hydrogen molecules can be consider as sphere of radius  $1 \text{ \AA}$ ) **3**
15. A box of  $1.00\text{m}^3$  is filled with nitrogen at 1.5 atm at 300K. The box has a hole of an area  $0.010 \text{ mm}^2$ . How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm. **5**

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**Answer**

1. b. 0.943 N

**Explanation:**  $F = 500 \times \frac{\Delta p}{\Delta t}$   
 $= 500 \times \frac{2mv \cos 45^\circ}{\Delta t}$   
 $= \frac{500 \times 2 \times 5 \times 10^{-3} \times 8 \times 1}{30 \times 1.41}$   
 $F = 0.943 \text{ N}$

2. b. 13.3 m/s

**Explanation:**  $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$   
 $v_{rms} = \sqrt{\frac{25 + 64 + 144 + 144 + 144 + 196 + 196 + 289 + 400}{9}} = \sqrt{\frac{1602}{9}}$   
 $v_{rms} = 13.3 \text{ m/sec}$

3. c. 3R

**Explanation:**  $C_V = \frac{1}{2} Rf$

an atom in a solid though has no degree of freedom for translational and rotational motion. It has 3 x 2 = 6 degree of freedom due to vibration along 3 axis.

$$C_V = \frac{6}{2} R = 3R$$

4. a.  $2.1 \times 10^{-16} \text{ J}$

**Explanation:**  $E = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10 \times 10^6 = 2.07 \times 10^{-16} \text{ J}$   
k = boltzman constant

5. a.

$$P = nm\overline{v_x^2}$$

**Explanation:** Pressure is due to collision of molecule with the wall of container. Collision by molecule to the wall of container assumed to be perfectly elastic. By the principle of conservation of momentum, the momentum imparted to the wall in the collision =  $2mv_x$

the number of molecules hitting the wall in time  $\Delta t = nAv_x\Delta t$

$n$  = no. of molecule per unit volume

$A$  = Area of container

The total momentum transferred to the wall by these molecules in  $\Delta t$  time =

$$2mv_x(nAv_x\Delta t)$$

Pressure  $P = F/A =$  rate of momentum transfer per unit area

$$P = \frac{2mv_x(nAv_x\Delta t)/\Delta t}{A} = nmv_x^2$$

all molecules in a gas do not have the same velocity; there is a distribution in velocities. The above equation therefore, stands for pressure due to the group of molecules with speed  $v_x$  in the x-direction and  $n$  stands for the number density of that group of molecules. The total pressure due to all groups

$$P = nm\overline{v_x^2}$$

6. 197 gm gold has number of atoms =  $6.023 \times 10^{23}$

$$1 \text{ gm gold will have number of atoms} = \frac{6.023 \times 10^{23}}{197}$$

$$39.4 \text{ gm gold has number of Au atoms} = \frac{39.4 \times 6.023 \times 10^{23}}{197}$$
$$= 1.2 \times 10^{23} \text{ atoms}$$

7. When electric fan is switched on, first electrical energy is converted into mechanical energy and then mechanical energy is converted into heat. The heat energy will increase the Kinetic energy of air molecules; hence temperature of room will increase.

8. Temperature remaining unchanged, density of a given gas is directly proportional to the pressure of the gas.

Mathematically, if  $T =$  constant, then  $\rho \propto P$ .

9. At low temperature, a diatomic gas has three translational and two rotational degrees of freedom, making total number of degrees of freedom equal to 5.

But at high temperature, as the gas molecule starts to vibrate which give two additional degrees of freedom. In that case total number of degrees of freedom becomes  $5+2 = 7$ .

10. As force of attraction between molecules disappears, then molecules will hit the wall with more speeds. As a result, rate of change of momentum will also increase (since mass remains same). Again from Newton's 2nd law of motion we know that

$F = \frac{\Delta p}{\Delta t}$ , where F is average force on the wall due to molecules,

$\Delta p$  is change in momentum and  $\Delta t$  is the time duration. Due to increase in  $\Delta p$ , force F will also increase. As a result, pressure,  $p = \frac{F}{A}$  will increase. Here, A is area of one wall, which remains constant throughout the whole process.

11. According to the kinetic theory of gases, pressure exerted by a gas is given by

$$P = \frac{1}{3V} mNv^2$$

where v = rms speed of gas molecules, V is volume of the gas, and mN is total mass of the gas

$$\therefore PV = \frac{1}{3} mNv^2$$

Multiplying and dividing by 2 on RHS, we get

$$PV = \left(\frac{2N}{3}\right) \left(\frac{1}{2}mv^2\right) = \left(\frac{2N}{3}\right) K \cdot E_{av}$$

The kinetic energy of a gas, as per kinetic theory, is directly proportional to its absolute temperature.

At a constant temperature, K.E. is constant. Thus,

$$PV = \text{constant}$$

This explains Boyle's law.

12. According to the kinetic theory of gases, the pressure p exerted by one mole of an ideal gas is

$$P = \frac{1}{3} \times \frac{M}{V} \times C^2 \quad M = \text{Molar mass of the gas}$$

$$\text{or } PV = \frac{1}{3} MC^2 \quad V = \text{Volume of the gas}$$

Since  $PV = RT$  (for 1mole of an ideal gas at absolute temperature T)

$$\text{or } \frac{1}{3} MC^2 = RT \quad (R = \text{Universal gas constant})$$

$$\therefore C^2 = \frac{3RT}{M} \quad (T = \text{Absolute temperature})$$

So,  $C \propto \sqrt{T}$ , [since R and M are constants for the gas]

$$\text{Also, } \frac{1}{3} MC^2 = RT \dots (i)$$

Dividing equation (i) by number of molecules of the gas = N

$$\frac{1M}{3N} C^2 = \frac{R}{N} T \quad (K = \text{Boltzmann's constant})$$

$$\Rightarrow \frac{1}{3} mC^2 = KT \quad [m = M/N = \text{mass of one single molecule of the gas}]$$

$$\text{or } \frac{1}{2} mC^2 = \frac{3}{2} KT$$

Since,  $\frac{1}{2} mC^2 =$  average kinetic energy of translation per molecule of the gas

$$\text{So, } \frac{1}{2} mC^2 \propto T \dots (ii)$$

$$\text{as } \frac{3}{2} K = \text{constant}$$

Hence it is proved from equation (ii) that the average kinetic energy of translation per molecule of an ideal gas is directly proportional to the absolute temperature of the gas.

13. Isothermal bulk modulus is defined as volume times negative partial derivative of pressure with respect to volume at constant temperature:

$$K = -V \left( \frac{\partial P}{\partial V} \right)_T$$

An ideal gas satisfies the following equation of state:

$$PV = nRT$$

So the pressure for an ideal gas is given by:

$$P = \frac{nRT}{V}$$

The partial derivative of pressure w.r.t. volume at constant temperature is:

$$\left( \frac{\partial P}{\partial V} \right)_T = \left( \frac{\partial (nRT/V)}{\partial P} \right)_T$$

$$= nRT d(1/V) / dV$$

$$= -nRT / V^2$$

$$= \frac{-nRT/V}{V}$$

$$= -P/V$$

Hence,  $K = -V (-P/V) = P$

14. Volume of 1 molecule of hydrogen

$$= \frac{4}{3} \pi r^3 = \frac{4}{3} \times 3.14 \times (10^{-10})^3 \text{ m}^3$$

$$r = 1 \text{ \AA} = 10^{-10} \text{ m (Given)}$$

$$\therefore \text{Volume of 1 molecule of hydrogen} = 4 \times 1.05 \times 10^{-30} \text{ m}^3 = 4.20 \times 10^{-30} \text{ m}^3$$

$$\text{Number of moles in 0.5 g of } H_2 \text{ gas} = \frac{0.5}{2} = 0.25 \text{ mole } [ \because H_2 \text{ has 2 mole } ]$$

$$\therefore \text{Volume of } H_2 \text{ molecules of .25 mole}$$

$$= 0.25 \times 6.023 \times 10^{23} \times 4.2 \times 10^{-30} \text{ m}^3 \text{ (since 1 mole of } H_2 = 6.023 \times 10^{23} \text{ number of molecules)}$$

$$= 1.05 \times 6.023 \times 10^{+23-30}$$

$$= 6.324 \times 10^{-7} \text{ m}^3$$

$$\text{Hence net volume 0.5 gm of } H_2 \text{ molecules, } V \approx 6.3 \times 10^{-7} \text{ m}^3$$

Now for ideal gas at constant temperature, according to Boyle's law,

$$P_i V_i = P_f V_f \text{ (} P_i, V_i, P_f \text{ and } V_f \text{ are initial pressure, initial volume, final pressure and}$$

final volume respectively;  $P_i = 1 \text{ atm}$ ,  $P_f = 100 \text{ atm}$ ,  $V_i = \text{volume of the cube}$ )

$$\therefore V_f = \frac{P_i V_i}{P_f} = \frac{1}{100} \times (3 \times 10^{-2})^3$$

[ $\because \text{vol. of cube } V_i = (\text{side})^3 \text{ and } P_i = 1 \text{ atm at NTP}$ ]

$$\Rightarrow V_f = \frac{27 \times 10^{-6}}{100} = 2.7 \times 10^{-5-2} = 2.7 \times 10^{-7} \text{ m}^3$$

Hence on compression, the final volume of the gas,  $V_f$  becomes of the same order of molecular volume,  $V'$ . As the molecules lie very close to each other, so intermolecular forces can not be ignored, i.e. the gas can not be treated as an ideal gas. Therefore one is not justified assuming the ideal gas law.

15. Initial volume of the given box =  $1 \text{ m}^3 = V_1$

Initial pressure =  $1.5 \text{ atm} = P_1$

Final pressure = Initial pressure - pressure reduced by what amount =  $1.5 - 0.1 = 1.4 \text{ atm} = P_2$

Air pressure outside the box =  $P_2 = 1 \text{ atm}$

Initial temperature  $T_1 = 300 \text{ K}$

Final temperature  $T_2 = 300 \text{ K}$

A = area of hole =  $0.01 \text{ mm}^2 = 0.01 \times 10^{-6} \text{ m}^2 = 10^{-8} \text{ m}^2$

initial pressure difference between the box and the outside atmosphere,

$$\Delta P = (1.5 - 1) \text{ atm} = 0.5 \text{ atm}$$

mass of a  $N_2$  gas molecule =  $\frac{0.028 \text{ Kg}}{6.023 \times 10^{23}} = 46.5 \times 10^{-27} \text{ Kg}$  (= molar mass /

Avogadro's number)

Boltzmann's constant,  $K_B = 1.38 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1}$

Let  $\rho_{n1}$  is the initial number of  $N_2$  gas molecule per unit volume per unit time. Now, in time  $\Delta t$  let  $v_{tx}$  is the speed of the molecules along x axis.

Number of molecules colliding in time  $\Delta t$  on a wall of the cube

$$= \frac{1}{2} \rho_{n1} [(v_{tx}) \Delta t] A, \text{ A being area}$$

$\frac{1}{2}$  is multiplied as other  $\frac{1}{2}$  molecule will strike to the opposite wall of the cube.

$$v_{rms}^2 (N_2 \text{ molecule}) = v_{tx}^2 + v_{ty}^2 + v_{tz}^2$$

$$\therefore |v_{tx}| = |v_{ty}| = |v_{tz}|$$

$$\text{Then } v_{rms}^2 = 3v_{tx}^2$$

$$\text{KE of the nitrogen gas molecule} = \frac{3}{2} K_B T$$

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} K_B T$$

$$\Rightarrow m \times 3v_{tx}^2 = 3K_B T$$

$$\Rightarrow v_{tx} = \sqrt{\frac{K_B T}{m}} \dots (A)$$

Number of  $N_2$  gas molecules striking to a wall in  $\Delta t$  time =  $\frac{1}{2} \rho_{n1} \sqrt{\frac{K_B T}{m}} \Delta t \times A$ , outward [putting the value of  $v_{tx}$  from equation (A).]

Temperature inside the box and air are equal to T

The number of air molecule striking to hole in time  $\Delta t$

$$\text{inward} = \frac{1}{2} \rho_{n2} \sqrt{\frac{K_B T}{m}} \Delta t \times A$$

$\rho_{n2}$  = number of air molecules per unit volume striking the wall in unit time

Net number of molecules (going outward)

$$\frac{1}{2} \rho_{n1} \sqrt{\frac{K_B T}{m}} \Delta t A - \frac{1}{2} \rho_{n2} \sqrt{\frac{K_B T}{m}} \Delta t A.$$

Net number of molecules going out from hole in  $\Delta t$  time

$$= \frac{1}{2} [\rho_{n1} - \rho_{n2}] \sqrt{\frac{K_B T}{m}} \cdot \Delta t \times A \dots (i)$$

Ideal gas equation for  $\mu$  moles of the gas,  $P_1 V = \mu R T \Rightarrow \mu = \frac{P_1 V}{R T}$

As for box  $\frac{\mu}{V} = \frac{P_1}{R T}$  ( $\mu$  = No. of moles of gas in box)

$$\rho_{n1} = \frac{N(\text{Total number of molecules in box})}{\text{volume of box}} = \frac{\mu N_A}{V}, N_A \text{ being Avogadro's number.}$$

$$= \frac{P_1 N_A}{R T} \dots (ii) \text{ (putting the value of } \frac{\mu}{V} \text{) per unit volume}$$

Let after time T, pressure reduced by 0.1 atm and becomes

$$P'_2 = (1.5 - .1) = 1.4 \text{ atm}$$

Then final new density of  $N_A$  molecule  $\rho'_{n1}$  (say)

$$\text{Therefore, } \rho'_{n1} = \frac{P_2 N_A}{R T} \text{ per unit volume (iii)}$$

Net number of molecules going out from volume V

$$= (\rho_{n1} - \rho'_{n1}) V = \frac{P_1 N_A}{R T} V - \frac{P'_2 N_A}{R T} V$$

$$= \frac{N_A V}{R T} [P_1 - P'_2] \dots (iv) \text{ (using equations ii, iii)}$$

$P'_2$  = final pressure of box.

From equation (i), total number of molecules going out in time  $\tau$  from hole

$$= \frac{1}{2} [\rho_{n1} - \rho_{n2}] \sqrt{\frac{K_B T}{m}} \times \tau \times A \dots (v)$$

$$\text{Again, } \rho_{n1} - \rho_{n2} = \frac{P_1 N_A}{R T} - \frac{P_2 N_A}{R T}$$

$$\therefore \rho_{n1} - \rho_{n2} = \frac{N_A}{R T} [P_1 - P_2] \dots (vi) \text{ (} P_2 \text{ = Pressure of air outside the box)}$$

Net number of molecules going out in time  $\tau$  from above (using equations v and vi)



$$= \frac{1}{2} \frac{N_A}{RT} [P_1 - P_2] \sqrt{\frac{K_B T}{m}} \cdot \tau \cdot A \dots\dots(\text{vii})$$

From equations (iv) and (vii),

$$\frac{N_A V}{RT} (P_1 - P_2) = \frac{1}{2} \frac{N_A}{RT} (P_1 - P_2) \sqrt{\frac{K_B T}{m}} \cdot \tau \cdot A$$

$$\therefore \tau = \frac{N_A V}{RT} (P_1 - P_2) \times \frac{2RT}{N_A} \frac{1}{(P_1 - P_2)} \sqrt{\frac{m}{K_B T}} \cdot \frac{1}{A}$$

$$\therefore \tau = \frac{2(P_1 - P_2)}{(P_1 - P_2)} \cdot \frac{V}{A} \sqrt{\frac{m}{K_B T}}$$

$$= \frac{2[1.5 - 1.4]}{(1.5 - 1)} \frac{1}{10^{-8}} \sqrt{\frac{46.5 \times 10^{-27}}{1.38 \times 10^{-23} \times 300}} \quad (\text{As } P_2 = \text{atmospheric pressure} = 1 \text{ atm})$$

$$= \frac{2 \times 0.1}{0.5 \times 10^{-8}} \sqrt{\frac{4650 \times 10^{-27+23-2}}{138 \times 3}}$$

$$= 0.4 \times 10^{-8} \sqrt{\frac{775 \times 10^{-5}}{69}}$$

$$= 0.4 \times 10^8 \times 10^{-3} \times \sqrt{11.23} = 0.4 \times 10^5 \times 3.35$$

$$\therefore \tau = 1.34 \times 10^5 \text{ seconds.}$$

This is the required time in the question.

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