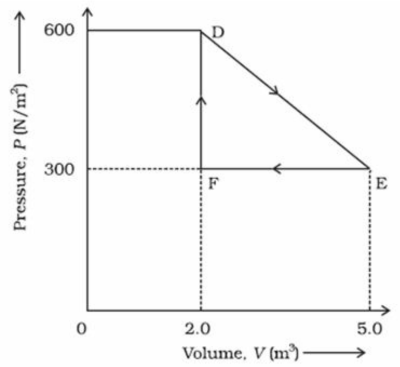
CBSE Test Paper 02

Chapter 12 Thermodynamics

1. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in Figure. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F **1**



   a. 500 J
   b. 480 J
   c. 450 J
   d. 470 J

2. A 1.0-kg bar of copper is heated at atmospheric pressure. If its temperature increases from 20° C to 50° C, what is the work done by the copper on the surrounding atmosphere? Given that for Cu 3α = 5.1 x 10^{-5} / C, $\rho_{Cu} = 8.92 \times 10^3 Kg/m^3$ **1**

   a. 1.7×10^{-2} J
   b. 1.8×10^{-2} J
   c. 1.6×10^{-2} J
   d. 1.9×10^{-2} J

3. In Thermodynamics Extensive variables depend **1**

   a. on the 'temperature' of the system
   b. on the 'size' of the system
   c. on the 'entropy' of the system

d. on the ‘pressure’ of the system

4. An ideal gas is enclosed in a cylinder with a movable piston on top. The piston has a mass of 8 000 g and an area of 5.00 cm^2 and is free to slide up and down, keeping the pressure of the gas constant. How much work is done as the temperature of 0.200 mol of the gas is raised from 20.0^{0}C to 300^{0}C? **1**

   a. 456 J
   b. 486 J
   c. 476 J
   d. 466 J

5. A steam engine delivers 5.4×10^8 J of work per minute and services $3.6 \times 10^9 J$ of heat per minute from its boiler. What is the efficiency of the engine? How much heat is wasted per minute? **1**

   a. 14 percent, 3.0×10^9 J
   b. 12 percent, 2.8×10^9 J
   c. 13 percent, 2.9×10^9 J
   d. 15 percent, 3.06×10^9 J

6. What is the efficiency of Carnot engine Working between ice point and steam point? **1**

7. If the temperature of the sink is increased, what will happen to the efficiency of Carnot engine? **1**

8. A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature lose from 27°C to 97°C. Calculate the work done and heat produced in the gas. Take γ as 1.5. **1**

9. Calculate the fall in temperature when a gas initially at 72°C is expanded suddenly to eight times its original volume. (γ = 5.3) **2**

10. State Kelvin-Planck statement of Second Law of Thermodynamics. What is the significance of this law? **2**

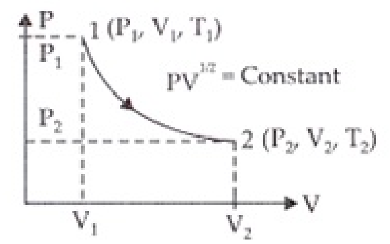
11. State Carnot's theorem. What is its significance? **2**

12. At 0^0C and normal atmospheric pressure, the volume of 1g of water increases from $1cm^3$ to 1.091 cm^3 on freezing. What will be the change in its internal energy? Normal atmospheric pressure is 1.013×10^5 N|m^2 and the latent heat of melting of ice is 80 cal/g. **3**

13. A perfect Carnot engine utilizes an ideal gas the source temperature is 500 K and sink temperature is 375 K. If the engine takes 600 kcal per cycle from the Source, Calculate **3**
    a. The efficiency of engine
    b. Work done per cycle
    c. Heat rejected to sink per cycle

14. Two Carnot engines A and B are operated in series. The first one A receives heat at 900 K and rejects it to a reservoir at temperature T. The second engine B operates on this reservoir and rejects heat to a reservoir at 400 K. Calculate temperature T when **3**
    i. efficiencies of both A and B are equal.
    ii. the work outputs of both A and B are equal.

15. Consider a P-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in Figure. **5**



    i. Find the work done when the gas is taken from state 1 to state 2.
    ii. What is the ratio of temperature $T_1/T_2,$ if $V_2 = 2V_1$?
    iii. Given the internal energy for one mole of gas at temperature T is $(\frac{3}{2})RT$, find the heat supplied to the gas when it is taken from state (1) to (2) with $V_2 = 2V_1$.

Answer

1. c. 450 J
   Explanation: work done by the gas is equal to area under P-V curve

2. a. 1.7 × 10^{-2} J
   Explanation: $\Delta V = 3\alpha V_i \Delta T = 5.1 \times 10^{-5} \times 30 \times V_i = 1.5 \times 10^{-3} V_i$
   $\Delta V = 1.5 \times 10^{-3} \frac{m}{\rho_{Cu}} = 1.5 \times 10^{-3} \frac{1}{8.92 \times 10^3} = 1.7 \times 10^{-7} W = P\Delta V$
   $= 1.01 \times 10^5 \times 1.7 \times 10^{-7} = 1.7 \times 10^{-2} J$

3. b. on the ‘size’ of the system
   Explanation: The variables which depend on the size or amount of the substance are called extensive variables.
   Example: Mass, volume etc

4. b. 466 J
   Explanation: in isobaric process
   $W = nR(T_f - T_i)$
   $W = 0.2 \times 8.31(300 - 20) = 466J$

5. d. 15 percent, 3.06$\times 10^9$ J
   Explanation: $\eta = \frac{\text{WorkDone}}{\text{HeatExtracted}} = \frac{5.4 \times 10^8}{3.6 \times 10^9}$ = 0.15 = 15 %
   Heat wasted = Heat Extracted - Work Done
   Heat Wasted = $3.6 \times 10^9 - 5.4 \times 10^8 = 3.06 \times 10^9 J$

6. η = $1 - \frac{T_2}{T_1} \times 100$ = $\frac{(373-273)}{373} \times 100$= 26.8%

7. If the temperature of the sink is increased, then
   Efficiency, $\eta = 1 - \frac{T_2}{T_1}$
   By increasing (T_2), the efficiency of the Carnot engine will decrease.

8. It is given that, T_i = 27°C = 27 + 273 = 300 K

T_f = 97°C = 97 + 273 K = 370 K,

Also γ = 1.5 (given)

Threfore, Work done in adiabatic compression is given by:
W $= \frac{R}{1-\gamma}(T_i - T_f) = \frac{8.31}{1-1.5}(300 - 370)$ = 1163.4 Joules
Also Heat Produced during the process, H = $\frac{W}{J} = \frac{1163.4}{4.2}$ = 277 calories

9. $T_1V_1^{v-1} = T_2V_2^{v-1}$
$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{v-1} = 345\left(\frac{x}{8x}\right)^{2/3}$
$= 345 \times \frac{1}{4} = 86.25K$

10. According to the Kelvin-Planck statement, "it is impossible for any device that operates on a cycle to receive heat from a single reservoir and produces a net amount of work".
This law is significant because it gives a fundamental limitation to the efficiency of a heat engine as well as the coefficient of performance of a refrigerator. In simple terms, it says that the efficiency of a heat engine can never be unity. It implies that when heat Q_1 is taken out from a hot reservoir, some heat Q_2 must be released to the cold reservoir and Q_2 can never be zero. Also, it implies that for a refrigerator, the coefficient of performance can never be infinite.

11. Carnot's theorem states: Carnot's rule, is a principle that specifies limits on the maximum efficiency any heat engine can obtain. The efficiency of a Carnot engine depends solely on the temperatures of the hot and cold reservoirs. The formula for this maximum efficiency is $\eta_{max} = \eta_{Carnot} = 1 - \frac{T_C}{T_H}$
where T_C is the absolute temperature of the cold reservoir, T_H is the absolute temperature of the hot reservoir.

    i. All heat engines between two heat reservoirs are less efficient than a Carnot heat engine operating between the same reservoirs.
    ii. Every Carnot heat engine between a pair of heat reservoirs is equally efficient, regardless of the working substance employed or the operation details.

12. Mass of water = m = 1 g
L_f = Latent heat of melting of ice = 80 cal/g

$Q = -(mL_f) = -80$ cal [Negative sign is assigned to Q because it is given out by water]

During freezing, the water expands against atmospheric pressure. Hence, external work (W) is done by water :-

$$\mathrm{W} = \mathrm{P} \times \Delta \mathrm{V}$$

$P = 1.013 \times 10^5$ N/m^2;

V_1 = Initial volume = 1 cm^3

V_2 = final volume = 1.91 cm^3

$\Delta V = V_2 - V_1 = 1.091 - 1 = 0.091$ cm^3 = o.o91 × 10^{-6} m^3

So, W = $\left(1.013 \times 10^5\right) \times \left(0.091 \times 10^{-6}\right)$ ....(i)

W = 0.0092 J

Since, 1 cal = 4.2J so,

$W = \frac{0.0092}{4.2} = 0.0022\ cal$.........(ii)

The work has been done by ice, so it will be taken positive.

Acc. to first law of thermodynamics,

$$\Delta Q = \Delta U + W$$

So, $\Delta U = Q - W$

= (-80) - (-0.0022) (using (i) & (ii))

ΔU = - 80.0022 cal

Negative sign indicates that internal energy of water decreases on freezing.

13. Temperature of source T_1 = 500 K

    Temperature of sink T_2 = 375 K

    Heat input from the source Q_1 = 600 kcal

    a. $\eta = 1 - \frac{T_2}{T_1} = \frac{125}{500} = 0.25$

       ∴ Efficiency of carnot engine will be = $\eta \times 100$ = 25 %

    b. Let W be the work done per cycle

       Then, $\eta = \frac{W}{Q_1}$

       $W = \eta Q_1 = 0.25 \times 600\ k\ \text{cal} = 150\ \text{k cal}$

    c. Let Q_2 be the heat given to sink

For cyclic process $\Delta U = 0$

Hence, according to first law of thermodynamics

$\Delta Q = \Delta W$

W =Q_1 - Q_2

Q_2 = Q_1 - W = 600 - 150

Q_2 = 450 kcal

14. i. Efficiency of A = efficiency of B

$\eta_A = \eta_B$

$\Rightarrow 1 - \frac{T}{900} = 1 - \frac{400}{T}$

$\Rightarrow T^2 = 900 \times 400$

$T = 600K$

ii. Suppose the first engine take Q_1 heat as input at temperature, $T_1 = 800K$ and gives out heat Q_2 at temperature T_0. The second engine receive Q_2 as input and give is out heat Q_3 at temperature $T_3 = 300K$ to the sink.

Work done by first (A) engine = work done by second (B) engine.

Therefore Q_1 - Q_2 = Q_2 - Q_3

Dividing both sides by Q_1

$1 - \frac{Q_2}{Q_1} = \frac{Q_2}{Q_1} - \frac{Q_3}{Q_1}$

$\Rightarrow 1 - T/T_1 = \frac{Q_2}{Q_1}\left(1 - \frac{Q_3}{Q_2}\right)$

$\Rightarrow 1 - T/T_1 = \frac{T}{T_1}(1 - T_3/T)$

$\Rightarrow T_1/T - 1' = 1 - \frac{T_3}{T} \Rightarrow \frac{T_1}{T} + \frac{T_3}{T} = 2$

$\Rightarrow \frac{1}{T}(T_1 + T_3) = 2 \Rightarrow T = \frac{T_1+T_3}{2}$

$\Rightarrow T = \frac{900+400}{2} = 650\text{K}$

15. A thermodynamic cycle consists of a linked sequence of thermodynamic processes that involve transfer of heat and work into and out of the system, while varying pressure, temperature, and other state variables within the system, and that eventually returns the system to its initial state.

$\therefore PV^{1/2}$ = constant = K(given) or $P_1V_1^{1/2} = P_2V_2^{1/2} = K$ and $P = K/V^{1/2}$

a. Work done for process from 1 to 2

$$WD = \int_{V_1}^{V_2} P.dv = \int_{V_1}^{V_2} \frac{K}{V^{1/2}} dV = K \int_{V_1}^{V_2} V^{-(1/2)} dV$$

$$WD = K\left[\frac{V^{1/2}}{\frac{1}{2}}\right]_{V_1}^{V_2} = 2K\left[\sqrt{V_2} - \sqrt{V_1}\right]$$

WD from V_1 to $V_2, i.e., dW = 2P_1V_1^{1/2}\left[\sqrt{V_2} - \sqrt{V_1}\right]$

$= 2P_2V_2^{1/2}\left[\sqrt{V_2} - \sqrt{V_1}\right]$

b. from gas equation of ideal gas PV = nRT

$\Rightarrow T = \frac{PV}{nR} = \frac{P\sqrt{V}\sqrt{V}}{nR} = \frac{K\sqrt{V}}{nR}$

$T_1 = \frac{K\sqrt{V_1}}{nR}$ and $T_2 = \frac{K\sqrt{V_2}}{nR}$

$\frac{T_1}{T_2} = \frac{\frac{K\sqrt{V_1}}{nR}}{\frac{K\sqrt{V_2}}{nR}} = \frac{\sqrt{V_1}}{\sqrt{V_2}} = \sqrt{\frac{V_1}{2V_1}}\ (\therefore V_2 = 2V_1 given)$

$\therefore \quad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$ ...(ii)

required ratio is $1 : \sqrt{2}$.

c. Given that internal energy U of gas is

$U = \left(\frac{3}{2}\right)RT$

$\Delta U = \frac{3}{2}RdT = \frac{3}{2}R\left(T_2 - T_1\right)$

$\therefore T_2 = \sqrt{2}T_1$, from apart (b)

$\Delta U = \frac{3}{2}R\left[\sqrt{2}T_1 - T_1\right] = \frac{3}{2}RT_1(\sqrt{2} - 1)$

Form part (a) dW $= 2P_1V^{12}\left(\sqrt{V_2} - \sqrt{V_1}\right)$

$\because V_2 = 2V_1($ given $)$

So, $\sqrt{V_2} = \sqrt{2}\sqrt{V_1}$ then

$dW = 2P_1V_1^{1/2}\left(\sqrt{2}\sqrt{V_1} - \sqrt{V_1}\right)$

$= 2P_1V_1^{12}\sqrt{V_1}\left[\sqrt{2} - 1\right]$

$dW = 2P_1V_1(\sqrt{2} - 1)$

$dW = 2nRT_1(\sqrt{2} - 1)\ (\therefore P_1V_1 = nRT_1)$

$\therefore n = 1 \therefore dW = 2RT_1(\sqrt{2} - 1)$

$\therefore dQ = dW + dU = 2RT_1(\sqrt{2} - 1) + \frac{3}{2}RT_1(\sqrt{2} - 1)$

$= (\sqrt{2} - 1)RT_1\left[2 + \frac{3}{2}\right]$

$dQ = -(\sqrt{2}1)RT.$